Exercise sheet 4

(due Wed. 14.5.14)

Exercise 10. [Moreau-Yosida approximation] Let H be a Hilbert space and $\phi : H \to (-\infty, +\infty]$ be convex, proper, and lower semicontinuous.

(1) Let $\lambda > 0$ and $u \in H$. Prove that the functional

$$v \mapsto \frac{|u-v|^2}{2\lambda} + \phi(v)$$

has a minimum. Call this minimum $\phi_\lambda(u)$.

(2) Prove that $u \mapsto \phi_\lambda(u)$ is convex and $\phi_\lambda(u) \leq \phi(u)$.

(3) Show that ϕ_λ is differentiable (it is indeed $C^{1,1}$).

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