That which is not forbidden is compulsory. – T.H. White

That which is not forbidden is compulsory.

Unless it is extremely unlikely.

Free surface flows: Topological: Wave crater collapse

Axisymmetric:

$\delta \sim (t_o - t)^{2/3}$ length scales

$v \sim (t_o - t)^{-1/3}$ velocity scales

$Pr(v) \sim |v|^{-4}$ velocity probability distribution

Similarity solutions appear to exist

2-D like axisymmetric!

Quantum fluid flows: Topological: Vortex reconnection

$\delta \sim (t_o - t)^{1/2}$ length scales

$v \sim (t_o - t)^{-1/2}$ velocity scales

$Pr(v) \sim |v|^{-3}$ velocity probability distribution in turbulence

Similarity solutions exist

Fixed points with reconnection geometry exists

Safety!

Navier-Stokes flows: ?: turbulence

$Pr(v_i) \sim \exp(-v_i^2/\sigma_v^2)$

$Pr(\partial_j v_i) \sim \exp(-|v_i|/\sigma_s)$

$Pr(\epsilon)$ dissipation interesting!

$Pr(\Omega)$ enstrophy interesting!

Euler flows: ?: ?

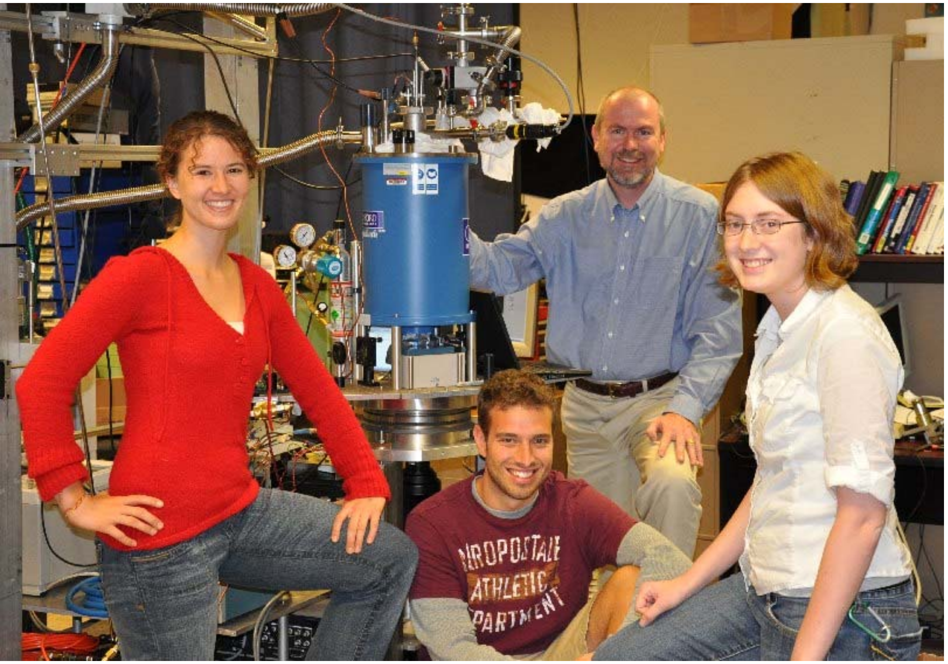
Fixed points associated with reconnection?

Danger!

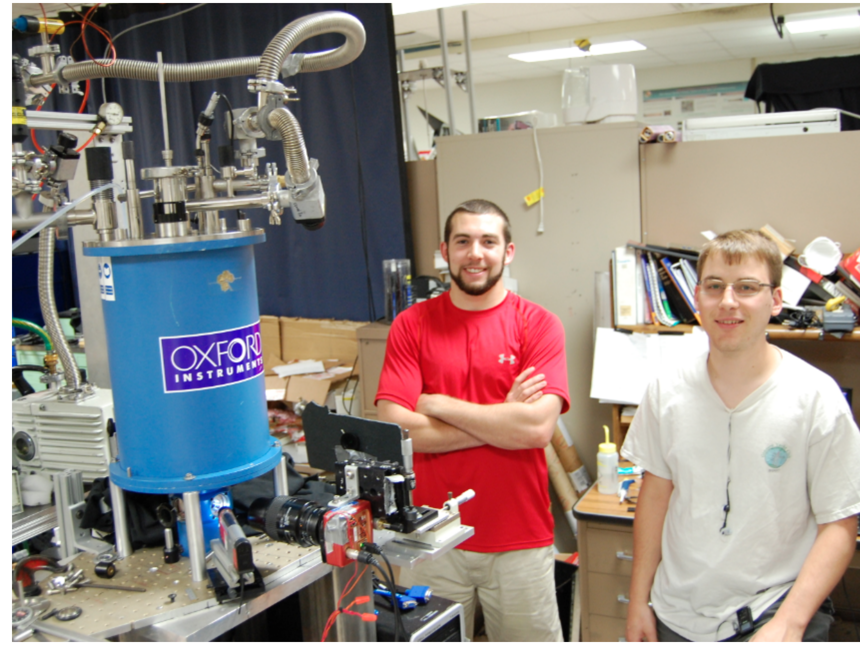
Singular events in fluid flow due to topological change

