News from low regularity GR

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Overview

Long-term project on

Lorentzian geometry and general relativity

with metrics of low regularity

jointly with

- ‘theoretical branch’ (Vienna & U.K.):
  Melanie Graf, James Grant, Günther Hörmann, Mike Kunzinger, Clemens Sämann, James Vickers
- ‘exact solutions branch’ (Vienna & Prague):
  Jiří Podolský, Clemens Sämann, Robert Švarc

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Low Regularity GR

What is is?

GR and Lorentzian geometry on spacetime manifolds $(M, \mathbf{g})$, where M is smoot **but g is non-smooth** (below $\mathcal{C}^2$)

Why is it needed?

1. Physics: Realistic matter models $\rightsquigarrow$ $\mathbf{g} \in \mathcal{C}^{1,1}$ (derivs. loc. Lip.)
2. Analysis: ivp $\mathbf{g} \in H^{5/2}(M), \mathcal{C}^1(\Sigma)$, recent big improvements

Where is the problem?

Physics and Analysis	vs.	**Lorentzian geometry**
want/need low regularity		needs high regularity

But isn't it just a game for silly mathematicians?

NO! Low regularity really changes the geometry!

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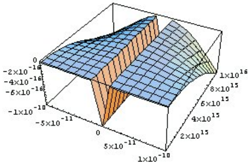
Completeness for impulsive gravit. waves

- exact models of violent but short pulses of gravitational radiation
- metric distributional or Lipschitz continuous
- various models, eg. gyratons [Frolov et al. 1995–]



$$ds^2 = h_{ij}dx^i dx^j - 2dudr + H(x)\delta_{\alpha,\beta}(u)du^2 + 2a_i(x)\vartheta_L(u)dudx^i$$

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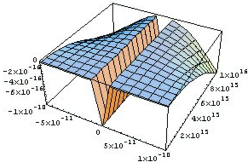

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How is ist done?

- distributional: regularisation techniques & fixed point arguments
- Lipschitz: Filippov's solution concept & use of specific geometry

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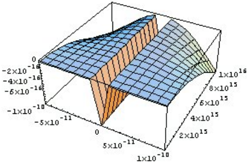

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Message

Analytically highly singular spacetimes are shown to be nevertheless **physically non-singular** hence good models.

Singularity Theorems in $\mathcal{C}^{1,1}$

Singularity thms: under suitable realistic conditions spacetimes develop singularities: **black holes** (Penrose), **big bang** (Hawking)

Theorem **[Penrose, 1965]**

A $\mathcal{C}^2$ -spacetime is future null geodesically incomplete, if

(i) $\mathrm{Ric}\,(X, X) \geq 0$ for every null vector X

(ii) There exists a non-compact Cauchy hypersurface S in M

(iii) There exists a trapped surface $\mathcal{T}$

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Theorem **[Kunzinger, S., Vickers, 2015]**

A $\mathcal{C}^{1,1}$-spacetime is future null geodesically incomplete, if

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Message: $\mathcal{C}^{1,1}$ is the natural regularity class

- Failure of $\mathcal{C}^2$ physically not **really** singular
- below $C^{1,1}$ unbounded curvature $\rightsquigarrow$ **really** singular

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How is it done?

- exponential map [KSS, 2014] & causality [KSSV, 2014]
- Regularisation adapted to causal structure [CG, 2012]
- replacement energy conditions for regularised metric [KSSV, 2015]

Some related Literature

[CG,12] P.T. Chruściel, J.D.E. Grant, *On Lorentzian causality with continuous metrics.* CQG 29 (2012)

[KSS,14] M. Kunzinger, R. Steinbauer, M. Stojković, *The exponential map of a $\mathcal{C}^{1,1}$-metric.* Diff. Geo. Appl.34(2014)

[KSSV,14] M. Kunzinger, R. Steinbauer, M. Stojković, J.A. Vickers, *A regularisation approach to causality theory for $\mathcal{C}^{1,1}$-Lorentzian metrics.* GRG 46 (2014)

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[PSSŠ,16] J. Podolský, C. Sämann, R. Steinbauer, R. Švarc, *The global uniqueness and C^1-regularity of geodesics in expanding impulsive gravitational waves.* arXiv:1602.05020, to appear in CQG

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[S,14] R. Steinbauer, *Every Lipschitz metric has C^1-geodesics.* CQG 31, 057001 (2014)

Vielen Dank fürs Zuhören!