International Conference on Generalized Functions

GF2022

Book of Abstracts

University of Vienna

September 19-23, 2022

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Talks are generally held in lecture hall **14**. If there are two names in one timeslot, the first talk will be held in lecture hall **14** and the second talk in lecture hall **16**.

Participants

Name	University	Country
Antonić, Nenad	University of Zagreb	HR
Ascanelli, Alessia	Università degli Studi di Ferrara	IT
Bargetz, Christian	Universität Innsbruck	AT
Bonino, Matteo	University of Turin	IT
Bryzgalov, Aleksandr	University of Vienna	AT
Cappiello, Marco	University of Turin	IT
Chung, Soon-Yeong	Sogang University	KR
Cordero, Elena	Università di Torino	IT
Coriasco, Sandro		IT
de Gosson, Maurice	Universität Wien	AT
Debrouwere, Andreas	Vrije Universiteit Brussel	BE
Dimovski, Pavel	Skopje	MK
Federico, Serena	University of Bologna	IT
Garetto, Claudia	Queen Mary University of London	GB
Giordano, Paolo	University of Vienna	AT
Grosser, Michael	Universität Wien	AT
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Lučić, Milica	University of Novi Sad	RS
Matsuyama, Tokio	Chuo University	JP
Mincheva-Kaminska, Svetlana	University of Rzeszow	PL

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Mitrovic, Darko	University of Vienna	AT
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Nugraheni, Sekar	University of Vienna	AT
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Ohanyan, Argam	University of Vienna	AT
Oparnica, Ljubica	University of Novi Sad	RS
Ortner, Norbert	Universität Innsbruck	AT
Parmeggiani, Alberto	University of Bologna	IT
Petersson, Albin	Linneaus University	SE
Pfeuffer, Christine	Martin-Luther-Universität Halle-Wittenberg	DE
Pilipovic, Stevan	Professor emeritus	RS
Prangoski, Bojan	Ss. Cyril and Methodius University in Skopje	MK
Rajter-Ciric, Danijela	University of Novi Sad	RS
Rottensteiner, David	Ghent University	BE
Ruzhansky, Michael	Ghent, Queen Mary University of London	BE
Scarpalezos, Dimitrios	Unversité Paris -Diderot	FR
Schindl, Gerhard	University of Vienna	AT
Seleši, Dora	University of Novi Sad	RS
Spreitzer, Christian	Pädagogische Hochschule Niederösterreich	AT
Steinbauer, Roland	University of Vienna	AT
Taniguchi, Koichi	Tohoku University	JP
Teofanov, Nenad	University of Novi Sad	RS
Todorov, Todor D.	California Polytechnic State University	US
Toft, Joachim	Linnaeus University	SE
Velinov, Daniel	Ss. Cyril and Methodius University in Skopje	MK
Vickers, James	University of Southampton	GB
Vojnovic, Ivana	University of Novi Sad	RS
Wahlberg, Patrik	Politecnico di Torino	IT
Zigic, Milica	University of Novi Sad	RS

Wednesday
11:30–12:30
HS 14

The initial value problem for p-evolution equations: regularity and asymptotic behavior of the solution

Alessia Ascanelli

University of Ferrara, Italy

We deal with the Cauchy problem

$$(*) \qquad \begin{cases} Pu(t,x) = f(t,x) & (t,x) \in [0,T] \times \mathbb{R}^n \\ u(0,x) = g(x) \end{cases}$$

for a class of anisotropic evolution operators P with $(t,x)-$depending coefficients, smooth and complex valued, except for the leading coefficients which are supposed to be real valued. We say that $(*)$ is well-posed in a functional space X if for every choice of the data $f \in C([0,T],X)$, $g \in X$, there exists a unique solution $u \in C([0,T],X)$. Several PDEs of phiscal interest belong to the class we are interested in, for instance Schrödinger equation and Korteweg-de Vries equation. Our model operator has in dimension 1 the form

$$P(t,x,D_t,D_x) = D_t + a_p(t)D_x^p + \sum_{j=0}^{p-1} a_j(t,x)D_x^j,$$

where $p \geq 2$ is the degree of evolution and $D = -i\partial$. The assumption that a_p is real valued guarantees that the principal symbol $\tau + a_p(t)\xi^p$ of P has a real root: this fulfills the necessary condition for well-posedness of $(*)$ in Sobolev spaces. Another well-known necessary condition to obtain some well-posedness result for $(*)$ is that $\Im a_j$ satisfy suitable decay conditions as $|x| \to \infty$. We look for sufficient conditions on the a_j leading to a well-posed Cauchy problem either in Sobolev spaces or spaces of functions with Gevrey regularity.
After a general survey on the topic, focusing on both regularity and asymptotic behvior of the solution, we will analyze in depth the cases $p = 2$ and $p = 3$, considering also semilinear equations.
The results discussed in the talk come from some joint works with A.Arias Jr and M.Cappiello, C.Boiti and L.Zanghirati, M.Cicognani and M.Reissig.

Homogenisation and microlocal energy propagation for the wave equation revisited

Wednesday
18:00–18:30
HS14

Nenad Antonić

University of Zagreb

We investigate the asymptotic behaviour of the wave equation on $\langle 0,T\rangle \times \Omega$

$$\begin{cases} (\rho_n(t,\mathbf{x})u_n')' - \mathsf{div}\,(\mathbf{A}_n(t,\mathbf{x})\nabla u_n) = f_n \\ u_n = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } \langle 0,T\rangle \times \Gamma \\ u_n(0,\cdot) = u_n^0 \\ (\rho_n u_n')(0,\cdot) = u_n^1 \quad , \end{cases}$$

where the coefficients are in $\mathrm{BV}(0,T;\mathrm{L}^\infty(\Omega))$, the matrices $\mathbf{A}_n$ not necessarily symmetric.

The problem exhibits two different comportments, one where the oscillation in the coefficients is dominant, therefore leading to the homogenisation problem, and the other, where the propagation of oscillations generated by initial conditions is dominant, which can be approached by using H-measures [1].

This problem was studied in [2,4], with coefficients independent of time, while the homogenisation with time-dependent coefficients (albeit only with symmetric $\mathbf{A}_n$) was considered in [3].

This is a joint work with Matko Grbac.

References

[1] Nenad Antonić, Martin Lazar: *Microlocal energy density for hyperbolic systems* in: *Applied Mathematics and Scientific computing*, pp. 179–190, Z. Drmač et al. (eds.), Kluwer Academic/Plenum Publishers, 2003.

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Monday
17:30–18:00
HS 14

On projective descriptions of spaces of distributions

Christian Bargetz

Universität Innsbruck

We present a number projective descriptions of spaces of smooth functions and distributions. These representations provide a description of the topology of these spaces by seminorms which are a combination of classical norms and multiplication and convolution with certain functions.

This is joint work with Eduard A. Nigsch and Norbert Ortner.

References

[1] C. Bargetz, E. A. Nigsch, N. Ortner, Projective descriptions of spaces of functions and distributions. Preprint (accepted for publication in Mathematische Nachrichten, arxiv:2109.14448)

Tuesday
16:30–17:00
HS 16

Numerical and Graphical Tools to Visualize GSF

Aleksandr Bryzgalov

University of Vienna, Faculty of Mathematics

Using [1] as a theoretical basis, we prepared an easy-to-use framework in Wolfram Mathematica [2] with numerical and graphical tools to visualize Generalized Smooth Functions (GSF). We implemented the gauge $\rho = [\rho_\varepsilon]$ with the $\mathrm{d}\rho$ notation and Cauchy-Dirac GSF, i.e. GSF of the form $f(x) = [\varphi(x_\varepsilon, p_\varepsilon)] \in {}^\rho\tilde{\mathbb{R}}$, where $\varphi \in C^\infty$ is an ordinary smooth function and $p = [p_\varepsilon]$ is a vector of generalized parameters. For example, $f(x) = \mathrm{d}\rho \cdot \sin(\frac{x}{\mathrm{d}\rho})$ is a Cauchy-Dirac GSF that can be inputted as it is written thanks to an overloading of the product symbol $\cdot$ and of the symbol sin.

GSF can also be inputted as defined by cases in ε (for example, $f(x) = [f_{1\varepsilon}(x_\varepsilon)]$ if $\varepsilon = \frac{1}{n}$ for some $n \in \mathbb{N}$ and $f(x) = [f_{2\varepsilon}(x_\varepsilon)]$ otherwise) or GSF defined by smoothly interpolating two (or more) given ε-depending smooth functions.

GSF can also be inserted, modified and visualized using the common notations for sum, product, composition, powers, etc. of previously defined GSF. Such implementation allows one to also compute specific manipulations with δ and H functions, like $f(x) = \delta(\delta(\delta(x))) + \cos(H(\delta(x)))$. We also present some results about the implementation of the Picard-Lindelöf theorem for PDE.