Dan Lathrop

University of Maryland
National Science Foundation



Kristy (Gaff) Johnson   **Matt Paoletti**   **Kaitlyn Tuley**

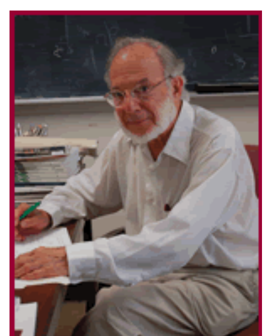


Chris Boughter and David Meichle

Enrico Fonda

Greg Bewley

K.R. Sreenivasan

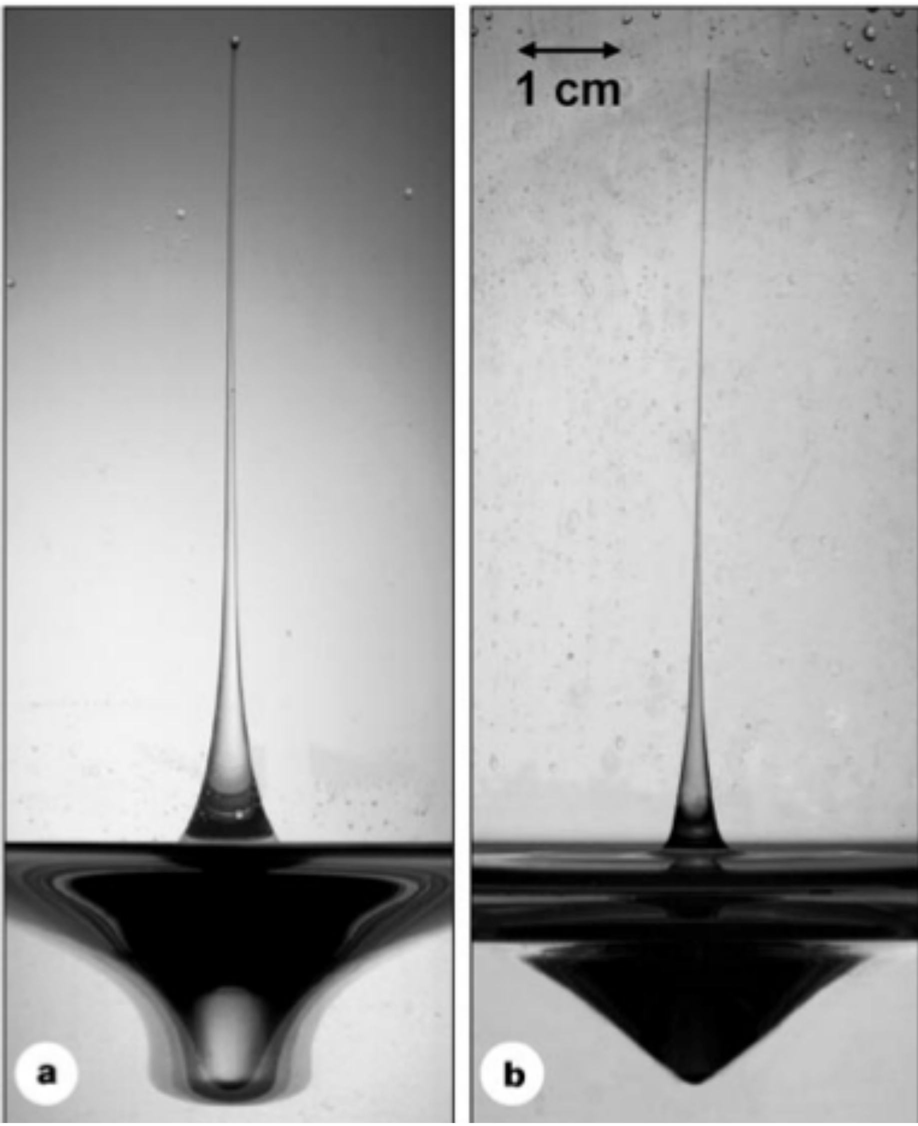


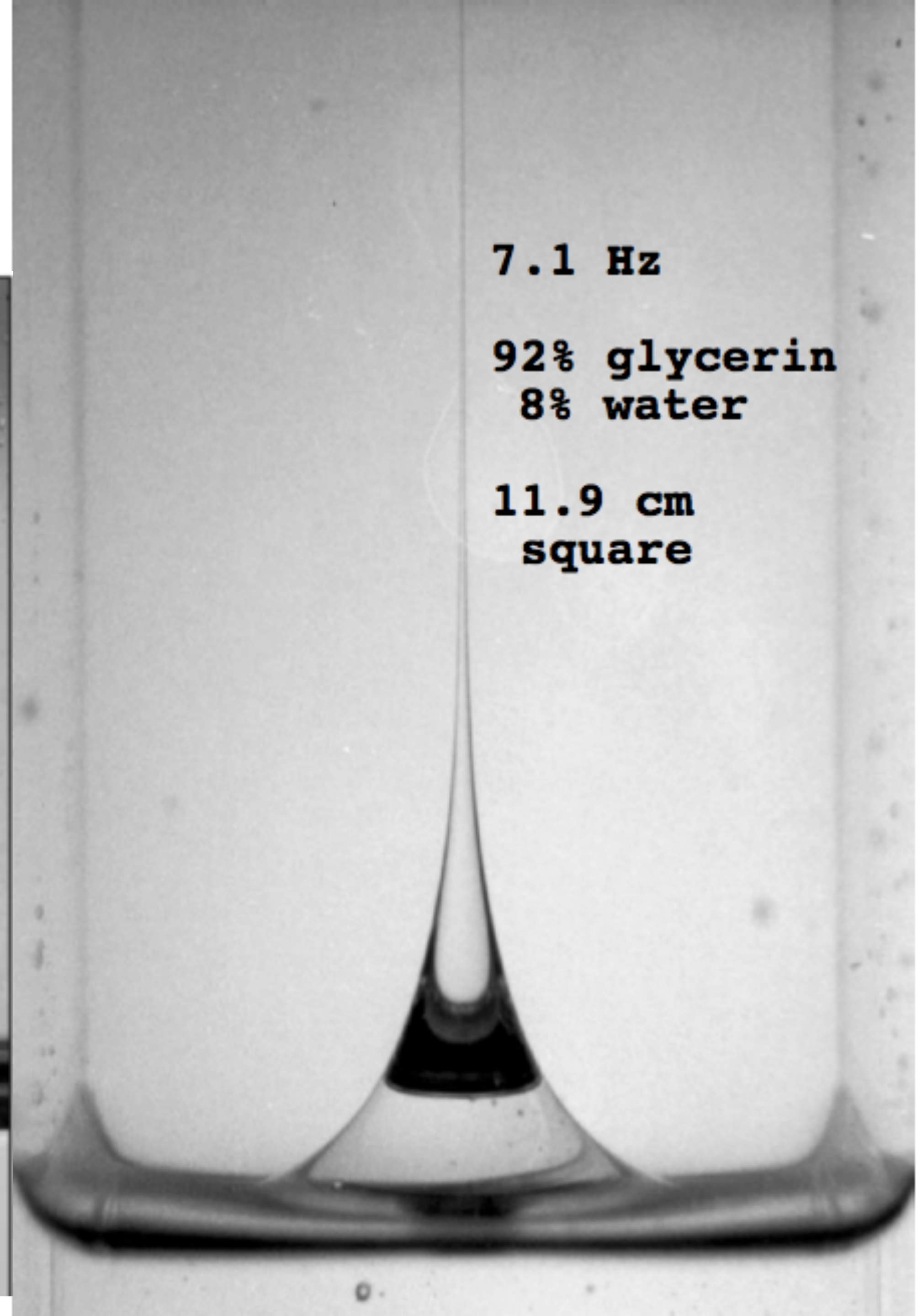
Michael Fisher

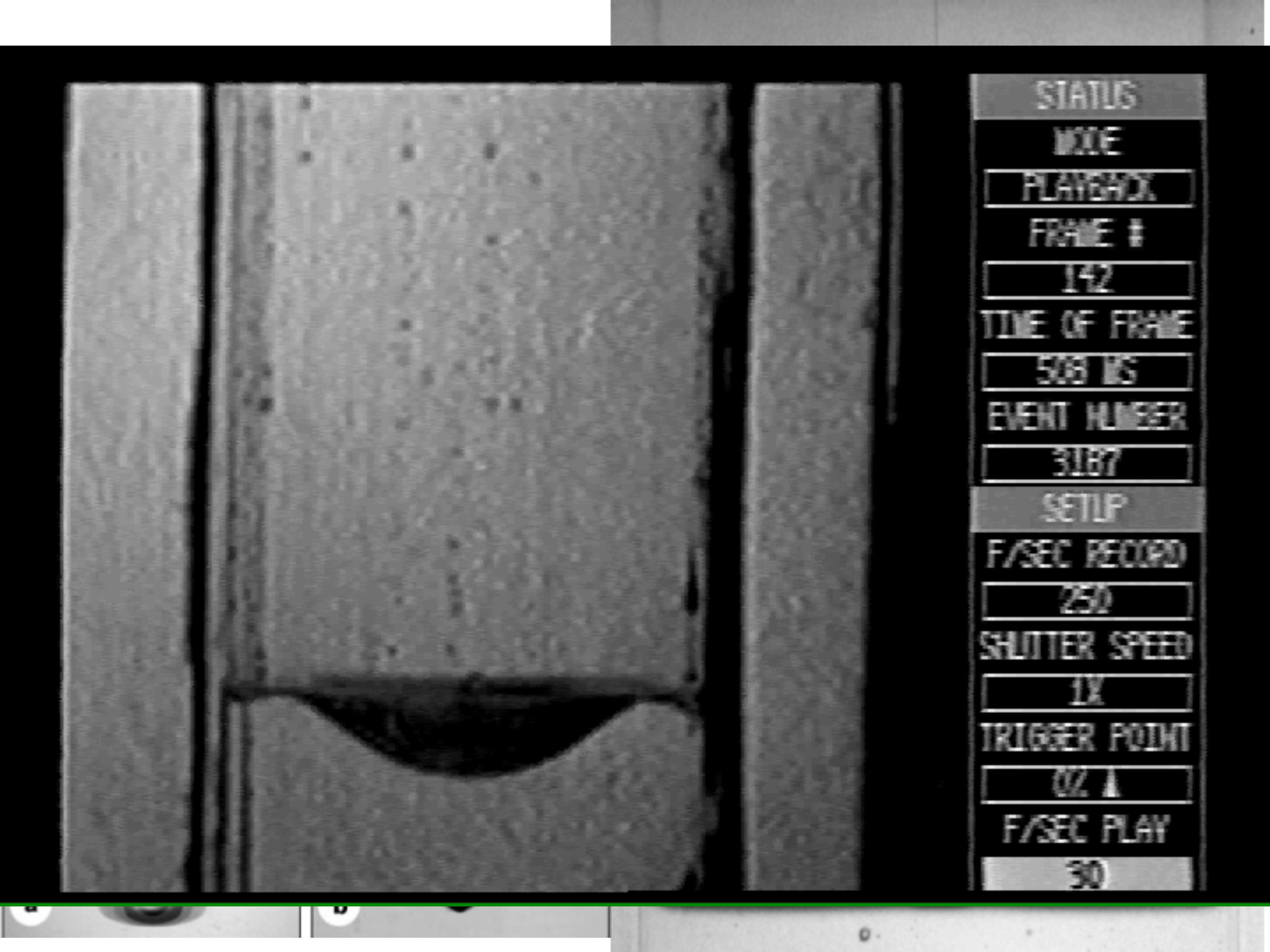
Outline:

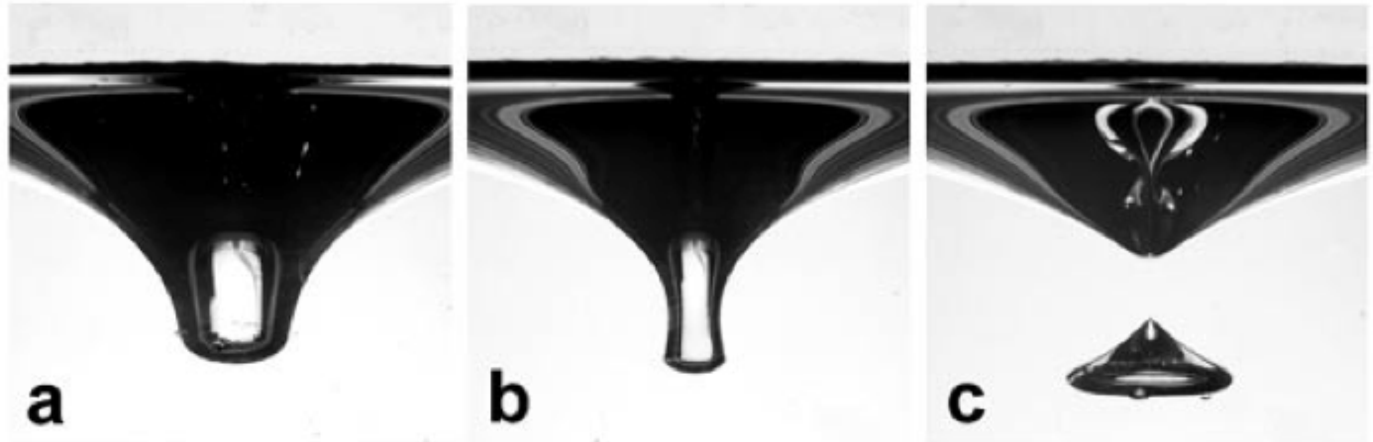
1) **Free surface singularities**
2) **Quantum vortices**
3) **Navier-Stokes turbulence**
4) **Euler flow**

