INFORMATION ON THE COURSE ON NONLINEAR EVOLUTION EQUATIONS

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ABSTRACT. I am collecting here some information and on the course on *Nonlinear Evolution Equations*, SS14, Uni Wien. I will be indebted to anyone pointing me out typos etc.

0.1. **VERY IMPORTANT.** If someone still needs to sign up for the course please contact me before March 11 as I have to turn in the list.

0.2. **Aim.** The focus of the course is that of providing a hands-on introduction to some themes in evolution equations. We will be dealing with time-dependent PDE problems originating form different applications. Among the many possible choices, I will be concentrating on variational evolutions, i.e. evolution problem driven by functionals.

0.3. **Tentative program.** Here is a list of topics that I would like to present. The plan could of course be adjusted to the interests of the class.

- **Intro.** Motivation and examples.
- **Tools.** Metric and function spaces. Calculus of variations. Variational formulations. Bochner integration.
- **Gradient flows.** Classical convex theory. Curves of maximal slope in metric spaces.
- **Doubly nonlinear equations.** Discretization and approximation. Existence. Variational characterizations.
- **Rate-independent flows.** Hysteresis. Energetic and local solutions. Variational formulations and existence.

0.4. **Timing.** The class will be meeting weakly on Wednesdays 16.00-18.15 in the Seminar Room SR10 at the Uni in Oskar-Morgenstern-Platz 1.

First lecture 5.3, last lecture 25.3. There will be no class on 12.3. (Rector's day), 2.4, 16.4 and 23.4 (Easter break).

0.5. **Audience.** The course is offered to Master students in Analysis and Applied Math at the Uni and PhD students at Uni and TU (DK *Dispersive Dissipative*). Anybody interested is of course welcome.

0.6. **Prerequisites.** The reference frame for the course are PDEs and applied functional analysis. These themes will be probably familiar to many in the class and could serve as mild prerequisites. Among these one could list

- Banach spaces, dual, separability, reflexivity, uniform convexity,
- Hibert spaces, projections, Riesz, Lax-Milgram, bases
- L^p, $W^{k,p}$,
- Variational formulations of some elliptic problem, weak existence, regularity

0.7. **Materials.** Most of what I would like to present is fairly (sometimes very) classical. As such, it can be found in many texts. In the bibliography you find the text that I have used for the course.

I will make a personal selection of the topics. I shall try to keep track of what we do by regularly updating this file into a script.

0.8. **Language.** English.

0.9. **Evaluation.** The final grade will be the result of participation in class (1/3), home assignments (1/3), and the finals (1/3). The finals will be oral exams on two themes from the course (you choose one).

Alternatively one can choose to study one of the following papers:

list of papers:

- E. DiBenedetto, R. E. Showalter. Implicit degenerate evolution equations and applications. *SIAM J. Math. Anal.* 12 (1981), no. 5, 731751.
- P. Colli, A. Visintin. On a class of doubly nonlinear evolution equations. *Comm. Partial Differential Equations* 15 (1990), no. 5, 737756.
- A. Mielke, F. Theil. On rate-independent hysteresis models. *NoDEA Nonlinear Differential Equations Appl.* 11 (2004), no. 2, 151189.
- M. G. Crandall, T. M. Liggett. Generation of semi-groups of nonlinear transformations on general Banach spaces. *Amer. J. Math.* 93 (1971) 265298.
- R. Rossi, G. Savaré. Gradient flows of non convex functionals in Hilbert spaces and applications. *ESAIM Control Optim. Calc. Var.* 12 (2006), no. 3, 564614.
- A. Visintin. Variational formulation and structural stability of monotone equations. *Calc. Var. Partial Differential Equations* 47 (2013), no. 1-2, 273317.
- N. Ghoussoub, L.Tzou. A variational principle for gradient flows. *Math. Ann.* 330 (2004), no. 3, 519549.

- T. Roubíček. Direct method for parabolic problems. *Adv. Math. Sci. Appl.* 10 (2000), no. 1, 5765.
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0.10. **Contact.** By email: ulisse.stefanelli@univie.ac.at

1. Functional analytic toolbox

- Compactness in Banach spaces
- BV spaces
- Convex functions, lower semicontinuity, direct method, subdifferential, conjugate, Fenchel
- Vector-valued functions, Bochner integral

References

Ambrosio05 [1] L. Ambrosio, N. Gigli, G. Savaré. *Gradient flows in metric spaces and in the space of probability measures*. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2008. Second edition.

Barbu76 [2] V. Barbu. *Nonlinear semigroups and differential equations in Banach spaces*. Noordhoff International Publishing, Leyden, 1976.

Brezis73 [3] H. Brézis. *Operateurs maximaux monotones et semi-groupes de contractions dans les espaces de Hilbert*. Math Studies, Vol.5, North-Holland, Amsterdam/New York (1973).

Brezis [4] H. Brezis. *Analyse fonctionnelle. Théorie et applications*. Collection Mathématiques Appliquées pour la Maîtrise. Masson, Paris, 1983.

Evans [5] L. C. Evans. *Partial differential equations*. Second edition. Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 2010.

Mielke05 [6] A. Mielke. *Evolution in rate-independent systems (ch. 6)*. In C. Dafermos and E. Feireisl, editors, *Handbook of Differential Equations, Evolutionary Equations*, 2: 461-559. Elsevier B.V., 2005.

showalter97 [7] R. E. Showalter. *Monotone operators in Banach space and nonlinear partial differential equations*. Mathematical Surveys and Monographs, 49. American Mathematical Society, Providence, RI, 1997.

Yosida [8] K. Yosida. *Functional analysis*. Reprint of the sixth (1980) edition. Classics in Mathematics. Springer-Verlag, Berlin, 1995.