Graphical tools of Wolfram Mathematica possess the ability to animate the plots of the GSF changing the ε parameter. This kind of "experiments" could be really helpful to build strong intuition in the study of infinite/infinitesimal behavior with respect to $\varepsilon \to 0^+$.

Joint work with: Paolo Giordano (University of Vienna, Faculty of Mathematics)

References

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[2] `https://www.wolfram.com/mathematica/`

Time-periodic Gelfand-Shilov spaces and global hypoellipticity for evolution operators

Wednesday
17:30–18:00
HS 14

Marco Cappiello

University of Turin

Global hypoellipticity for evolution operators in a periodic setting is a widely studied problem whose origin dates back to the pioneering works by Greenfield and Wallach [5,6]. In particular, Gevrey global hypoellipticity for vector fields on the torus has been studied by a large number of authors, see e.g. [1,3,4] and the references therein. Here we consider a class of evolution operators of the form $L = D_t + c(t)P(x, D_x)$ where $c(t)$ is a complex valued function defined on the one-dimensional torus $\mathbb{T}$ and $P(x, D_x)$ is a normal, globally elliptic differential operator on $\mathbb{R}^n$. We analyze the global hypoellipticity of L in spaces of functions whose elements are Gevrey regular, periodic in t and admit an exponential decay for $|x| \to \infty$. With this purpose we introduce a class of time-periodic Gelfand-Shilov spaces on $\mathbb{T} \times \mathbb{R}^n$ and characterize the Fourier coefficients of their elements corresponding to the eigenfunctions of P. A similar characterization is obtained for the dual space. By discretizing the equation $Lu = f$ and using this characterization, we state necessary and sufficient conditions for the global hypoellipticity of the operator L. These results are obtained in collaboration with Fernando de Ávila Silva (Federal University of Parana), cf. [2].

References

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[6] S. J. Greenfield, N. R. Wallach, *Hypoelliptic vector fields and continuous fractions*, Proc. Amer. Math. Soc. **31** (1972) no. 1 , 115-118-253.

Wednesday
16:30–17:00
HS 14

A Necessary and Sufficient Condition for the Global Existence of Nonlinear Reaction-Diffusion Equations on $\mathbb{R}^N$

Soon-Yeong Chung

Department of Mathematics, Sogang University, Seoul, Korea

A necessary-sufficient condition for the existence or nonexistence of global solutions to the following reaction-diffusion equations

$$\begin{cases} u_t = \Delta u + \psi(t)u^p, & \text{in } \mathbb{R}^N \times (0, t^*), \\ u(\cdot, 0) = u_0 \geq 0, & \text{in } \mathbb{R}^N, \end{cases}$$

has not been known and remained unsolved for a few decades. In fact, only for the cases $\psi(t) \equiv 1$, $\psi(t) = e^{\beta t}$, or $\psi(t) = t^\sigma$, some necessary and sufficient conditions for the existence of the global solution to the equation have been studied (see [1]-[4]).

The purpose of this paper is to resolve this problem completely, even for more general source $\psi(t)f(u)$ as follows:

There is no global solution to the equation for any nonnegative
and nontrivial initial data $u_0 \in C_0(\mathbb{R}^N)$ if and only if

$$\int_0^\infty \psi(t) \frac{f\left(\epsilon \left\|S(t)u_0\right\|_\infty\right)}{\left\|S(t)u_0\right\|_\infty} dt = \infty$$

for every $\epsilon > 0$, for every nonnegative nontrivial initial data $u_0 \in C_0(\mathbb{R}^N)$.

Here, $(S(t))_{t\geq 0}$ denotes the heat semigroup on $\mathbb{R}^N$.

** This lecture is based on a joint work with my student J. Hwang.*

References

[1]X. Bai, S. Zheng, and W. Wang, Critical exponent for parabolic system with time-weighted sources in bounded domain, J. Funct. Anal. 265 (2013).

[2]H. Fujita, On the blowing up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$, J. Fac. Sci. Univ. Tokyo Sect. I 13 (1966).

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Symplectic Analysis of Operators: Applications to Schrödinger equations and wave fronts

Monday
09:00–10:00
HS 14

Elena Cordero

University of Torino

We introduce a *symplectic time-frequency analysis*. The standard time-frequency representation *Short-time Fourier Transform* (STFT) is replaced by the $\mathcal{A}$-*Wigner distribution* defined by

$$W_{\mathcal{A}}(f,g) = \mu(\mathcal{A})(f \otimes \bar{g}),$$

where $\mathcal{A}$ is a $4d \times 4d$ symplectic matrix and $\mu(\mathcal{A})$ is an associate metaplectic operator. Basic examples are given by the so-called τ-Wigner distributions. We find subclasses of symplectic matrices $\mathcal{A}$ that can be used to define modulation spaces: the STFT originally used to introduce such spaces, cf. [1], is substituted for the $\mathcal{A}$-Wigner $\mu(\mathcal{A})(f \otimes \bar{g})$ [2]. Further, symplectic representations can be efficiently employed in the study of pseudodifferential operators, paving the way for a new understanding of quantization procedures [3]. Finally, we deduce micro-local properties for pseudodifferential operators in terms of the $\mathcal{A}$-Wigner wave front sets. In fact, modulation spaces and $W_{\mathcal{A}}$ representations are the frame for a new definition of wave front set, providing a sharp result for propagation of micro-singularities in the case of Schrödinger equations with quadratic Hamiltonians [4].

This is a joint project with Gianluca Giacchi (University of Bologna) and Luigi Rodino (University of Torino).

References

[1] H. G. Feichtinger, Modulation spaces on locally compact abelian groups, In "Wavelets and Their Applications", M. Krishna, R. Radha, S. Thangavelu, editors, Allied Publishers, (2003).

[2] E. Cordero and L. Rodino, Characterization of modulation spaces by symplectic representations and applications to Schrödinger equations. arXiv:2204.14124v1

[3] E. Cordero and L. Rodino, Wigner analysis of operators. Part I: pseudodifferential operators and wave fronts. Appl. Comput. Harmon. Anal., 58:85–123, (2022).

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Monday
18:00–18:30
HS 14

Wodzicki Residue for operators on non-compact manifolds

Sandro Coriasco

Dipartimento di Matematica "G. Peano"
Università degli Studi di Torino (Italy)

Wodzicki Residue was first considered by Wodzicki in 1984 [20], in the setting of pseudo-differential operators on closed manifold, while studying the meromorphic continuation of the zeta function of elliptic operators: the latter had been originally defined by Seeley [19]. Wodzicki Residue turns out to be a trace on the algebra of classical operators modulo smoothing operators. Moreover, if the dimension of the closed manifold is larger then one, it is the unique trace on such algebra, up to multiplication by a constant (the situation in dimension one is different, as a consequence of the fact that, in such a case, S^*M is not connected, cfr. Kassel [12]). In 1985, Guillemin [10] independently defined the so-called Symplectic Residue, equivalent to Wodzicki Residue, with the aim of "finding a soft proof of Weyl formula".

Wodzicki Residue, sometimes called non-commutative trace, gained a growing interest in the years, also in view of the links with non-commutative geometry and Dixmier trace, see, e.g., Connes [5], Ammann and Bär [1], Kalau and Walze [11], Kastler [13]. The concept has been extended to different situations: manifolds with boundary by Fedosov et al. [8], conic manifolds by Schrohe [18] and Gil and Loya [9], operators with log-polyhomogeneous symbols by Lesch [14], anisotropic operators on $\mathbb{R}^n$ by Boggiatto and Nicola [3], Heisenberg calculus by Ponge [17], holomorphic families of pseudo-differential operators by Paycha and Scott [16]. The concept of residue was also extended to hypoelliptic operators by Connes and Moscovici [6], where applications to the local index formula are given, to compact Lie groups by Cardona [4], and to Lorentzian spaces by Dang and Wrochna [7]. Wodzicki Residue in the case of SG-calculus on $\mathbb{R}^n$ was defined by Nicola [15], on manifolds with ends by Battisti and Coriasco [2].

I will discuss the extension of the notion of Wodzicki Residue to algebras of operators on certain non-compact manifolds. Part of this talk concerns joint work with M. Bonino.

References

[1] Ammann, B., Bär, C., The Einstein-Hilbert action as a spectral action. In Noncommutative geometry and the standard model of elementary particle physics (Hesselberg, 1999), Lecture Notes in Phys. 596, pp. 75–108. Springer, Berlin, (2002).

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Linear topological invariants for kernels of convolution and partial differential operators

Friday
10:30–11:30
HS 14

Andreas Debrouwere

Vrije Universiteit Brussel (VUB)

We give an overview of results concerning linear topological invariants for smooth and distributional kernels of convolution and partial differential operators. As a motivation, we explain how such results may be used to study the problem of parameter dependence of solutions of such operators.

