Exercise sheet 5

(due Wed. 4.6.14)

Exercise 11. [Again Moreau-Yosida]

(1) Let $\phi(u) = u^+ = \max\{0, u\}$ in $\mathbb{R}$ and $\lambda > 0$. Compute
$$\phi_\lambda(u), \quad \phi'_\lambda(u), \quad (\phi_\lambda)_\lambda(u), \quad (\phi_\lambda)'_\lambda(u), \quad \ldots$$
(2) Let $H = H^1_0(\Omega)$, $\lambda > 0$, and $\phi : H \to \mathbb{R}$ be the Dirichlet integral
$$\phi(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 \mathrm{d}x.$$
Compute ϕ_λ and $\mathrm{D}\phi_\lambda$.

Exercise 12. [Asymptotic behavior]

(1) Let H be Hilbert, $\phi : H \to [0, \infty]$ be convex with compact sublevels and $u^0 \in D(\phi)$. We have proved that, for all $T > 0$, the gradient flow
$$u' + \partial\phi(u) \ni 0 \text{ a.e. in } (0, T), \quad u(0) = u^0.$$
Check that indeed such solution can be uniquely extended for all times (namely there exists $u : [0, \infty) \to H$ solving the gradient flow on $(0, T)$ for all $T > 0$).
(2) Prove that there exists a sequence $t_n \to \infty$ such that $\lim_n u(t_n) = u_\infty$ for some $u_\infty \in D(\phi)$.
(3) Characterize u_∞ (What is u_∞ solving?)

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