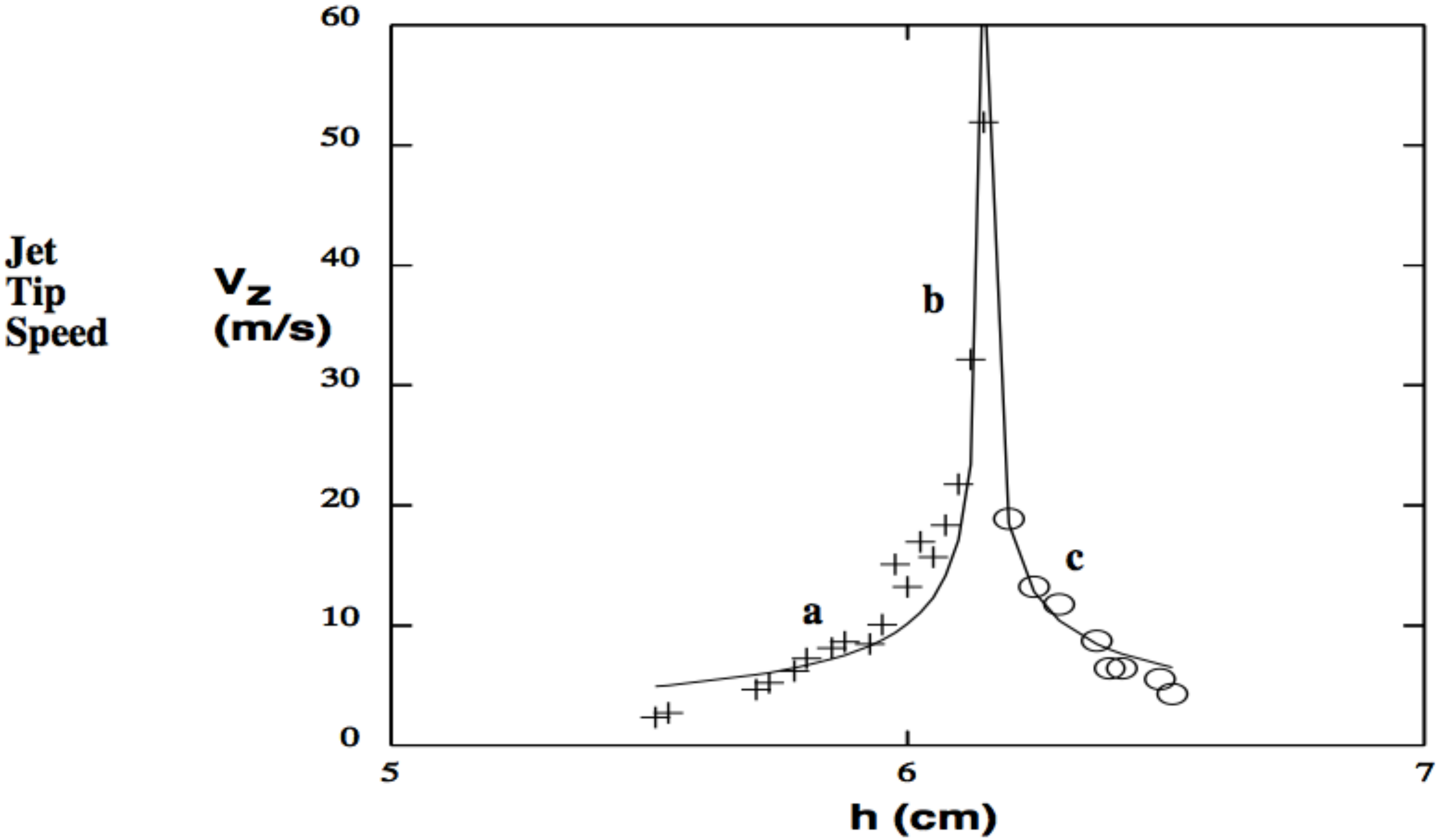
Zeff et al., Nature 2000



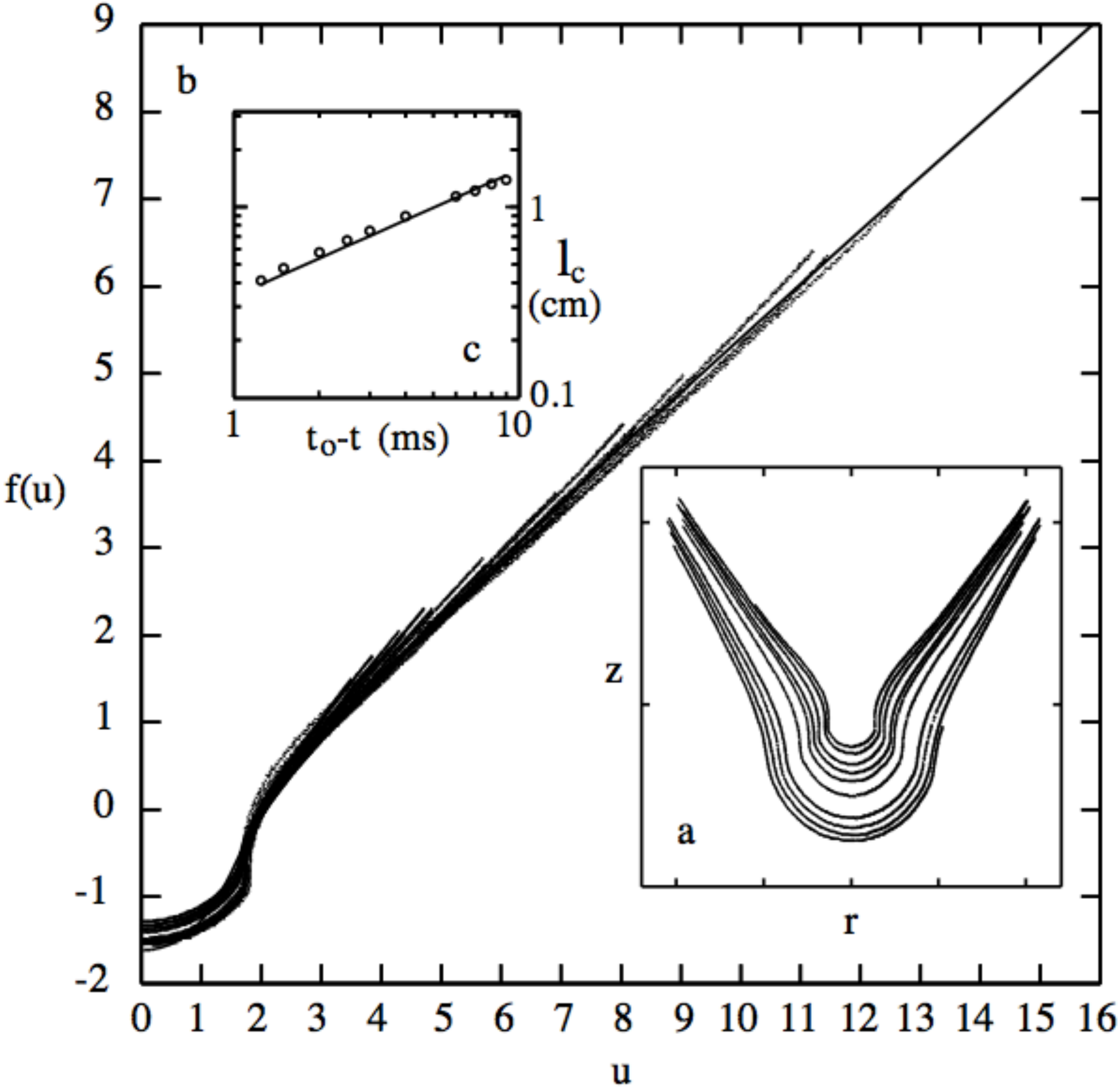



STATUS
MODE
PLAYBACK
FRAME #
142
TIME OF FRAME
568 MS
EVENT NUMBER
3187
SETUP
F/SEC RECORD
250
SHUTTER SPEED
1X
TRIGGER POINT
0%
F/SEC PLAY
30


a
b
c


Jet
Tip
Speed
v_z
(m/s)
60
50
40
30
20
10
0
5
6
7
h (cm)
a
b
c

Test of Similarity Solution



$f(u) = z/\tau^{2/3}$

$u = r/\tau^{2/3}$

Similarity Solution $t < 0$

velocity $\vec{u} = \vec{\nabla}\phi$

height $h(r,t)$

$$\nabla^2\phi = 0 \tag{1}$$

$$\frac{\partial h}{\partial t} + (\vec{u}\cdot\vec{\nabla})h = u_z \tag{2}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + \frac{\sigma}{\rho}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0 \tag{3}$$

Similarity Ansatz arount $\tau = (t_\circ - t) = 0$

$$\begin{aligned} h(r,t) &= \tau^\alpha f\left[r/\tau^\alpha\right] = f\left[u\right] \\ \phi(r,z,t) &= \tau^\gamma g\left[r/\tau^\alpha, z/\tau^\alpha\right] = \tau^\gamma g\left[u,v\right] \end{aligned}$$

Similarity Equations

$\alpha=2/3 \quad \gamma=1/3$

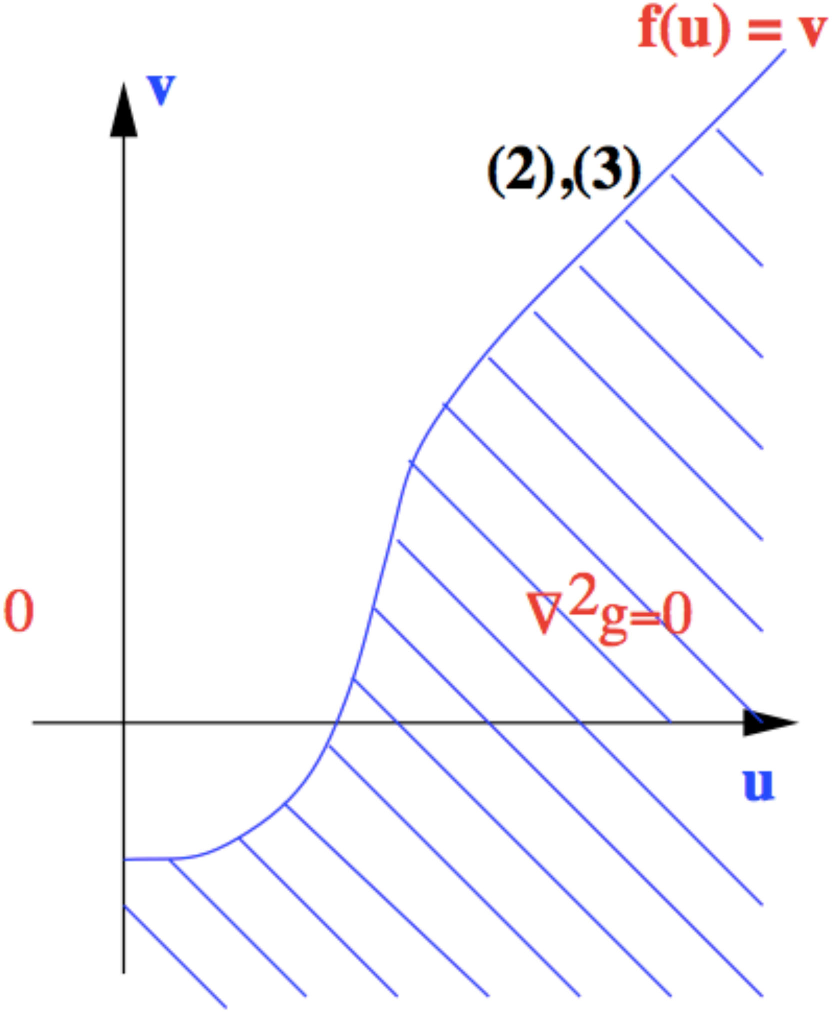
Time drops out of problem

$$u = r / (-t)^{2/3}$$
$$f(u) = v = z / (-t)^{2/3}$$
$$g(u,v) = (-t)^{1/3}\, \phi(r,z,t)$$

(1) $\nabla^2 g = 0$

(2) $\dfrac{\partial f}{\partial u} = \dfrac{g_v - 2f/3}{g_u - 2u/3}$

(3) $-\dfrac{g}{3} + \dfrac{2u}{3} g_u + \dfrac{2f}{3} g_v + \dfrac{1}{2}(\nabla g)^2 + \kappa = 0$