This talk is based on joint work with Thomas Kalmes (University of Chemnitz).

Toeplitz Density Operators and Quantum Mechanics

Monday
10:30–11:30
HS 14

Maurice de Gosson

University of Vienna

Density operators are positive semidefinite operators with trace one representing the mixed states of quantum mechanics. In this talk we define and study a subclass of density operators which we call Toeplitz density operators. They correspond to quantum states obtained from a fixed function ("window") by phase space translations, and reduce in the simplest case to the anti-Wick operators considered long ago by Berezin and extensively studied by Cordero, Boggiatto, Pilipovic, and others. The rigorous study of the Toeplitz density operators requires the use of classes of functional spaces defined by Feichtinger.

Wiener Amalgam Spaces of Quasianalytic Ultradistributions

Wednesday
14:00–14:30
HS 14

Pavel Dimovski

Faculty of Technology and Metallurgy, Ss. Cyril and Methodius University of Skopje, North Macedonia

We define Wiener amalgam spaces of (quasi)analytic ultradistributions whose local components belong to a general class of translation and modulation invariant Banach spaces of ultradistributions and global components are either weighted L^p or a weighted $\mathcal{C}_0$ spaces. We provide a discrete characterisation via so called uniformly concentrated partitions of unity. Finally, we study the complex interpolation method and we identify the strong duals for most of these Wiener amalgam spaces.

Joint work with Bojan Prangoski, Faculty of Mechanical Engineering, Ss. Cyril and Methodius University of Skopje, North Macedonia.

On some variable coefficient Schrödinger operators on the torus

Tuesday
14:00–14:30
HS 14

Serena Federico

University of Bologna

In this talk we will discuss the local well-posedness of the nonlinear initial value problem for some variable coefficient Schrödinger operators on the torus. Due to their fundamental role in the resolution of the aforementioned problem, we shall first consider the validity of Strichartz estimates in our setting. Next, the application these inequalities will give local well-posedness results with minimum regularity requirement on the initial datum. This is a joint work with Gigliola Staffilani.

References

[1] S. Federico and G. Staffilani, Sharp Strichartz estimates for some variable coefficient Schrödinger operators on $\mathbb{R} \times \mathbb{T}^2$, Mathematics in Engineering 2022, 4 (4): 1-23.

Hyperbolic Cauchy problems with multiplicities and singular coefficients

Tuesday
09:00–10:00
HS 14

Claudia Garetto

Queen Mary University of London

In this talk I will present some recent results on hyperbolic equations with multiplicities and singular coefficients. The well-posedness of the corresponding Cauchy problem will be obtained in different functions spaces and suitable notions of generalised solutions will be introduced.

Consequences of neglecting infinitesimals (and infinities) from mathematics

Friday
09:00–10:00
HS 14

Paolo Giordano

University of Vienna, Faculty of Mathematics

In neuroscience it is well-known that all of us have some kind of cognitive bias. In mathematics, Grothendieck identified this mindset with some kind of lacking of freedom from our peers. The banishing of infinitesimal and infinite numbers from mathematics sometimes resulted in this kind of behavior. For this reason, V.I. Arnol'd in [1] wrote: "Nowadays, when teaching analysis, it is not very popular to talk about infinitesimal quantities. Consequently, present-day students are

not fully in command of this language. Nevertheless, it is still necessary to have command of it".

I present several examples where a "standard" point of view can lead to a lack of powerful mathematical results, such as the closure of generalized functions with respect to composition, "compact" but unbounded sets, infinitesimal solutions of singular differential equations, a Fourier transform applicable to non-tempered generalized functions, Hadamard well-posedness of every ordinary smooth Cauchy problem for PDE, Cauchy-Kowalevski theorem valid for PDE defined by compactly supported generalized functions (such as, e.g., the Dirac-delta or a smooth bump function), and several examples of mathematical modeling in mathematical physics. I close with a list of, hopefully interesting, open problems in the theory of generalized smooth functions.

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Wednesday
16:00–16:30
HS 16

Completeness results for sets of shifted dilates in invariant Banach spaces of tempered distributions

Anupam Gumber

University of Vienna

In [5], V. Katsnelson investigated that for certain Hilbert spaces $\mathcal{H}$ of functions, continuously embedded into $\left(\boldsymbol{L}^2(\mathbb{R}), \|\cdot\|_2\right)$ satisfying a few additional (invariance) conditions has the property that the set of shifted and dilated Gaussian's g span a dense subspace of $\mathcal{H}$, with $g(t) = \exp(-\pi t^2)$, $t \in \mathbb{R}$.

We show that well-established methods from the theory of Banach modules and time-frequency analysis allow to derive completeness results for the collection of shifted and dilated version of a given (test) function in a quite general setting. As it turned out, the setting of the paper [2] appeared to be most appropriate, which

is quite similar to the setting of so-called standard spaces as used in papers on compactness ([3]) or double module structures ([1]).

While the basic ideas show strong similarity to the arguments used in a recent paper by V. Katsnelson [5] we extend his results in several directions, both relaxing the assumptions and widening the range of applications. There is no need for the Banach spaces considered to be embedded into $\big(\boldsymbol{L}^2(\mathbb{R}),\, \|\cdot\|_2\big)$, nor is the Hilbert space structure relevant. We choose to present the results in the setting of the Euclidean spaces, because then the Schwartz space $\boldsymbol{\mathcal{S}}'(\mathbb{R}^d)$ $(d \geq 1)$ of tempered distributions provides a well-established environment for mathematical analysis. We also establish connections to modulation spaces and Shubin classes $\big(\boldsymbol{Q}_s(\mathbb{R}^d),\, \|\cdot\|_{\boldsymbol{Q}_s}\big)$, showing that they are special cases of Katsnelson's setting (only) for $s \geq 0$. This talk is based on a recent joint work with Hans G. Feichtinger [4].

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Generalized functions applied to models of natural risks

Tuesday
16:00–16:30
HS 16

Maximilian F. Hasler

Université des Antilles

Natural hazards or risks like hurricanes, tsunami and other shock waves including earthquakes, heat waves and other shocks such as financial breakdowns or even pandemics, and many others can quite obviously be associated to singularities. Such singularities arise because of non-linearities – as long as the response of the physical system is linear, small perturbations will have a proportionally small result. Therefore, algebras of generalized functions are a natural and necessary framework to tackle these very important problems described by nonlinear differential equations.

We briefly review earlier work on this subject, focusing on our most recent contributions, but also earlier results by ourselves and other approaches. This includes

theorems about existence, evolution and propagation of weak and strong singularities (vertices, jumps, ...) in various types of nonlinear differential equations such as nonlinear wave equations and Navier-Stokes type equations from fluid dynamics. As already alluded to, these are most conveniently formulated and studied in the framework of Colombeau type algebras of generalized functions.

In addition to such more theoretical results, we also give several distinct examples of explicit algebraic and numerical computations performed on systems of equations derived in this framework, such as predictions of hurricane tracks.

A Necessary and Sufficient Condition for the Existence of Global Solutions to Discrete Semilinear Parabolic Equations on Networks

Tuesday
17:00–17:30
HS 14

Jaeho Hwang

Research Institute for Basic Science, Sogang University, Seoul, Republic of Korea

In his pioneering paper [1], Meier considered the equations

$$u_t = \Delta u + \psi(t)u^p, \quad \text{in } \Omega \times (0, t^*), \tag{1}$$

where Ω is a general (bounded or unbounded) domain in $\mathbb{R}^N$, under the Dirichlet boundary condition and obtained the following results:
For $p > 1$ and $\psi \in C[0, \infty)$,

(i) If $\limsup_{t\to\infty} \|S(t)u_0\|_\infty^{p-1} \int_0^t \psi(\tau)d\tau = \infty$ for all $u_0 \in C_0(\Omega)$, then there is no global solution to (1) for any initial data.

(ii) If $\int_0^\infty \psi(\tau)\|S(\tau)u_0\|_\infty^{p-1}d\tau < \infty$ for some $u_0 \in C_0(\Omega)$, then there exists a global solution to (1) for sufficiently small initial data.

Here, $(S(t))_{t\geq 0}$ is the heat semigroup with the Dirichlet boundary condition on $\partial\Omega$.

However, Meier's criterion is not satisfactory, because it was not given in a form of a necessary and sufficient condition. In fact, obtaining the necessary and sufficient condition has remained as an open problem for a few decades. In this talk, we discuss the necessary-sufficient condition for the existence and non-existence of global solutions of the equations (1) on networks for more general source term $\psi(t)f(u)$, under various boundary conditions.

* This is a joint work with my advisor S.-Y. Chung.

