Exercise sheet 3

(due Wed. 23.4.14)

Exercise 7. [Weak convergence] Give an example of $u_n \in L^2(0,1)$ so that u_n is weakly convergent but not strongly convergent. For such a sequence, find $f, g : \mathbb{R} \to \mathbb{R}$ continuous so that $f \circ u_n$ is strongly convergent, $(g \circ u_n)^3$ is weakly but not strongly convergent.

Exercise 8. [Strong L^p convergence] Let $u_n \to u$ strongly in $L^1(0,1)$ and $\{u_n\}$ be bounded in $L^p(0,1)$ for some $p > 1$. Prove that $u_n \to u$ strongly in $L^q(0,1)$ for all $q < p$ and give a counterexample for $q = p$.

Exercise 9. [First-order operators] Let $H = L^2(0,1)$ and define $D(A) = \{u \in H^1(0,1) \mid u(0) = 0\}$ and $A : H \to H$ defined as $A(u) = u'$. Prove that A is maximal monotone and check that the equation $u' + f(u) = g$ has unique (strong) solution for all $f : \mathbb{R} \to \mathbb{R}$ monotone, Lipschitz continuous, and coercive (in $\mathbb{R}$) and any $g \in L^2(0,1)$.

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