Asymptotics

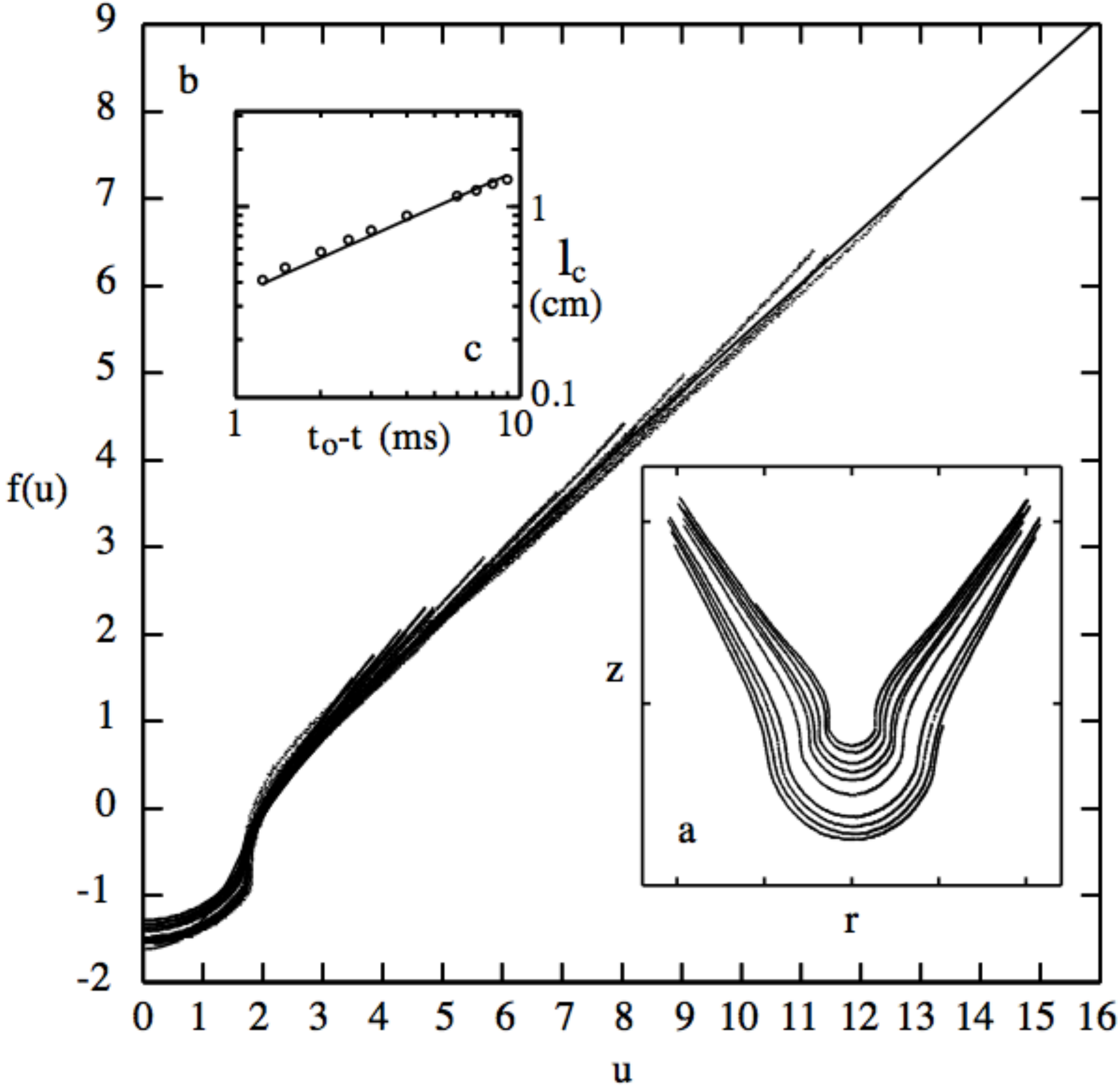
large radius --> cone

$f \sim u$ **angle free**

$g \sim u^{1/2}$

small radius --> regular

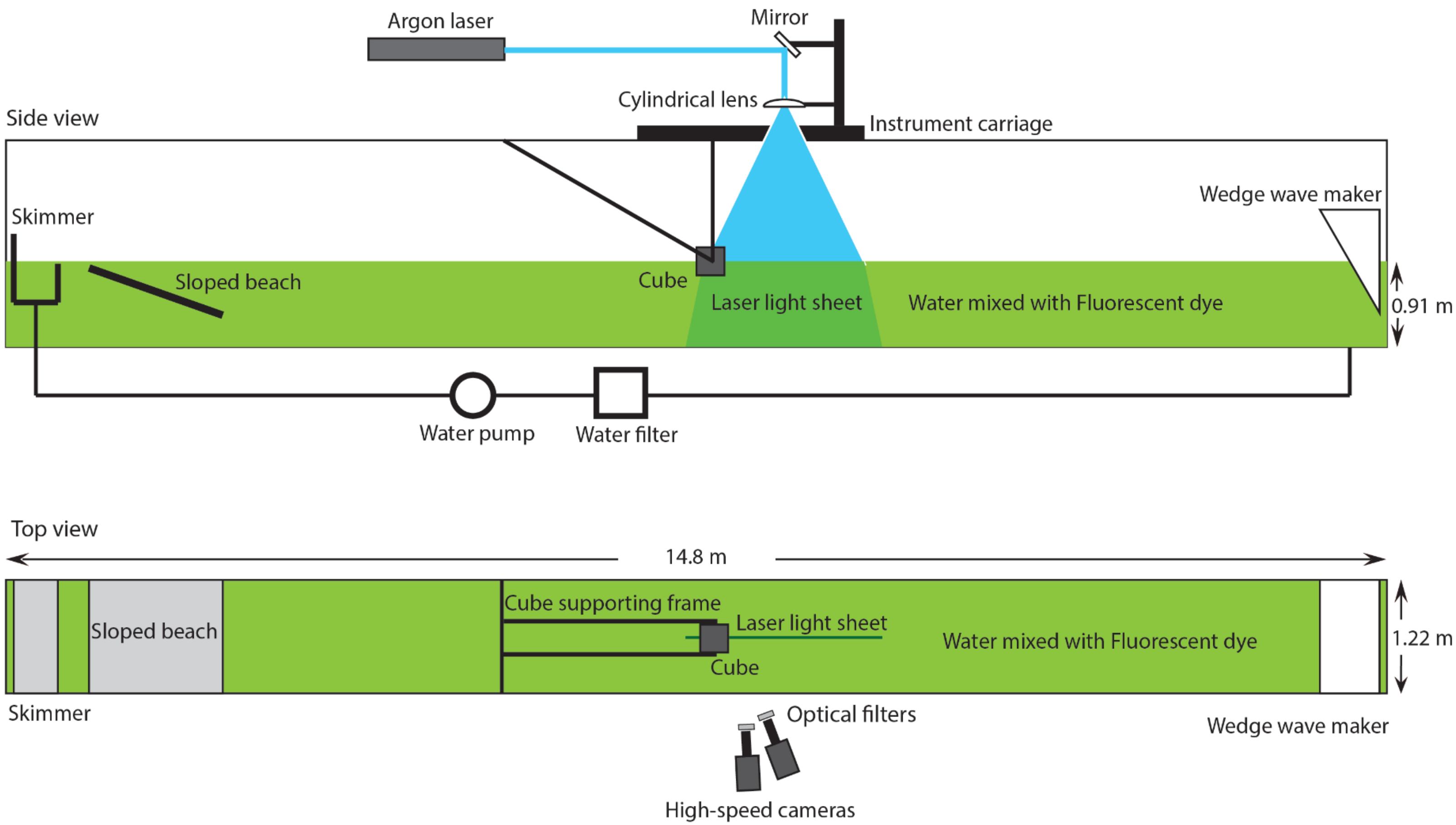
Test of Similarity Solution



$f(u) = z/\tau^{2/3}$

$u = r/\tau^{2/3}$

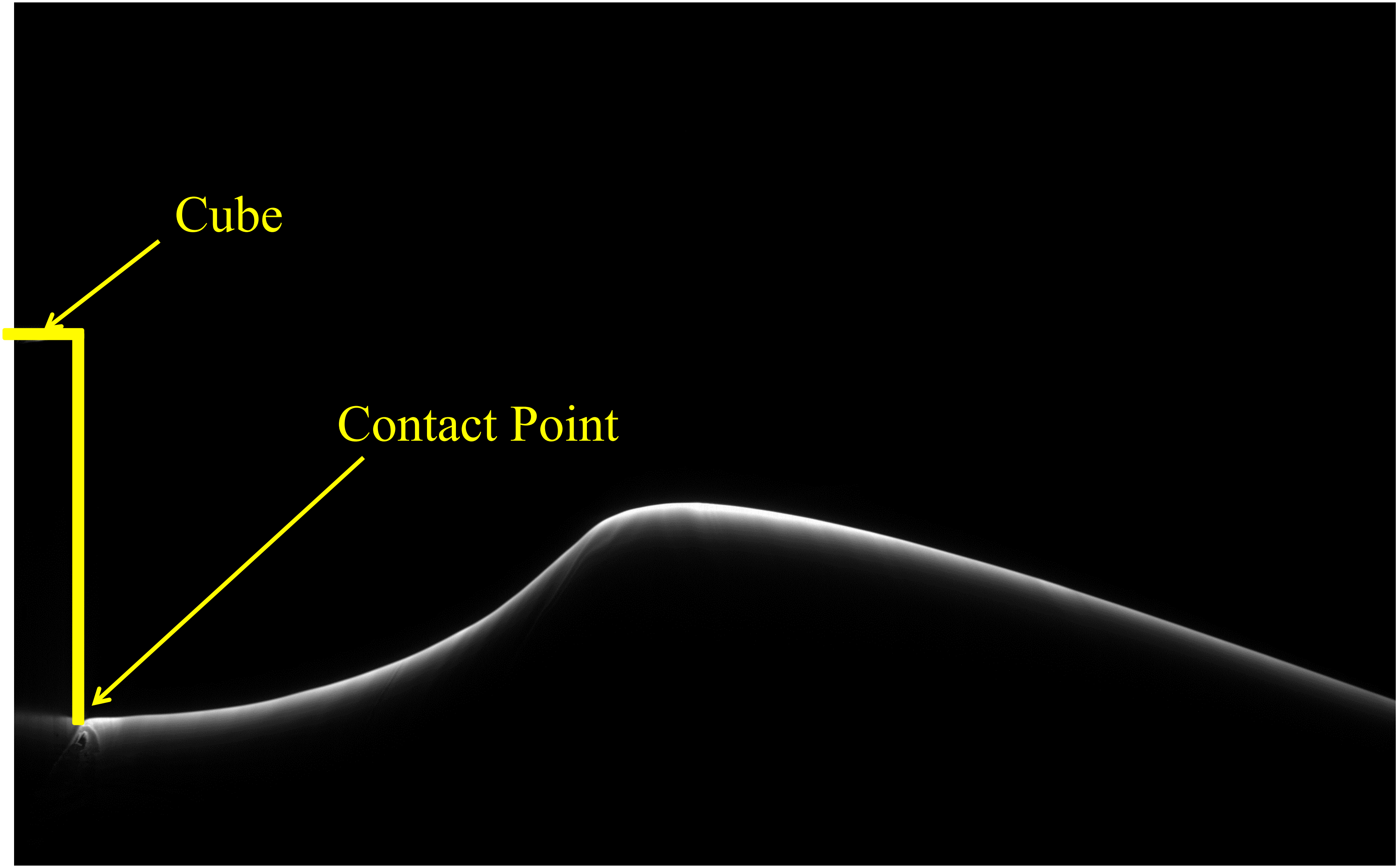
- **2D wave collapse: An Wang, James H. Duncan and Daniel P. Lathrop**



- The cube (L = 30.5 cm) is rigidly mounted at center plane of the wave tank

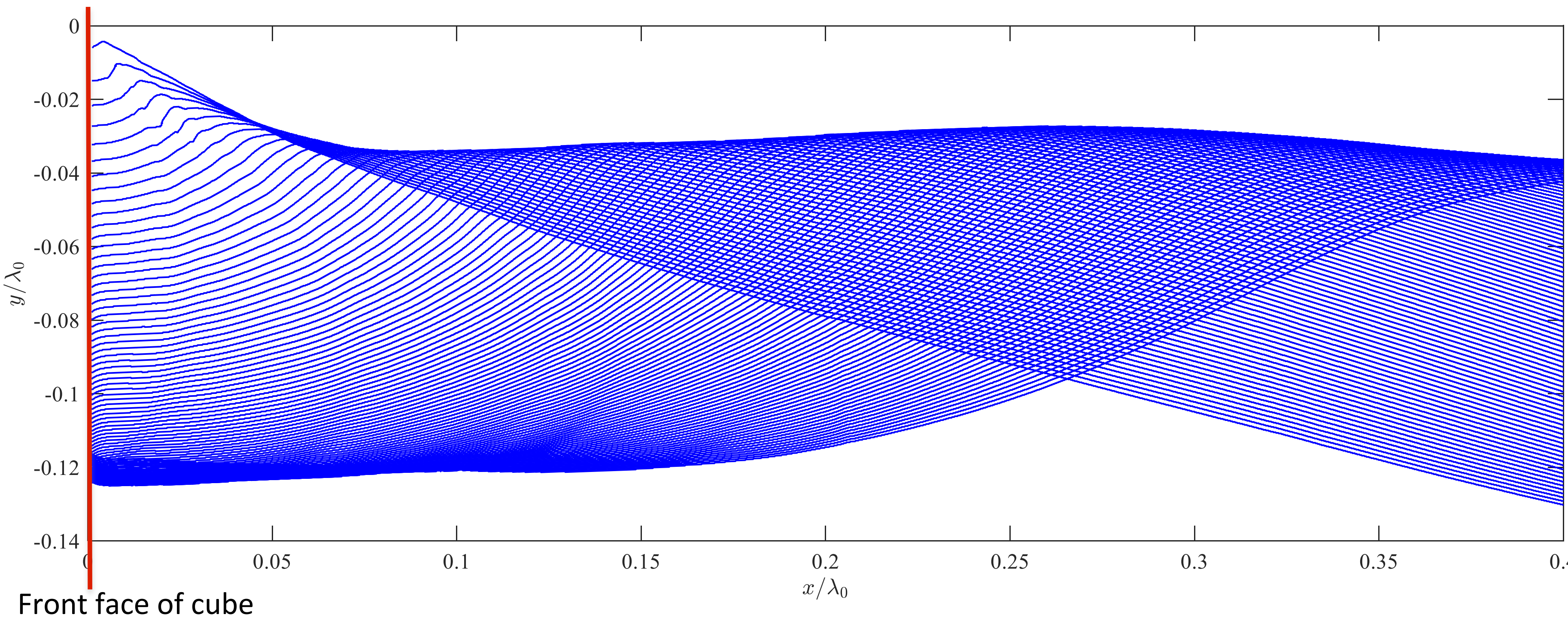
High-speed Movie of Wave Impact

(1500 pps, played at 15 fps, field of view 53 cm)



Water Surface Profiles

- The profiles are equally spaced in time, measured at a frame rate of 1500 pps and plotted every 5 frames.
- The last profile represent the **moment of impact** (the water surface between the contact point and the crest collapse to a point and the surface becomes a straight line.)