References

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Derivation of the heat and wave equation in the setting of generalized smooth funcions

Wednesday
14:00–14:30
HS 16

Kevin Islami

University of Vienna

We present rigorous versions of the usual deductions of the heat and wave equations for generalized smooth functions (GSF). GSF theory is a branch of Colombeau theory where generalized functions share a number of fundamental properties with smooth functions, in particular with respect to composition and nonlinear operations. After highlighting what are the main inconsistencies in the classical informal derivations, we present Taylor's formula with nilpotent infinitesimals (i.e. without remainder), we prove that Fourier's law is equivalent to the product of the mass density, specific heat capacity and the derivative of the temperature of a chosen body. This can further be expressed in a different way using the divergence theorem and obtaining the classical heat equation. After this part, we present the theorem about the deduction of the wave equation, which explains the equivalence between the wave equation for a string (up to second order infinitesimals) and the assumption that the corresponding angular oscillation is a nilpotent infinitesimal number of fourth order.

Joint work with: Paolo Giordano, University of Vienna, Faculty of Mathematics.

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The Hahn-Banach theorem in ${}^{\rho}\widetilde{\mathbb{R}}$-modules and its applications

Wednesday
14:30–15:00
HS 16

Djamel eddine Kebiche

University of Vienna

We first review the present state of the art concerning the Hahn-Banach extension property: for any ${}^{\rho}\widetilde{\mathbb{R}}$-linear map g satisfying

$$\forall x \in \mathcal{F} \subseteq \mathcal{G} : g(x) \leq p(x), \tag{2}$$

where $\mathcal{G}$ is an ${}^{\rho}\widetilde{\mathbb{R}}$-module, $\mathcal{F}$ is an ${}^{\rho}\widetilde{\mathbb{R}}$-submodule and p is an ${}^{\rho}\widetilde{\mathbb{R}}$-sublinear map, there exists an ${}^{\rho}\widetilde{\mathbb{R}}$-linear map f that extends g and satisfies (1) for every $x \in \mathcal{G}$. Firstly,

theorem 10.1 of [4] shows that under some set-theoretic assumptions (continuum hypothesis) there exists a continuous ${}^{\rho}\widetilde{\mathbb{R}}$-linear map ϕ defined on an ${}^{\rho}\widetilde{\mathbb{R}}$-submodule of ${}^{\rho}\widetilde{\mathbb{R}}$ that cannot be extended to a continuous ${}^{\rho}\widetilde{\mathbb{R}}$-linear map $\psi : {}^{\rho}\widetilde{\mathbb{R}} \longrightarrow {}^{\rho}\widetilde{\mathbb{R}}$. When $\mathcal{G}$ is a Hilbert ${}^{\rho}\widetilde{\mathbb{R}}$-module and if $\mathcal{F}$ is closed and edged the extension property holds, [1]. Moreover, when $L \subset {}^{\rho}\widetilde{\mathbb{R}}$ is a subfield and $(E, ||\cdot||)$ is an ultrametric normed L-linear space, any continuous L-linear functional on some L-linear subspace V of E, can be extended, [2]. We also mention that the Hahn-Banach extension property holds for non-Archimedean normed linear spaces over a field $\mathbb{K}$, [3].
We then show that the extension property holds when $\mathcal{G}$ is a Colombeau space $\mathcal{G}_E$ based on a net of locally convex topological vector spaces $(E_\varepsilon, \{p_\varepsilon\}_n)$, when $\mathcal{F}$ is internal, and when the maps g and p have representatives, [5].
Regarding the geometric forms of the Hahn-Banach theorem, we proved that one could not extend the separation of convex sets property to subsets of $\mathcal{G}$ without any additional assumption. This was proved by finding two non-empty strongly disjoint convex sets A, $B \subseteq {}^{\rho}\widetilde{\mathbb{R}}$, where A is an open set (in the sharp topology), that cannot be separated by an hyperplane. In our paper, we have already found a sufficient condition that makes the separation property of convex sets possible in $\mathcal{G}_E$. Roughly speaking, we proved that the separation property holds e.g. when A is a strongly internal set and B is an internal set.
Joint work with: Paolo Giordano, University of Vienna, Faculty of Mathematics.

References

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Classical Spaces of Distributions as Amalgam Spaces

Wednesday
14:30–15:00
HS 14

Tillmann Kleiner

University of Stuttgart

Convolutor spaces $F = \mathcal{O}'_C(\mathcal{D}, E)$ are studied where E is a solid locally convex space of Radon measures that is continuously invariant with respect to convolution with test functions. A characterization of its locally convex structure suggests to interpret F as an amalgam space with local component $\mathcal{D}'$ and global component E. It is demonstrated that a general class of distribution spaces, that contains $\mathcal{D}'_{L^p}$ and its weighted variants, can be represented as an amalgam space F. Inheritance properties of the mapping $E \mapsto \mathcal{O}'_C(\mathcal{D}, E)$ are obtained such as continuity or hypocontinuity of convolution of distributions as bilinear mapping involving a triple of spaces [1]. The results apply to maximal domains of sets of convolution operators, understood in terms of convolution of distributions [2].

This talk presents joint work with R. Hilfer.

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Qualitative properties of hyperbolic operators with finite time propagation of singularities

Thursday
11:30–12:30
HS 14

Irina Kmit

Humboldt University of Berlin

We will discuss a class of (nonlinear) initial-boundary value problems for first order 1D (integro)-differential strictly hyperbolic systems which are distinguished by finite time propagation of singularities. We characterize the last property in terms of boundary and/or equation coefficients. This class of problems exhibits a number of interesting mathematical phenomena such as the smoothing effect for solutions [3], the superstability property [1], robustness of exponential dichotomy [2], the Fredholm property and the absence of small divisors in periodic problems [4]. Our analysis has applications to solving inverse problems and finding global classical solutions to quasilinear problems [2].

References

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Global Controllability for Quasilinear Nonnegative Definite System of ODEs and SDEs

Wednesday
16:00–16:30
HS 14

Sanja Konjik

University of Novi Sad

We consider exact and averaged control problem for a system of quasilinear ODEs and SDEs with a nonnegative definite symmetric matrix of the system. The strategy of the proof is the standard linearization of the system by fixing the function appearing in the nonlinear part of the system and then applying the Leray–Schauder fixed point theorem.We shall also need the continuous induction arguments to prolong the control to the final state which is a novel approach in the field. This enables us to obtain controllability for arbitrarily large initial data (so-called global controllability).

Fully nonlinear mean field games

Tuesday
17:00–17:30
HS 16

Miłosz Krupski

University of Zagreb, University of Wrocław

Mean field games are systems of coupled Hamilton–Jacobi and Fokker–Planck equations, which can be used to model e.g. population dynamics. In this talk I will present a new, fairly general derivation of such games from a stochastic model involving Lévy processes, as well as methods (and challenges) used to construct solutions.

Joint work with Indranil Chowdhury and Espen Jakobsen.

A spectral method approach for solving SPDEs and the related control problems

Friday
11:30–12:00
HS 16

Tijana Levajković

TU Vienna

We present a spectral method approach for solving linear and nonlinear stochastic partial differential equations where the coefficients, initial and boundary conditions might be highly singular, i.e., they are generalized stochastic processes. We consider nonlinearities of Wick-type and a quadratic cost functional for the related optimal control problem. Particularly, we apply a polynomial chaos expansion (PCE) method, a spectral method based on the tensor product of deterministic orthogonal polynomials as a basis in the space of square integrable stochastic processes. Using the PCE method combined with operator theory, semigroup theory, and theory of deterministic partial differential equations, we prove that the stochastic equations under consideration have unique solutions in appropriate (weighted) spaces of stochastic processes. In an analogous way we prove the existence and uniqueness of the related optimal control. The solutions are given in explicit forms and provide a novel theoretical and numerical framework for treating these problems. The novelty also relies on the application of splitting methods to approximate the so-called optimal state. Numerical experiments show the potential of our method.

On the reflexivity properties of Banach bundles and Banach modules

Tuesday
16:00–16:30
HS 14

Milica Lučić

Department of Mathematics and Informatics, Faculty of Sciences,
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The theory of $L^p(\mathfrak{m})$-normed $L^\infty(\mathfrak{m})$-modules was introduced by Gigli in [2]. It holds that for every measurable Banach bundle $\mathbf{E} : X \twoheadrightarrow \mathbb{B}$, the spaces of its L^p-sections, $\Gamma_p(\mathbf{E})$, $p \in (1, \infty)$ have a structure of $L^p(\mathfrak{m})$-normed $L^\infty(\mathfrak{m})$-modules. More surprisingly, in the case of separable normed modules the converse implication holds as well: every separable $L^p(\mathfrak{m})$-normed $L^\infty(\mathfrak{m})$-module is isomorphic to $\Gamma_p(\mathbf{E})$ for some measurable Banach $\mathbb{B}$-bundle $\mathbf{E}$, as proved in [1] by S. Di Marino, D. Lučić and E. Pasqualetto.

