THE GEOMETRIC COMPLEX OF A MORSE–BOTT–SMALE PAIR AND AN EXTENSION OF A THEOREM BY BISMUT–ZHANG

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Abstract

Let M be a closed Riemannian manifold and E a flat vector bundle over M equipped with a fiber metric. In this situation one has a Ray–Singer torsion T_{an}^M — a super determinant of the deRham differential. Suppose X is a Morse–Bott–Smale vector field on M with critical manifold Σ. Every connected component $S \subseteq \Sigma$ has a Ray–Singer torsion T_{an}^S on its own. The Morse–Bott complex is a filtered graded complex computing $H^*(M;E)$. Using the differentials in the associated (finite dimensional) spectral sequence one defines a combinatoric torsion T_{comb}. Our main result is the following localization formula for the analytic torsion:

$$\log T_{\text{an}}^M = \sum_{S\subseteq\Sigma} (-)^{\text{ind}(S)} \log T_{\text{an}}^S + \log T_{\text{comb}} + \log T_{\text{met}} + \int_M \theta \wedge (-X)^*\Psi$$

Here T_{met} is the volume of the integration isomorphism from deRham cohomology to Morse–Bott cohomology. In the last term Ψ is the Mathai–Quillen form and θ is a closed one form which measures to what extend the fiber metric on E is not parallel. If X is Morse–Smale then Σ is discrete and the formula reduces to a theorem of Bismut–Zhang. The proof we give is based on the Bismut–Zhang theorem.