Collapse in 2–D   Wave–wall interaction

(1) $\nabla^2\phi = 0$

(2) $\frac{\partial h}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial h}{\partial x} = \frac{\partial \phi}{\partial z}$

(3) $\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 = 0$

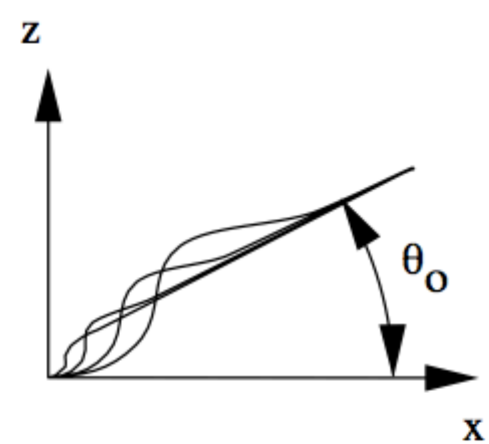
$h(x,t) = |t|^{\alpha} f(x|t|^{\alpha})$

$\phi(x,z,t) = |t|^{\alpha} g(x|t|^{\alpha}, z|t|^{\alpha})$

family of exponents allowed $0 < \alpha < \infty$ $\gamma = 2\alpha - 1$

$g = a_0 r^{\gamma/\alpha}\cos(\gamma\theta/\alpha) + a_{-1} r^{-1}\cos(\theta) + a_{-2} r^{-2}\cos(2\theta)$

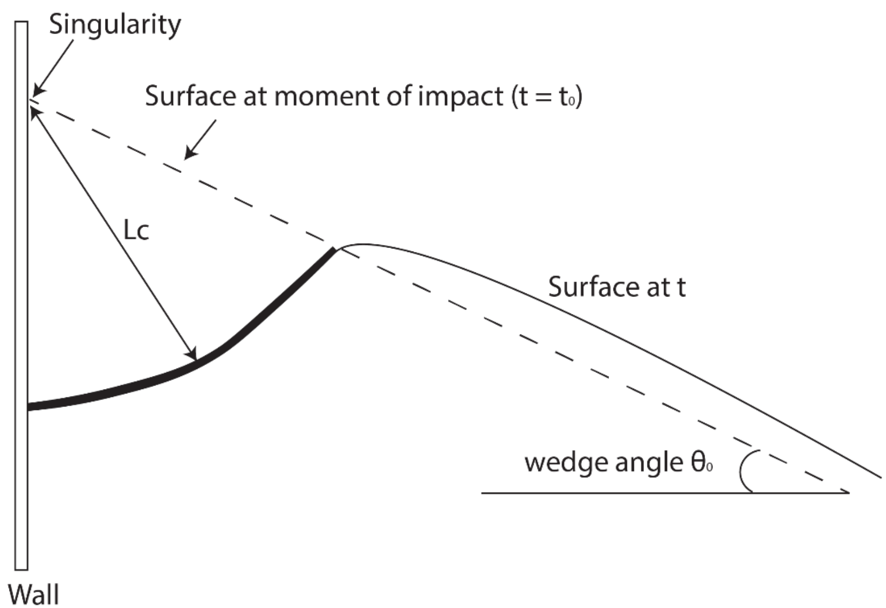
f wedge–like at large x, wedge angle θ_0

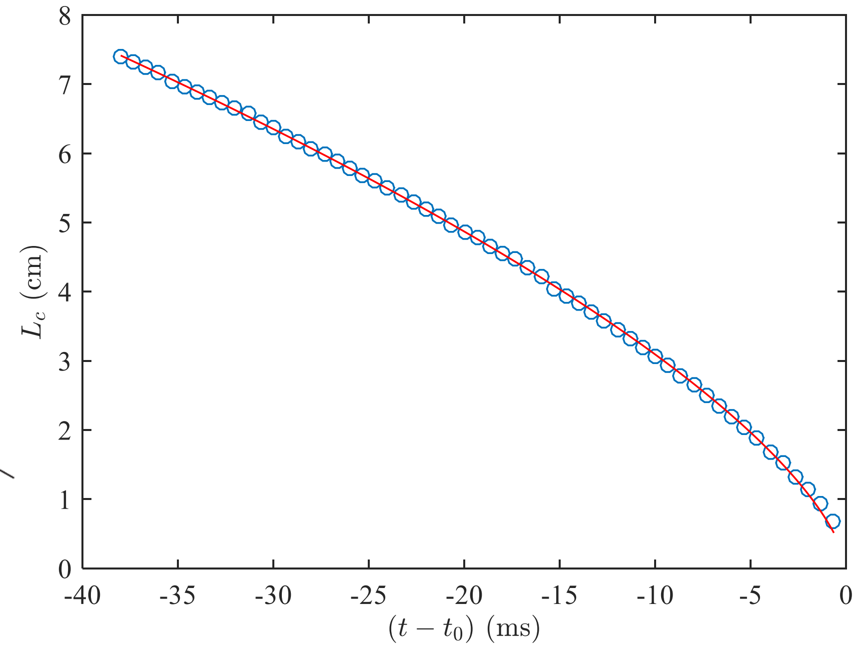


Bounded kinetic energy requires

for $\alpha > 1/2$ $a_0 = 0 \Rightarrow$ specifies θ_0

Collapse to Zero of the Length Scale

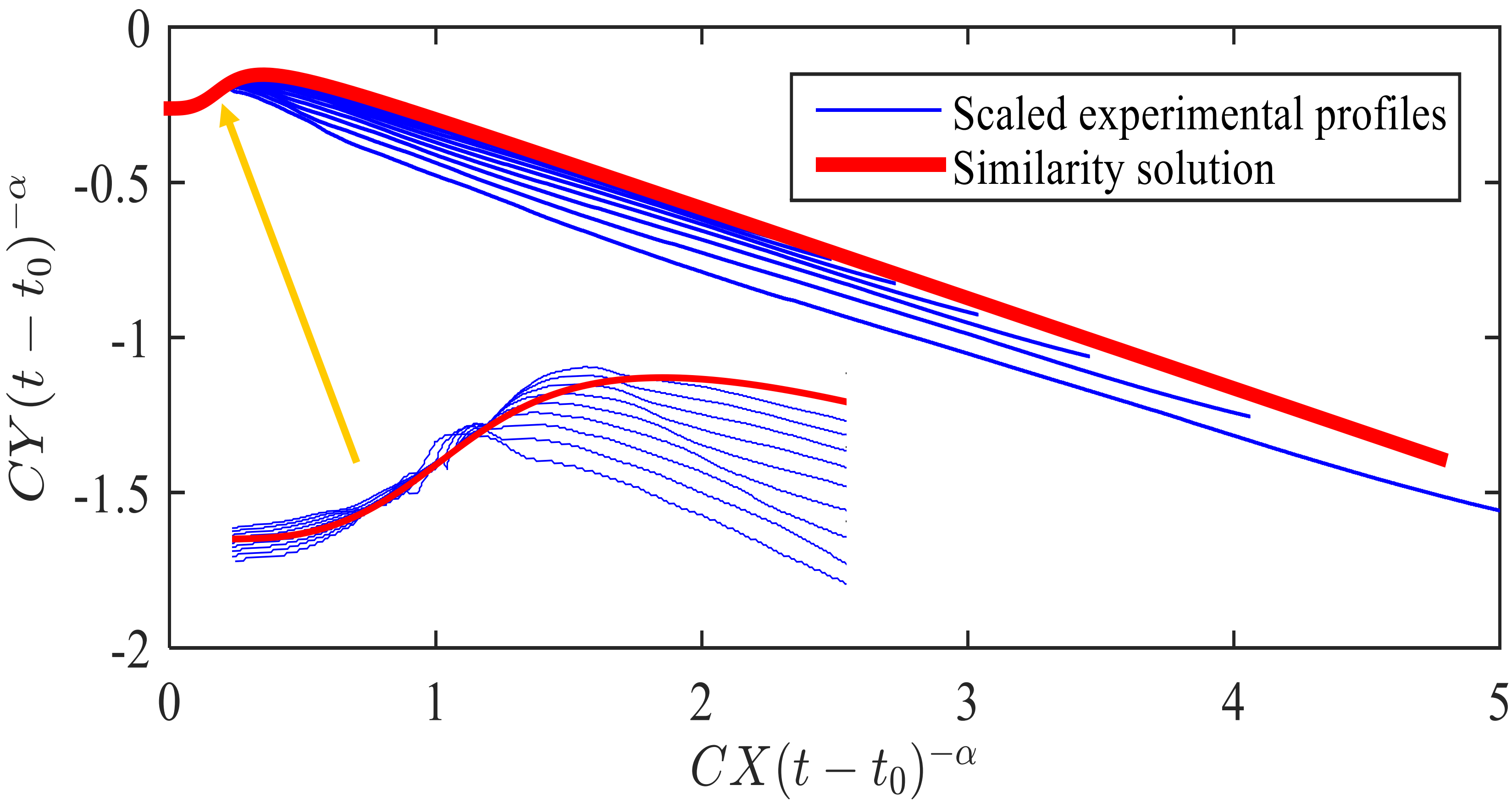




- Length scale is defined as the average distance from the origin (singularity point) to points on the curved portion of the surface (between contact point and crest).
- The decay of length scale follows the power law with exponent $\alpha = 0.655$
- The wedge angle remains nearly a constant about 15°

Comparison

From $t = 38$ ms to $t = 0$ ms (57 frames)



Quantum Fluids

A state of matter with long range quantum order

Type of synchronization

partial phase sync of the individual atomic wave functions

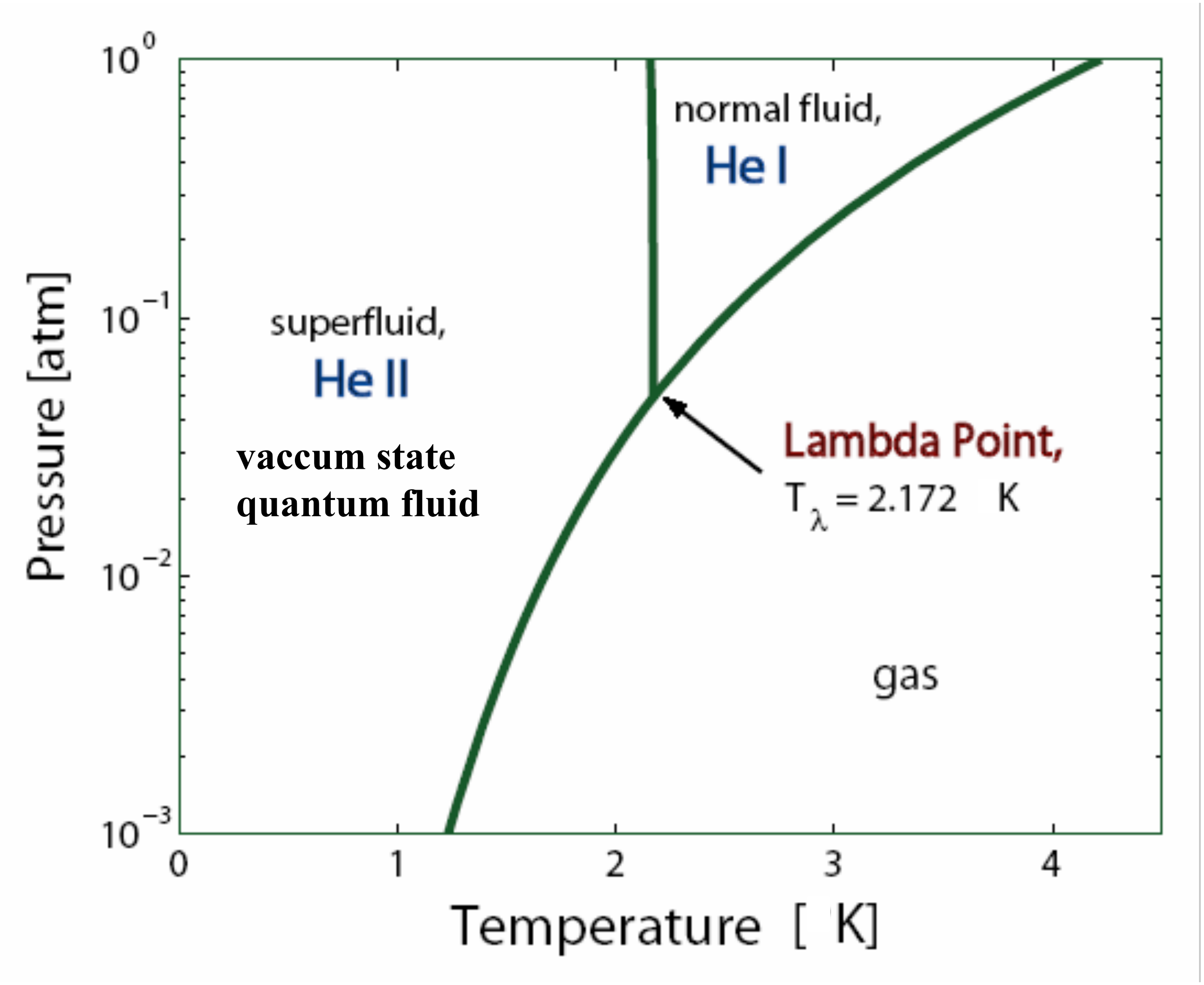
E.g.