In this talk we consider some important properties of measurable Banach bundles (i.e. its fibers), such as Hilbertianity, uniform convexity and reflexivity. Our main aim is to answer the questions whether these properties of the fibers of a measurable Banach bundle carry over to its space of sections, and vice versa. One of our main statements is that the space of L^p-sections, $p \in (1, \infty)$ of every separable Banach bundle having reflexive fibers is reflexive. In order to prove this statement we give

a characterization of the dual of the space of sections. We also prove that a Banach bundle has uniformly convex fibers (with a common modulus of convexity) if and only if the space of its L^p-sections is uniformly convex for every $p \in (1, \infty)$. These statements are generalisations of well-known corresponding results related to Lebesgue-Bochner spaces, which are particular instances of spaces of sections (of constant Banach bundles).

This talk is is based on collaborative work ([3]) with Enrico Pasqualetto and Ivana Vojnović.

References

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Tuesday
11:30–12:30
HS 14

On wave equation outside trapping obstacles and local energy decay in odd space dimensions

Tokio Matsuyama

Chuo University

In this talk we inform the appropriate cut-off resolvent estimates for the Dirichlet Laplacian on exterior domains. The geometrical assumptions on domains are rather general, for example, non-trapping condition is not imposed. The first key assumption guarantees the result on propagation of singularities, the second is the smallness of the symplectic measur of trapped sets in the cotangent bundle, and the third concerns the upper bound of the sojourn time. As a by-product of these estimates, the local energy decay estimate for solutions to the initial-boundary value problem for wave equation in the case of odd space dimensions is obtained. This talk is based on the joint work with Professor Vladimir Georgiev (Università di Pisa).

On the product in generalizad Gelfand-Shilov spaces

Thursday
14:00–14:30
HS 14

Svetlana Mincheva-Kamińska

University of Rzeszów

A. Debrouwere and J. Vindas in [3] discuss the completeness of some classes of weighted inductive limits of spaces of ultradifferentiable functions and study their duals. The authors define locally convex topologies in these spaces and give necessary and sufficient conditions for them to be ultrabornological. These results are applied to the spaces of convolutors for Gelfand-Shilov spaces. A. Debrouwere and L. Neyt in [1] study weighted (PLB)-spaces of ultradifferentiable functions defined via a weight function and characterize these of the considered spaces which are ultrabornological in terms of the defining weight system. Furthermore the authors determine the multiplier spaces of the Gelfand-Shilov spaces.

On the other hand, A. Debrouwere and J. Vindas in [2] provide a projective description for a class of generalized Gelfand– Shilov spaces of Roumieu type. Using the approach presented in [4], we define some classes of delta nets and give several definitions of the product of ultradistributions and tempered ultradistributions. On the base of the results of Debrouwere, Neyt and Vindas, we prove the equivalence of several of the given definitions of the product in Gelfand-Shilov spaces.

References

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Tuesday
14:30–15:00
HS 14

Velocity averaging for diffusive transport equations with discontinuous flux

Darko Mitrovic

University of Vienna, Austria

The following theorem is stated in [Theorem C, 1], but it is left unproven:

Theorem Let $1 < p \leq 2$ and let $h_\varepsilon \in L^p(R^+ \times R^{d+1})$ such that $\|h_\varepsilon\|_{L^p(R^+\times R^{d+1})} \leq c < \infty$. Assume also that the family (g_ε) belongs to the compact subset of $L^p(R^+ \times R^{d+1})$ and that the following PDE is satisfied in the weak sense

$$\begin{aligned}&\partial_t h_\varepsilon + \mathrm{Div}_x\big(a(\lambda)h_\varepsilon\big) - \sum_{i,j=1}^{d} a_{ij}(\lambda)\frac{\partial^2 h_\varepsilon}{\partial x_i \partial x_j}\\ &= (-\Delta_{t,x}+1)^{1/2}(-\Delta_\lambda+1)^{r/2} g_\varepsilon, \quad t \in R^+,\ x \in R^d,\ \lambda \in R,\end{aligned}$$

and a and $A = (a_{ij})$ are smooth. If the non-degeneracy conditions

$$(\forall \xi \in R^d \setminus \{0\}) \qquad \mathrm{meas}\big\{\lambda \in K \subset\subset R :\ \langle a(\lambda)\,|\,\xi\rangle = \langle A(\lambda)\xi\,|\,\xi\rangle = 0\big\} = 0\,,$$

are fulfilled, then for every $\rho \in C_c^r(R)$ the family $(\int_R h_\varepsilon(\cdot,\lambda)\rho(\lambda)d\lambda)$ belongs to a compact subset of $L^p_{loc}(R^+ \times R^d)$.

We prove the theorem in the L^q-setting, $q \geq 2$, and generalise the result on equations with x-dependent discontinuous flux a.

This is a joint work with Marko Erceg, University of Zagreb, Croatia, and Marin Misur, Argo AI, Munich, Germany.

References

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Singular shock solutions to conservation law with discontinuous fluxes

Wednesday
17:00–17:30
HS 14

Marko Nedeljkov

University of Novi Sad

We analyse a fairly general case of a discontinuous flux in one point for a conservation law. There are some cases when existence of a weak solution fails. Shadow waves ([4]), representing singular shocks defined in [2], are used to obtain a solution in these cases. Let us note that use of such waves explains so called "overshoot" obtained in numerical experiments (see [1]).

This talk is based on a joint work with Tanja Krunić ([3]).

References

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Weighted (PLB)-spaces of ultradifferentiable functions and multiplier spaces

Friday
11:30–12:30
HS 14

Lenny Neyt

Ghent University

(PLB)-spaces, i.e. countable intersections of countable unions of Banach spaces, arise naturally in functional analysis. Classical examples are the space of distributions, the space of real analytic functions and the space of multipliers of the Schwartz space. In order to be able to apply functional analytic tools such as De Wilde's open mapping and closed graph theorems or the theory of the derived projective limit functor, it is important to determine when such spaces are ultrabornological. Note that this is a non-trivial matter as the projective limit of a spectrum of ultrabornological spaces is not necessarily again ultrabornological. In a series of two papers [1,2] we showed a full characterization of when weighted (PLB)-spaces of ultradifferentiable functions are ultrabornological in terms of the defining weight

system by using tools from time-frequency analysis. This generalizes Grothendieck's classical result that the space of slowly increasing smooth functions is ultrabornological to the context of ultradifferentiable functions. Furthermore, we determined the multiplier spaces of the Gelfand-Shilov spaces and, by using the above result, characterized when such spaces are ultrabornological. In particular, we obtained that the multiplier space of the Gelfand-Shilov space $\Sigma_s^r(\mathbb{R}^d)$ of Beurling type is ultrabornological, whereas the one of the Gelfand-Shilov space $\mathcal{S}_s^r(\mathbb{R}^d)$ of Roumieu type is not. In this talk we will give an overview of these results.

The talk is based on collaborative works with Andreas Debrouwere.

References

[1] A. Debrouwere, L. Neyt, *Weighted (PLB)-spaces of ultradifferentiable functions and multiplier spaces*, Monatsh. Math. **198** (2022), 31–60.

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Monday
15:00–15:30
HS 14

Sequence space representations for spaces of smooth functions and distributions via Wilson bases

Eduard A. Nigsch

TU Wien

We provide explicit sequence space representations for the test function and distribution spaces occurring in the Valdivia-Vogt structure tables by making use of Wilson bases generated by compactly supported smooth windows. Furthermore, we show that these kind of bases are common unconditional Schauder bases for all separable spaces occurring in these tables. Our work implies that the Valdivia-Vogt structure tables for test functions and distributions may be interpreted as one large commutative diagram.

This is joint work with Christian Bargetz (Universität Innsbruck) and Andreas Debrouwere (Vrije Universiteit Brussel).

References

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On the Bourgain–Morrey spaces

Tuesday
16:30–17:00
HS 14

Toru Nogayama

Chuo University

The Morrey space was introduced by C.B.Morrey Jr. in 1938 to investigate the local behavior of solutions to second order elliptic partial differential equation in [2]. Later many authors were investigated this function space. One of the feature of this space is that Morrey spaces cover Lebesgue spaces so that Morrey spaces describe nice properties more than Lebesgue spaces.

Meanwhile, the function spaces which we call "Bourgain–Morrey spaces" were introduced by Bourgain [1]. He used this space to refine the Stein-Tomas (Strichartz) estimates. Moreover, Moyua, Vargas, and Vega [3] also used this space and proved the almost everywhere convergence of the solution to the initial data of a dispersive equation. The Bourgain–Morrey space is applied to various partial differential equations. However, to the best of our knowledge, Bourgain–Morrey spaces have yet to be investigated from the viewpoint of harmonic analysis.

