Exercise sheet 6

(due Wed. 18.6.14)

Exercise 13. [Gradient nonlinearity] Let $\Omega \subset \mathbb{R}^n$ be nonempty, open, bounded, connected, and smoothly bounded and $f : \mathbb{R}^n \to \mathbb{R}$ be Lipschitz continuous. Prove that for all $u^0 \in H^1_0(\Omega)$ there exists a unique $u \in H^1(0,T;L^2(\Omega)) \cap L^\infty(0,T;H^1_0(\Omega))$ solving the problem

$$\begin{aligned} u_t - \Delta u + f(\nabla u) &= 0 \quad \text{in} \;\; \Omega \times (0,T), \\ u(\cdot,0) &= u^0 \quad \text{in} \;\; \Omega, \\ u &= 0 \quad \text{in} \;\; \partial\Omega \times (0,T), \end{aligned}$$

at least in the sense of distributions.

Exercise 14. [A reaction-diffusion system] Given Ω as above, discuss the reaction-diffusion system

$$\begin{aligned} u_t - 3\Delta u &= -(u - v)^3 - u, \\ v_t - \Delta v &= (u - v)^3 + 2u \end{aligned}$$

in $\Omega \times (0,T)$ along with initial and homogeneous Neumann boundary conditions.

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