- BEC atomic systems
- ^{4}He
- ^{3}He
- Cooper pair electrons in superconductors
- Physical vacuum

Quantum Turbulence –> turbulence in a quantum fluid

Why does it matter?

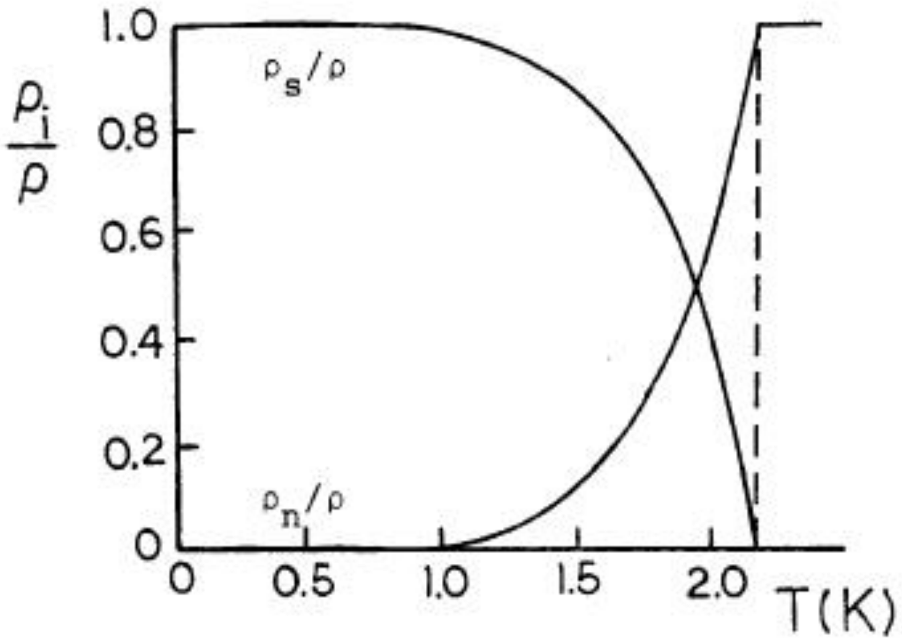
Background: Superfluid Helium



Two-Fluid Model

- Order parameter for superfluid helium is a complex field,

$$\Psi(\mathbf{x}) = Ae^{i\phi}$$



A is amplitude,

and ϕ is the phase

- Superfluid velocity given by

$$v_s = \kappa \nabla \phi \qquad \kappa = \frac{h}{m}$$

h = Planck's constant

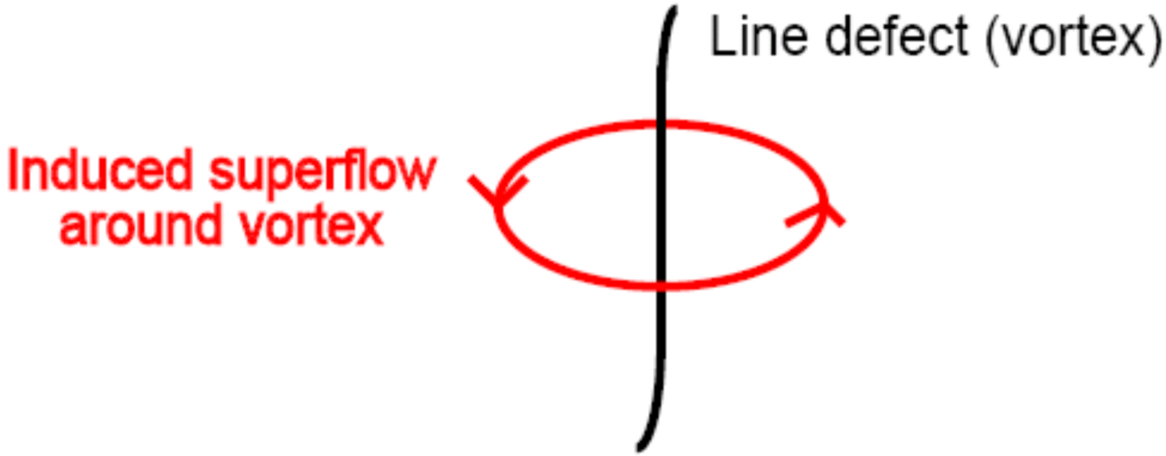
m = mass of helium atom

Quantized Vortices

- Lowest energy state: n=1, so ϕ wraps 2π around a defect
- Induces a superflow around the line:

$$\mathbf{v}_{\Phi} = \frac{\kappa}{r}$$

s is distance from defect



What is quantum turbulence?

An evolving set of quantized vortices:

Aperiodic

Large range of length scales
and curvatures

Rings

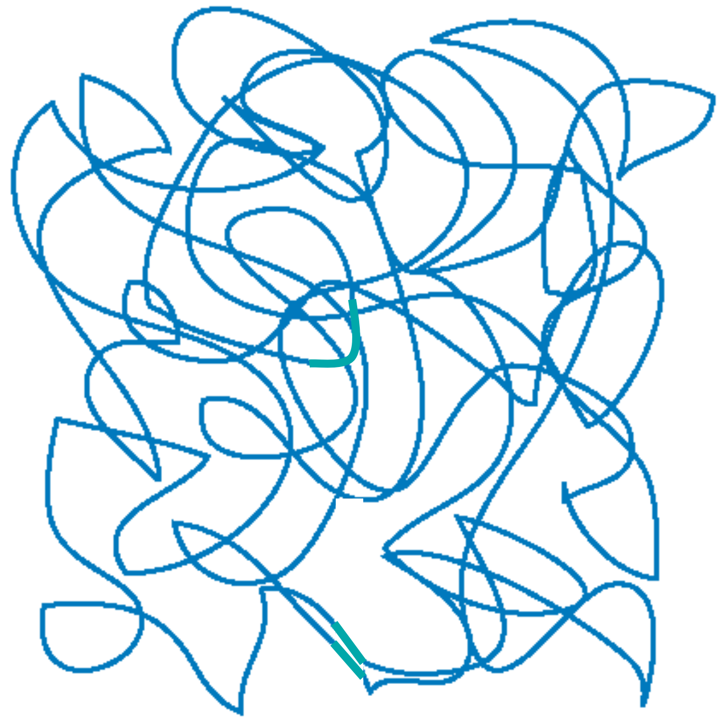
Vortices ending at walls

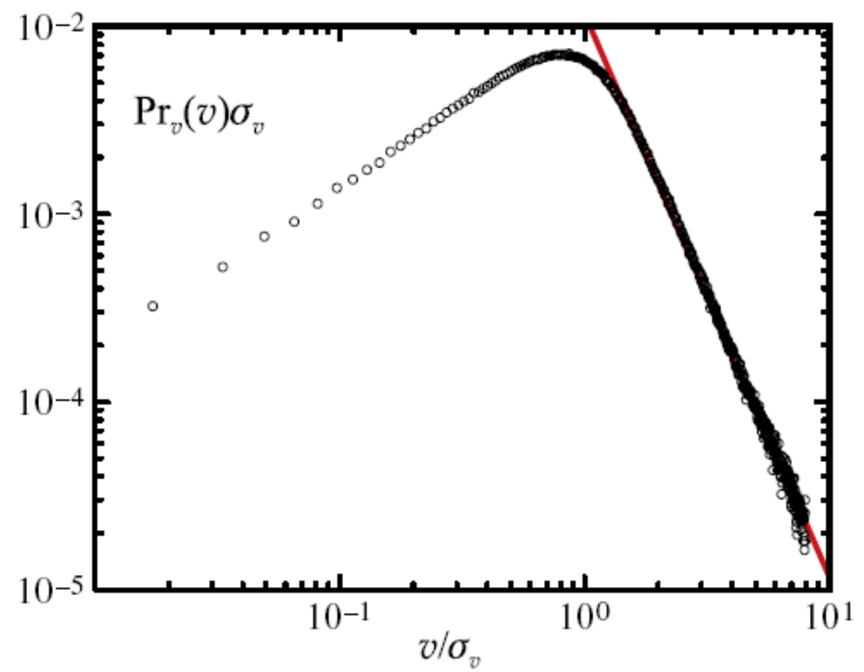
Knots

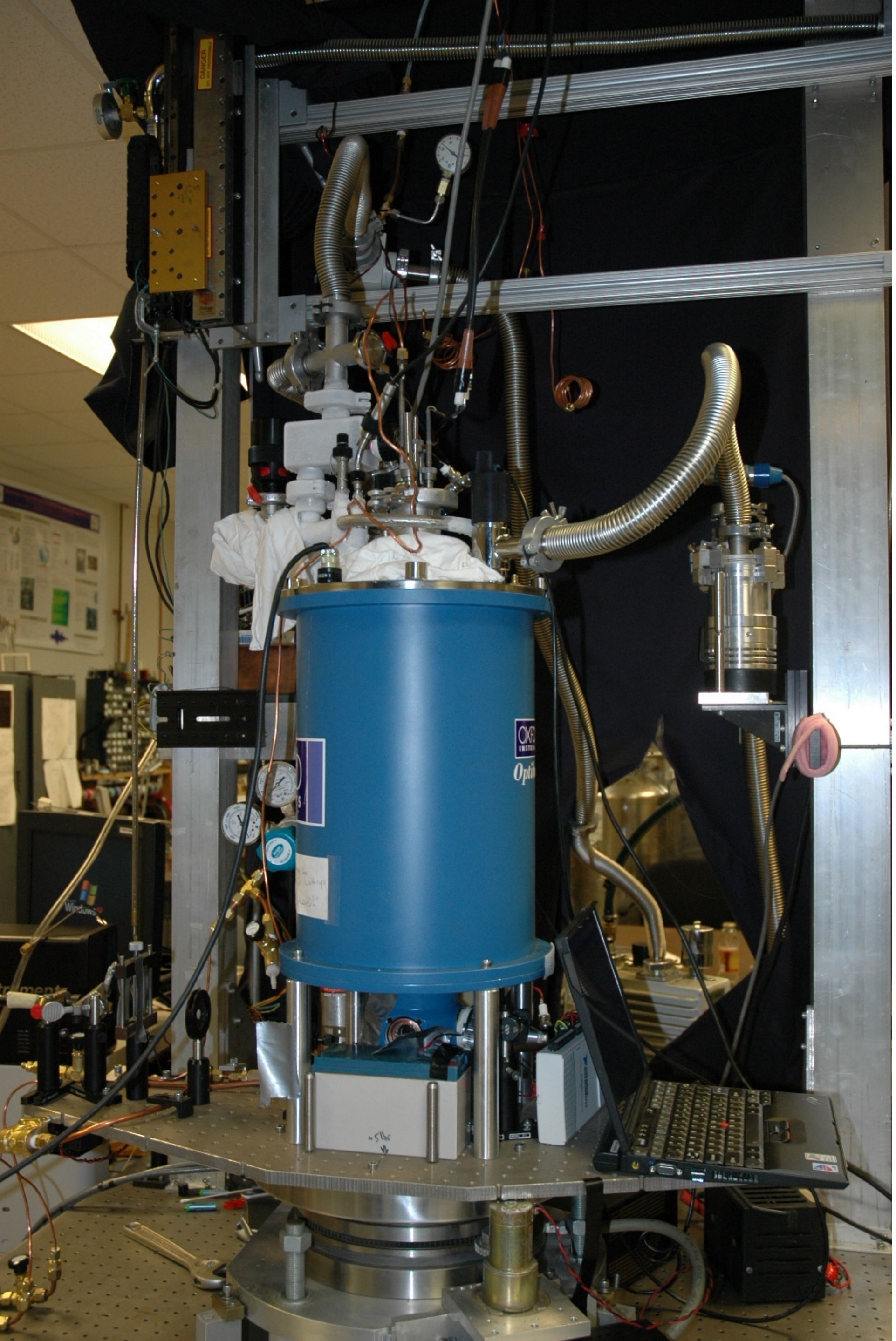
Quantum turbulence
is dominated by:

Reconnection

Ring collapse



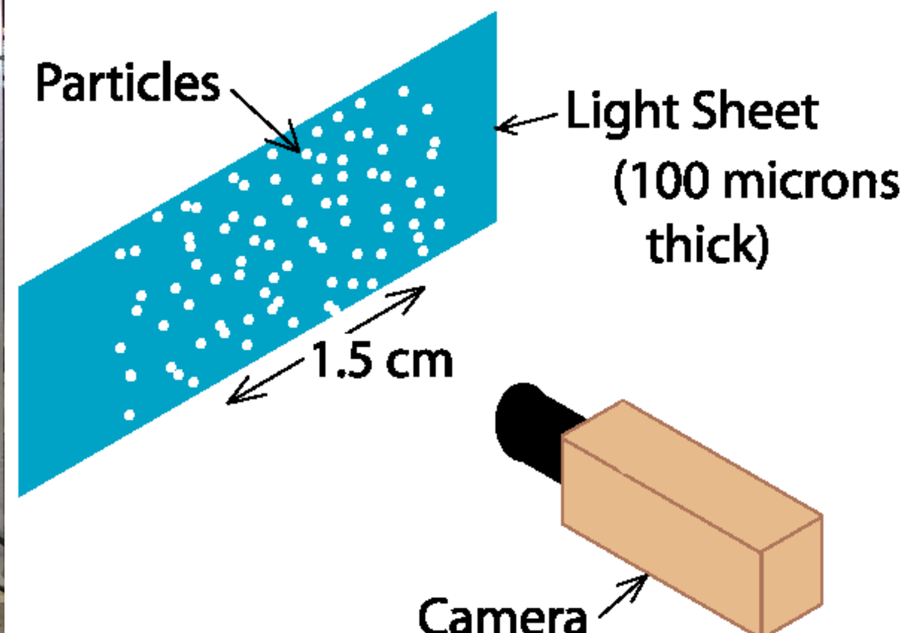




Superfluid helium Visualization

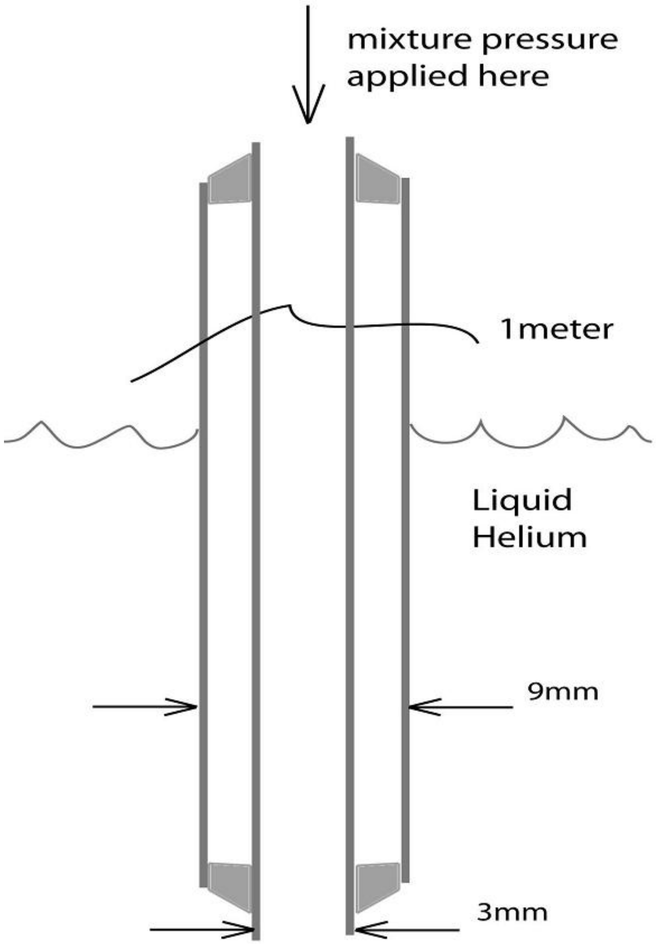
Apparatus

- **Optical cryostat**
- **Particle injector**
- **Laser sheet**
- **Low light camera**
- **T>1.6 K**

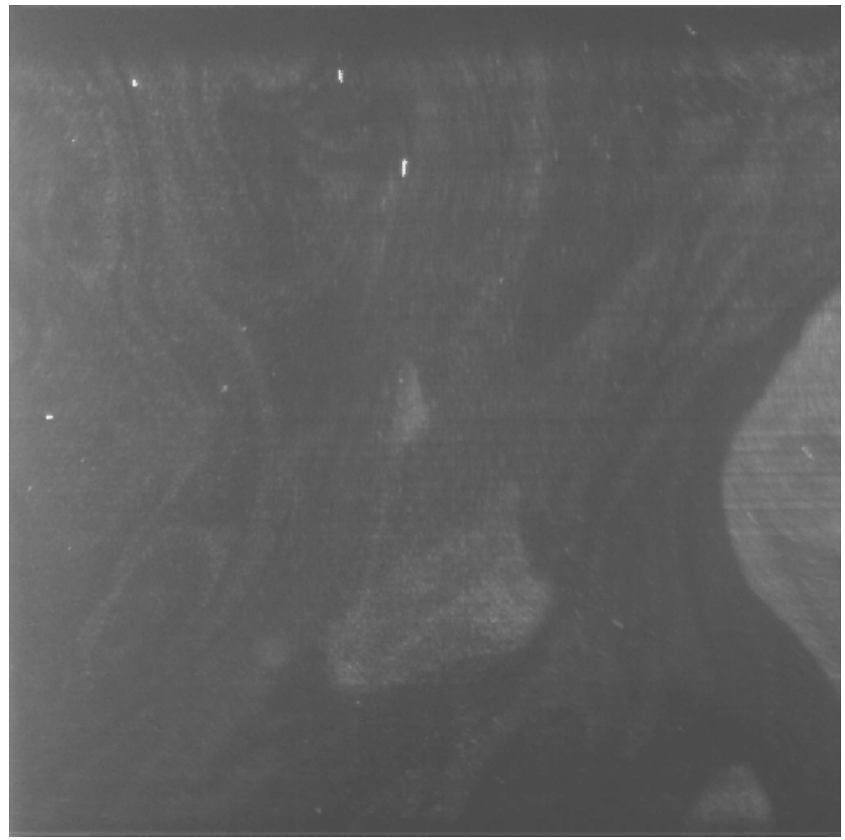


Particle Production

$1\ H_2 : \chi\ He \quad \chi >> 1$



$T > T_\lambda$



1/2
image
8 mm

Bewley et al., *Experiments in Fluids 2008*

Visualization of vortices - particle trapping

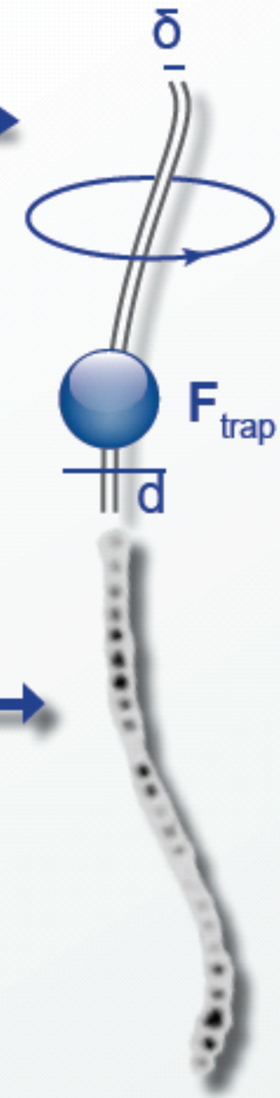
$$P = -\frac{\rho_s \kappa^2}{8\pi^2 r^2}$$

$$\mathbf{F} = \oint_{\partial\Omega} P \hat{n} \, dA$$

Decrease of energy $\Delta\varepsilon = \frac{\rho_s \kappa^2}{4\pi} d \ln \frac{d}{2\delta} = |\mathbf{F}_{trap}| d$

$$\frac{F_{trap}}{V} \propto \frac{\ln d}{d^3}$$

Particles get trapped on vortex cores



Ions in liquid He ⊖ 16 Å ⊕ 6 Å

Parks and Donnelly (1966), Williams and Packard (1974)

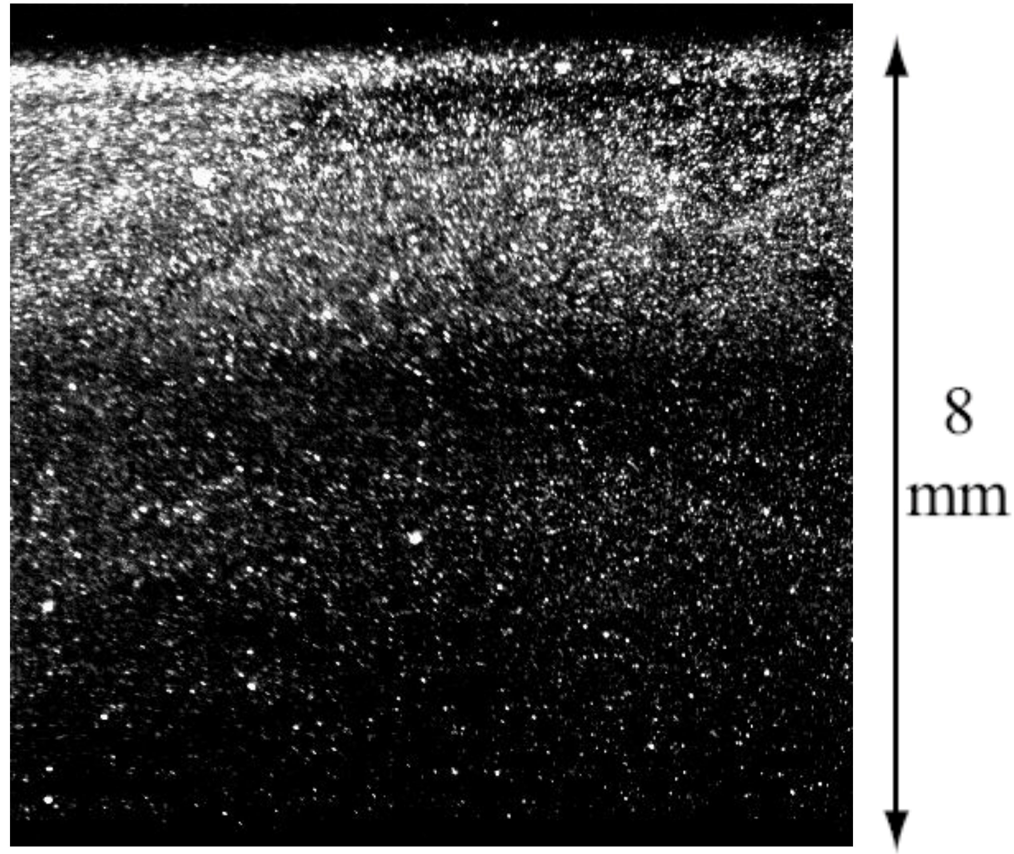
Solid hydrogen particles >1μm

Bewley, Lathrop, Sreenivasan, Nature 441 588 (2006)