In this talk, the results for the Bourgain–Morrey spaces from harmonic and functional analysis viewpoints are presented.

This is a joint work with Denny Ivanal Hakim (Bandung Institute of Technology), Naoya Hatano (Chuo University), and Yoshihiro Sawano (Chuo University).

References

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Image inpainting using the Cahn-Hilliard equation and its generalizations

Wednesday
15:00–15:30
HS 14

Andrej Novak

University of Zagreb, Faculty of Science, Zagreb, Croatia, Bijenička cesta 32,
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Digital image inpainting is the problem of reconstructing parts of an image based on information from surrounding areas such that the resulting changes are not trivially detectable by an ordinary observer. We approach this problem using the modified Cahn-Hilliard equation, where the standard double-well potential is replaced with the shock filter. Using fixed point arguments and Aubin-Lions lemma we prove the existence and uniqueness of the solution. In addition, we introduce a numerical method based on the convexity splitting idea to approximate the solutions of the considered problem. We apply this method to several binary images and demonstrate that this approach naturally extends image features and preserves their edges.

Generalized holomorphic functions

Friday
12:00–12:30
HS 16

Sekar Nugraheni

University of Vienna

Generalized smooth functions (GSF) theory is a branch of Colombeau theory where generalized functions share a number of fundamental properties with smooth functions, in particular with respect to composition and nonlinear operations. GSF allow us to prove a number of analogues of theorems of classical analysis: e.g., mean value theorem, intermediate value theorem, extreme value theorem, Taylor's theorem, local and global inverse function theorem, integrals via primitives, multidimensional integrals, theory of compactly supported functions, [1]. Our main aim is to update [3] with the present theory of GSF in order to develop a similar theory of generalized function of a complex variable.

We first see how to define a generalized holomorphic function (GHF) without already assuming the Cauchy-Riemann equations, but using a natural definition of complex differentiability. This goal can be accomplished as in [2] using two gauges to define a new notion of hyperlimit and of little-oh. On the other hand, we also use the better theory of integration of GSF, if compared with the one in Colombeau theory, to define and study the integral over a path of a GHF. We expect that several classical theorems of differential and integral calculus can be extended from the ordinary holomorphic case to the generalized holomorphic framework, and we present the results obtained so far.

Joint work with: Paolo Giordano, University of Vienna, Faculty of Mathematics.

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Solutions of low regularity to nonlinear wave equations

Tuesday
17:30–18:00
HS 14

Michael Oberguggenberger

University of Innsbruck

This talk addresses solutions of low regularity to nonlinear wave equations of the form

$$\partial_t^2 u - \Delta u = \pm u^p, \quad u(\cdot,0) = u_0,\ \partial_t u(\cdot,0) = u_1 \tag{3}$$

in space dimension $n \geq 1$ with initial data $u \in H^s(\mathbb{R}^n)$, $u_1 \in H^{s-1}(\mathbb{R}^n)$; $p > 1$ is an integer. It is known (see e.g. [1] and references therein) that there is a critical regularity, namely $s_{\text{crit}} = \max\left(\frac{n}{2} - \frac{2}{p-1}, \frac{n+1}{4} - \frac{1}{p-1}, 0\right)$ when $n \geq 2$ and $s_{\text{crit}} = \frac{1}{2} - \frac{1}{p}$ for $n = 1$, such that, roughly speaking, problem (3) is locally ill-posed for $s < s_{\text{crit}}$ and locally well-posed for $s \geq s_{\text{crit}}$. (For certain ranges of n, p, s additional restrictions have to be imposed.)

The purpose of this talk is twofold. First, it will be shown that for any $s < \frac{n}{2} - \frac{2}{p-1}$ there are solutions to (3) for which propagation along bicharacteristics fails: propagation along any non-characteristic direction is possible. These examples extend the list of anomalous solutions given in [2]. Second, in one space dimension, well-posedness can be achieved below s_{crit} for solutions in $H^s_\Gamma(\mathbb{R})$, the subspace of elements of $H^s(\mathbb{R})$ whose Fourier transform is supported in the half-line $\Gamma = [0, \infty)$. In this case, the power term $\pm u^p$ is understood in the sense of Hörmander's wave front set criterion. More specifically, problem (3) is locally well-posed in $H^s_\Gamma(\mathbb{R})$ when $p = 2$ and $s > -\frac{1}{2}$; $p = 2$ or $p = 3$ when $s > 0$; any integer $p \geq 1$ when $s = \frac{1}{2}$.

The talk is based on ongoing joint work with Heiko Gimperlein.

References

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Thursday
14:00–14:30
HS 16

Distributional Methods for Low Regularity Singularity Theorems

Argam Ohanyan

Department of Mathematics, University of Vienna, Austria

Singularity theorems have been central in General Relativity since the seminal work of Penrose in the 1960s [3]. They predict the existence of incomplete causal geodesics under fairly reasonable assumptions. Whether this is the correct notion of "physical singularity" has been the subject of debate, as causal geodesic incompleteness may be avoided by extending the spacetime in a less regular manner. This has motivated the study of singularity theorems for metrics which are below C^2. The regularity class $C^{1,1}$ has been studied extensively over the past decade. Recently, C^1-metrics have been considered by Graf [1] where the Hawking and the Penrose theorem are proven in this regularity, and the Hawking–Penrose theorem was established soon thereafter [2].

In this talk, we will focus on the finer analytic methods that go into the proofs of the C^1-singularity theorems. As a C^1-metric gives rise to an (order 1) distributional curvature, one needs to carefully formulate the energy conditions which, as a consequence, imply surrogate conditions for approximating smooth metrics. We will analyze in detail the corresponding focusing results which form the technical core of the singularity theorems in low regularity.

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Monday
17:00–17:30
HS 14

Some remarks on $\mathcal{LF}$-spaces

Norbert Ortner

University of Innsbruck

Theorem (V. S. Retakh 1970, J. Wengenroth 1996)**.** The *compact regularity* of the $\mathcal{LF}$-space $E = \varinjlim_k E_k$ (a countable union of Fréchet spaces $E_k \subset E_{k+1}$) is characerized by *condition (M)*:

(M) $\forall k\ \exists l > k\ \forall m > l$ and $E_k \hookrightarrow E_l \hookrightarrow E_m$ the topologies of E_l adn E_m coincide on every zero-neighbourhood of E_k.

We prove **Proposition 1**: If F is a *quasinormable Fréchet space* then its strong dual F'_b is a compactly regular $\mathcal{LB}$-space.

Application. $\mathcal{D}'_{L^p} = \varinjlim_m (1-\Delta_n)^m * L^p$ is compactly regular.

Proposition 2. $\frac{1}{p} + \frac{1}{q} = 1$. The topology $\kappa(\mathcal{D}_{L^p}, \mathcal{D}'_{L^q}$ of uniform convergence on compact sets of $\mathcal{D}'_{L^q}$ can be described by the seminorms

$$\mathcal{D}_{L^p} \ni \varphi \mapsto p_{\alpha,g}(\varphi) = \|g\partial^\alpha \varphi\|_p, \qquad \alpha \in \mathbb{N}_0^n,\ g \in \mathcal{C}_0.$$

Consequence of the **Theorem**: The compact regularity of an $\mathcal{LF}$-space implies its *completeness*.

Application. The spaces $\mathcal{O}_C$ (Grothendieck 1955) and $\mathcal{P}_C$ (Yoshinaga 1967) are *complete*.

On the hypoellipticity of Kohn's sums of squares of complex vector fields.

Wednesday
09:00–10:00
HS 14

Alberto Parmeggiani

University of Bologna

In this talk, I will at first review the known results of hypoellipticity of sums of squares of complex vector fields. I will then focus on an alternative way to prove a subellipticity result by J. J. Kohn which allows lower order perturbations and a generalization to a pseudodifferential setting.

Invariance of the Fredholm Index of Non-Smooth Pseudodifferential Operators

Monday
14:00–14:30
HS 14

Christine Pfeuffer

Martin-Luther University of Halle-Wittenberg

As nearly invertible operators Fredholm operators play an important role in the field of partial differential equations in order to obtain existence and uniqueness results. Hence great effort already was spent to get some conditions for the Fredholmness of pseudodifferential operators. However, there are very few results for the invariance of the Fredholm index of such operators.

In the smooth case Schrohe was able to show under certain conditions, that the Fredholm index of smooth pseudodifferential operators is invariant considered as a map between certain weighted Bessel potential spaces with symbols in the Hörmander-class $S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$.

In applications also non-smooth pseudodifferential operators occur. The goal of this talk is to show the invariance of the Fredholm index for non-smooth pseudodifferential operators with symbols in the class $C^{\tilde{m},s}S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$. To reach this aim

we use the main idea of the result from Rabier about the Fredholm index for non-smooth differential operators. The main difficulty is to prove a regularity result for non-smooth pseudodifferential operators needed in the proof.

The talk is based on a joint work with H. Abels.

Generalized functions over the spaces with Hörmander metrics. Some results and some open problems

Monday
11:30–12:30
HS 14

Stevan Pilipović

University of Novi Sad

We consider generalized functions over $W = V \times V'$ a sympletctic space with the positive-definite Borel measurable symmetric covariant 2-tensor field $W \ni X \mapsto g_X$ defining Hörmander admisible metric.

A joint work with Prangoski concerning ΨDOs over distribution spaces and the ones planned by Toft concerning ultradistribution spaces and global ΨDOs will be presented.

More precisely, the following results will be explained. 1. The ellipticity and the Fredholm property of a ΨDO acting on Sobolev spaces in the Weyl-Hörmander calculus are equivalent under relevant conditions. 2. The motivation for the geometric short-time Fourier transform and the characterisation of the Weyl-Hörmander classes $S(M, g)$ is given through the spectral invariance of some classes of matrix type operators. 3. The spaces of ultradistributions related to the Gevrey class $p!^s$ are introduced over W for $s > 1$.

Some open problems will be indicated.

Characterisation of the Weyl-Hörmander classes by time-frequency shifts

Wednesday
10:30–11:30
HS 14

Bojan Prangoski

Ss. Cyril and Methodius University in Skopje

Inspired by the characterisation of the Sjöstrand symbol classes in [2], Gröchenig and Rzeszotnik [1] gave the following characterisation of the Hörmander class $S^0_{0,0}(\mathbb{R}^{2n})$. A tempered distribution $a \in \mathcal{S}'(\mathbb{R}^{2n})$ belongs to $S^0_{0,0}(\mathbb{R}^{2n})$ if and only if for every $s > 0$ there is $C_s > 0$ such that

$$|\langle a^w \pi(X)\chi, \overline{\pi(\Xi)\chi}\rangle| \leq C_s(1 + |X - \Xi|)^{-s}, \quad \text{for all } X, \Xi \in \mathbb{R}^{2n};$$

here $\chi \in \mathcal{S}(\mathbb{R}^n)\backslash\{0\}$ and $\pi(X)\chi(y) = e^{2\pi i\xi y}\chi(y - x)$ for $X = (x, \xi) \in \mathbb{R}^{2n}$. In this talk, we present a generalisation of this result which characterises the elements of

the Weyl-Hörmander symbol classes $S(M,g)$ with g a Hörmander metric on $\mathbb{R}^{2n}$ and M a g-admissible weight. When g is the Euclidean metric and $M = 1$, $S(M,g)$ is just $S^0_{0,0}(\mathbb{R}^{2n})$ and, in this case, our result reduces to the above result of Gröchenig and Rzeszotnik [1].

The talk is based on collaborative works with Stevan Pilipović.

References

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Application of Colombeau semigroups of operators in solving some fractional PDEs

Tuesday
15:00–15:30
HS 14

Danijela Rajter-Ćirić

University of Novi Sad

We present an approach that uses the theory of Colombeau semigroups of operators in solving some fractional PDEs. We illustrate the application of developed theory by considering reaction-advection-diffusion equations with space fractional derivatives, inhomogeneous fractional evolution equations, time and time-space fractional wave equations with variable coefficients, and finally, deterministic and stochastic fractional heat equations with variable thermal conductivity and multiplicative noise. In all those cases, we prove that there exists a unique solution to the problem within a certain Colombeau generalized function space.

This talk is a result of joint work with Miloš Japundžić.

References

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Monday
14:30–15:00
HS 14

Pseudo-differential τ-calculus on graded groups

David Rottensteiner

Ghent University

We present a pseudo-differential τ-calculus for the Hörmander symbol classes on graded groups G that applies to a wide range of both novel and very well-known quantization schemes. The admissible quantizing functions τ, which determine the type of quantization, can be very general polynomial functions from G to G. Our calculus recovers the known asymptotic expansions of adjoint and composite symbols and mapping properties of the associated PsiDO's for, e.g., the Kohn-Nirenberg calculus on graded groups [1], hence its special case on $\mathbb{R}^n$, the Weyl calculus on $\mathbb{R}^n$, and more generally the τ-calculus on $\mathbb{R}^n$, for which $\tau(x) = \tau x$ for $\tau \in [0, 1]$.

Among the novel quantization schemes to which the calculus applies we discuss in detail one specific type, which for $G = \mathbb{R}^n$ coincides with the Weyl quantization. On a large class of Lie groups, which contains the Heisenberg group $\mathbb{H}_n$ and $\mathbb{R}^n$, this quantization scheme preserves many of the most relevant properties of the Euclidean Weyl quantization regarding self-adjoint operators, symplectic invariance, Poisson brackets, etc.

This is based on joint-work with Serena Federico and Michael Ruzhansky.

References

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Tuesday
10:30–11:3
HS 14

Very weak well-posedness for some classes of evolution equations

Michael Ruzhansky

Ghent University, Belgium, and Queen Mary University of London, UK

In this lecture we will give an overview of the recent work on the very weak well-posedness of evolution equations with (very) singular time- and space-dependent coefficients. In particular, we will focus on the hypoelliptic heat, wave, and Schrödinger equations with singular propagation speed and singular potentials.

Topologies and comparison of regularities for Debrouwere's algebra of generalized functions

Tuesday
14:00–14:30
HS 16

Dimitrios Scarpalezos

Université Paris Diderot

In his thesis Andreas Debrouwere introduced algebras in which Beurling and Roumieu ulradistributions could be embedded, and gave a notion of regularity such that if an ultradistribution is embedded into a regular element of the corresponding kind then it is ultradifferentiable.

We introduce an adapted sharp topology for those algebras such that the notion of well posed problems for equations with data in the algebra has a meaning, and introduce some criteria for that. We also introduce notions of strong and strict associations and prove that if a Beurling ultradistribution is strongly associated to a regular Debrouwer's generalized function then it is an ultradifferentiable function of the corresponding kind, and that if a Roumieu ultradistribution is strictly associated to a regular element of the corresponding kind then it is ultradifferentiable.

A comparison of two ways to generalize ultradifferentiable classes defined by weight sequences

Friday
11:30–12:30
HS 14

Gerhard Schindl

University of Vienna

In this talk we compare two recent developed methods to generalize classes of ultradifferentiable functions defined in terms of weight sequences.

The first one is the weight matrix approach, introduced by A. Rainer and G. Schindl, which deals with a (countable) family of weight sequences. This idea enables to treat both the weight sequence and the weight function setting (i.e. both classical settings) in a unified way.

The second method replaces the appearing geometric factor in the defintion by a more general expression involving a so-called exponent sequence. Such classes, for polynomially growing exponent sequences and particular weight sequences, have been introduced and studied in several papers by S. Pilipović, N. Teofanov and F. Tomić.

We prove that, under a mild growth restriction on the exponent sequence, the second method is a special case of the weight matrix approach and we also compare available results for both generalizations.

This is based on joint work with Javier Jiménez-Garrido (Universidad de Cantabria, Spain) and David Nicolas Nenning (University of Vienna).

Thursday
09:00–10:00
HS 14

Two approaches to generalized stochastic processes and singular stochastic differential equations

Dora Seleši

University of Novi Sad

In this talk I will present a comparison of two fundamental functional–analytical approaches to deal with singular stochastic differential equations. The first one is Hida's white noise theory, based on chaos expansions on rigged Hilbert spaces. The second one is the Colombeau approach making use of algebras of generalized stochastic processes. Both theories allow to deal with nonlinearities and singular coefficients in SDEs and SPDEs: either by Wick products and renormalization techniques or by ordinary products and regularization techniques.

References

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Tuesday
14:30–15:00
HS 16

On boundedness of composition operators acting on Besov spaces on the real line

Koichi Taniguchi

Tohoku University

In this talk, we consider the characterizations of a mapping φ from $\mathbb{R}$ to itself for which a linear operator

$$C_\varphi : f \mapsto f \circ \varphi$$

is bounded on the Besov space $B^s_{p,q}(\mathbb{R})$ with $s > 0$ and $0 < p, q \leq \infty$. If φ is nonsingular, i.e., $|\varphi(E)| = 0$ for any measurable null set $E \subset \mathbb{R}$, then C_φ is well-defined and is called a composition operator induced by φ. The composition operator is a basic operator appeared in various situations in mathematics and physics, and

its boundedness has been studied on various function spaces. For the boundedness of C_φ on $B^s_{p,q}(\mathbb{R})$ with lower derivative order $0 < s < 1$, the necessary and sufficient conditions on homeomorphisms φ have been clarified in G. Bourdaud and W. Sickel [2], G. Bourdaud [1] and references therein (the critical case $s = 1/p$ seems to be only partially solved). In contrast, there is almost no research on the higher order case $s > 1$, and the necessary and sufficient conditions have not been clarified to our knowledge. The purpose of this talk is to study the boundedness of C_φ in the case $s > 1$, and our main result is the necessary and sufficient conditions in the case $s > 1 + 1/p$. The proof uses the characterizations of the set of all pointwise multipliers of $B^s_{p,q}(\mathbb{R})$ given in V.K. Nguyen and W. Sickel [3]. This talk is based on the joint work with I. Ishikawa (Ehime University) and M. Ikeda (RIKEN/Keio University).

References

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Continuous frames for tensor product Hilbert spaces

Monday
16:30–17:30
HS 14

Nenad Teofanov

University of Novi Sad

The frame theory became an important part of harmonic analysis in the last 2 decades both in its discrete and continuous settings. In this lecture we will present recent results related to representation of dual frames for continuous frames and their natural extension to tensor products of Hilbert spaces. We proceed with the study of frame multipliers defined by a fixed multiplication pattern (the symbol/mask) inserted between the analysis and synthesis operators. In particular, we consider bilinear localization operators as examples of continuous frame multipliers.

A specific feature in the context of tensor products is the notion of partial trace. We will show how the corresponding partial trace theorem offers an interpretation of tensor product continuous frame multipliers as density operators for bipartite quantum systems.

This is joint work with Peter Balazs.

Non-Standard Version of Egorov's Algebra of Generalized Functions

Monday
14:00–14:30
HS 16

Todor D. Todorov

California Polytechnic State University, San Luis Obispo, California 93407, USA
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We consider a non-standard version of Egorov's algebra of generalized functions, which improves the properties of the generalized scalars and the embedding of Schwartz distributions in the original standard version. The embedding of distributions is similar to, but different from author's works in the past and independently done by Hans Vernaeve. The emphases is on the simplicity and potential for axiomatization.

References

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Fractional Fourier transform, harmonic oscillator propagators and Strichartz estimates

Monday
16:00–16:30
HS 14

Joachim Toft

Linnæus University, Växjö, Sweden

The talk is based on the content of [6]. We show that harmonic oscillator propagators and fractional Fourier transforms are essentially the same. We deduce continuity properties for such operators on modulation spaces, and apply the results to prove Strichartz estimates for the harmonic oscillator propagator when acting on modulation spaces. Especially we extend some results in [1,2,3,4]. We also show that general forms of fractional harmonic oscillator propagators are continuous on suitable Pilipović spaces. Especially we show that fractional Fourier transforms of any complex order can be defined, and that these transforms are continuous on any Pilipović space and corresponding distribution space, which are *not* a Gelfand-Shilov space or their distribution space. Parts of these investigations go back to [5].

The talk is based on a joint work with Divyang Bhimani and Ramesh Manna.

References

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(ω, ρ)-Periodic solutions of impulsive differential equations on Banach space

Tuesday
15:00–15:30
HS 16

Daniel Velinov

Ss. Cyril and Methodius University in Skopje

The (ω, c)-periodic functions ($c \in \mathbb{C}$) were introduced and extensively studied in the past few years by many mathematicians [1]. As a type of periodic functions, they are quite general and include the most exploited classes of periodic functions, such as periodic, antiperiodic, Bloch periodic and almost periodic. Recently, by M. Fe.ckan, K. Liu, J. R.Wang in [2], this notation was extended into (ω, ρ)- periodic functions, where $\rho : X \to X$ is a linear isomorphism. We present some results on the solutions of the nonhomogeneous linear impulsive equations and certain useful estimates for the further investigations. We use Banach fixed point theorem and Schauder fixed point theorem to show existence and uniqueness of the (ω, ρ)-solutions for certain impulsive differential equations [3].

The presented results are based on a joint work with Marko Kostić and Michal Fečkan.

References

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Friday
14:00–15:00
HS 14

Quantum Field Theory on low-regularity spacetimes

James Vickers

University of Southampton

In this talk I will describe the mathematics required in order to describe quantum fields on low-regularity spacetimes using algebraic quantisation. In particular I consider the case of a massless scalar field ϕ on a globally hyperbolic spacetime M with $C^{1,1}$ metric g. In the first part of the talk I will show how Colombeau regularisation methods can be used to show that the (classical) Cauchy problem for the wave equation is well-posed for initial data and sources in suitable Sobolev spaces. This enables one to construct low-regularity advanced and retarded Green operators $G^{\pm}$ as maps $G^{\pm} : H^1_{\text{comp}} \to H^2_{\text{loc}}$ where we note that these Sobolev spaces are independent of the choice of background metric used in the definition. In specifying the relevant function spaces we need to control the norms of both ϕ and $\Box_g \phi$ in order to ensure that $\Box_g \circ G^{\pm}$ and $G^{\pm} \circ \Box_g$ are the identity maps on those spaces. The causal propagator $G = G^+ - G^-$ is then used to define a symplectic form ω on a normed space $V(M)$ which is shown to be isomorphic to $\ker(\Box_g)$. This enables one to provide a locally covariant description of the quantum fields in terms of the elements of quasi-local C^*-algebras. In particular one can construct a functor QUANT from the space of globally hyperbolic manifolds with Green operators $G^{\pm}$ to the space of quasi-locla C^*-algebras that satisfies the Haag–Kastler axioms.

In order to construct a full quantum field theory in a low regularity spacetime, a suitable choice of quantum states must be made which in the algebraic quantisation method correspond to normalised positive linear functionals on the quasi-local C^*-algebra. The standard choice for the physical quantum states in the smooth case are given by the Hadamard states which are define by a microlocal spectrum condition on the wavefront set of the two point function. As a consequence of the construction these posses a natural Fock space structure which agrees with the standard construction for quantum fields on Minkowski space. In the low regularity setting

we require a generalisation of Hadamard states. A larger class of states, called adiabatic states of order N and characterised in terms of their Sobolev wavefront set, has been obtained by Junker and Schrohe. These states are natural candidates to replace the Hadamard states in spacetimes with limited regularity. We end the talk by outlining this procedure for the case of low-regularity ultrastatic spacetimes.

This is joint work with Günther Hörmann, Yaftet Sanchez Sanchez and Christian Spreitzer.

References

[1] G. Hörmann, Y. Sanchez Sanchez, C. Spreitzer and J.A. Vickers, Green operators in low regularity spacetimes and quantum field theory, Class. Quantum Grav. **37** 175009 (2020).

Generalised solutions to linear and non-linear Schrödinger-type equations

Monday
14:30–15:00
HS 16

Ivana Vojnović

Department of Mathematics and Informatics, Faculty of Sciences, University of Novi Sad

We consider a semi-linear Schrödinger equation of Hartree type in three spatial dimensions and its various approximations of singular, point-like perturbations. The corresponding nets of approximate solutions represent generalised solutions for the singular-perturbed Schrödinger equation. The behaviour of such nets is investigated.

We also study a generalised solution in the Colombeau algebra and for such a solution compatibility with the classical Hartree equation is established, in the sense of the Colombeau generalised solution theory.

This is joint work with Nevena Dugandžija and Alessandro Michelangeli.

References

[1] S. Albeverio, J.E. Fenstad, and R. Høegh-Krohn, Singular perturbations and nonstandard analysis, Trans. Amer. Math. Soc., 252 (1979).

[2] T. Cazenave, Semilinear Schrödinger equations, vol. 10 of Courant Lecture Notes in Mathematics, New York University Courant Institute of Mathematical Sciences, New York, (2003).

[3] A. Michelangeli, A. Olgiati, and R. Scandone, Singular Hartree equation in fractional perturbed Sobolev spaces, J. Nonlinear Math. Phys., 25 (2018).

Thursday
10:30–11:30
HS 14

Anisotropic global microlocal analysis

Patrik Wahlberg

Politecnico di Torino

We define an anisotropic Gabor wave front set for tempered distributions, parametrized by one positive parameter. This wave front set measures the lack of super-polynomial decay of the short-time Fourier transform along power type curves in phase space. With a corresponding anisotropic modification of Shubin's symbols for pseudodifferential operators we develop a pseudodifferential calculus, and show a microlocal and a microelliptic inclusion. Finally we show a result on propagation of singularities that we apply to a class of evolution PDEs.

This project is a collaboration with L. Rodino.

Wednesday
15:00–15:30
HS 16

Generalized integrated semigroups

Milica Žigić

Faculty of Sciences, University of Novi Sad

In this talk, we consider the abstract Cauchy problem $d/dt\ u = Au + f$, $u(0) = x$, where A is a net of generators of integrated semigroups, so that it allows A to have singularities, and the initial condition x and the inhomogeneous part f are generalized functions. The relations between generalized infinitesimal generators, their resolvents and integrated semigroups are investigated. The results are illustrated by the inhomogeneous wave equation with singular initial conditions and source function. This is a joint work with Stevan Pilipović.