Visualizing Superfluid Vortices in He II

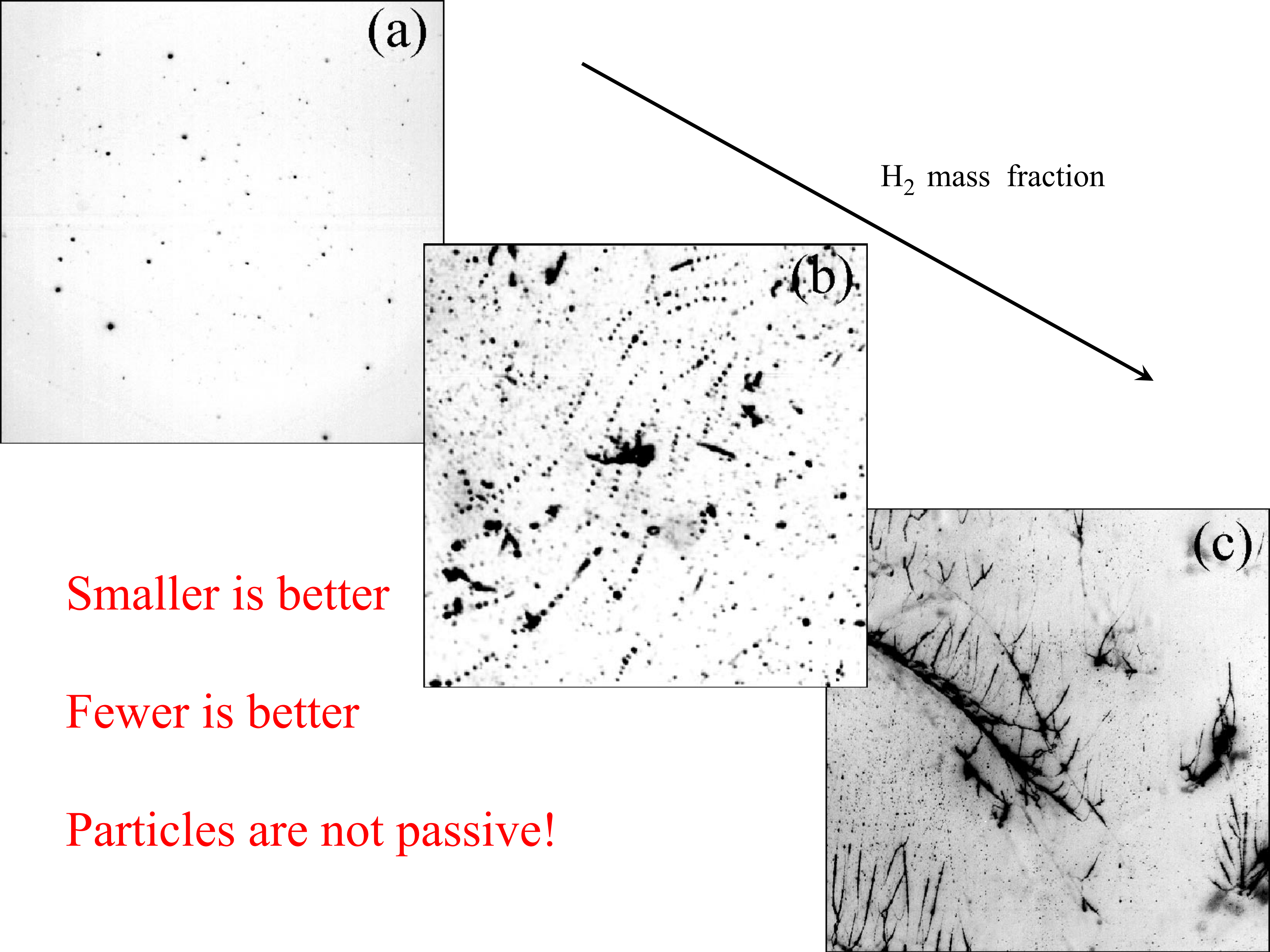
- Below T_λ hydrogen particles collect onto filaments

- Previous work has shown these filaments are particles trapped on the superfluid vortices (Bewley, *et al.*, *Nature* 2006)



Movie in real time
Begins 180 s after transition
T_λ - T ~ 50 mK

Sounds through transition caught with MEMS microphone



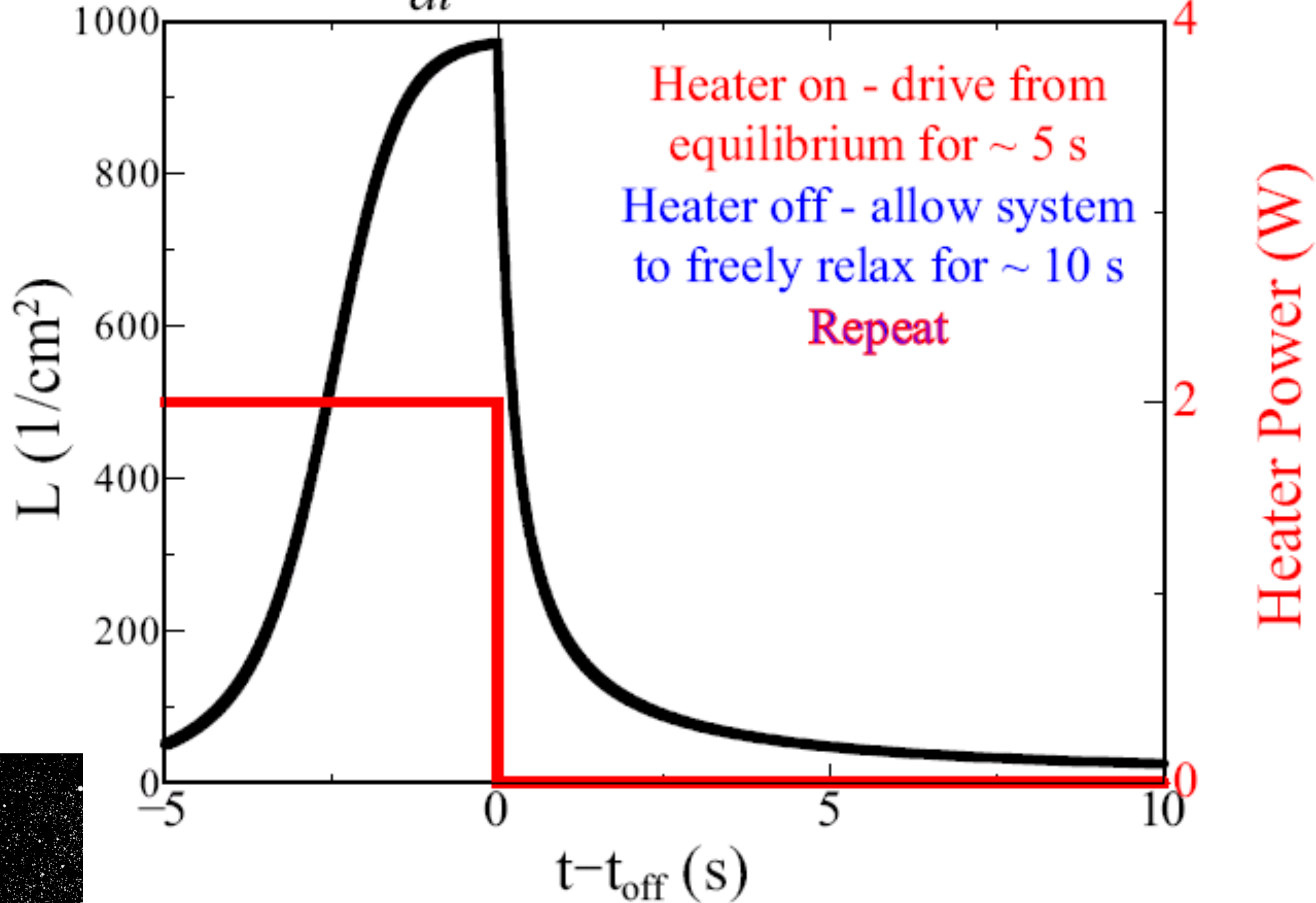
Smaller is better

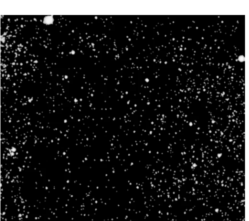
Fewer is better

Particles are not passive!

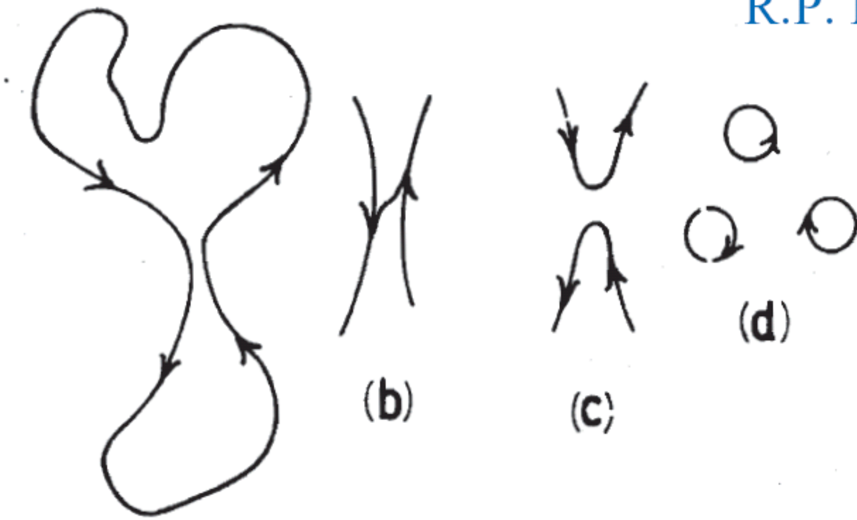
Pulsed Counterflow Experiments

$$\frac{dL}{dt} = \alpha \left| \mathbf{v}_{ns} \right| L^{3/2} - \beta \kappa L^2$$



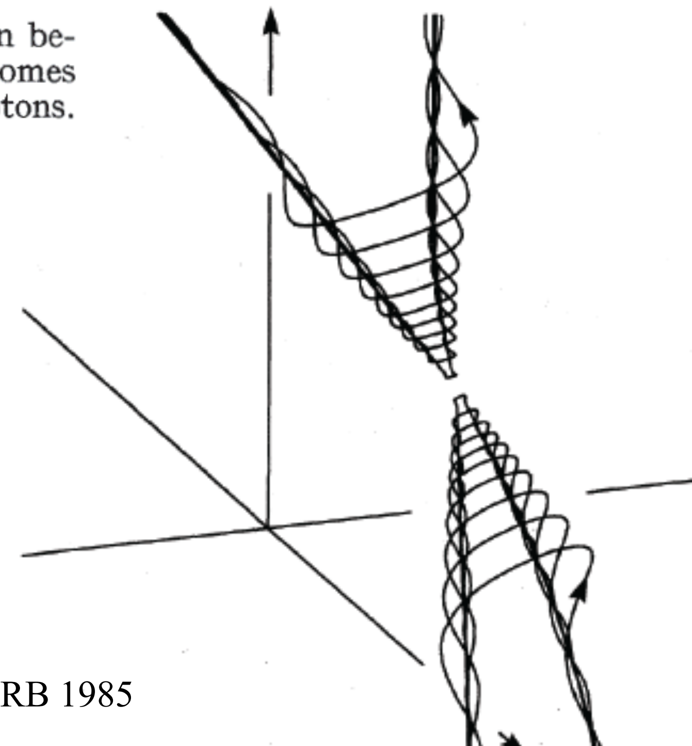


R.P. Feynman



Prog. Low Temp. Phys. 1, 17 (1955)

Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

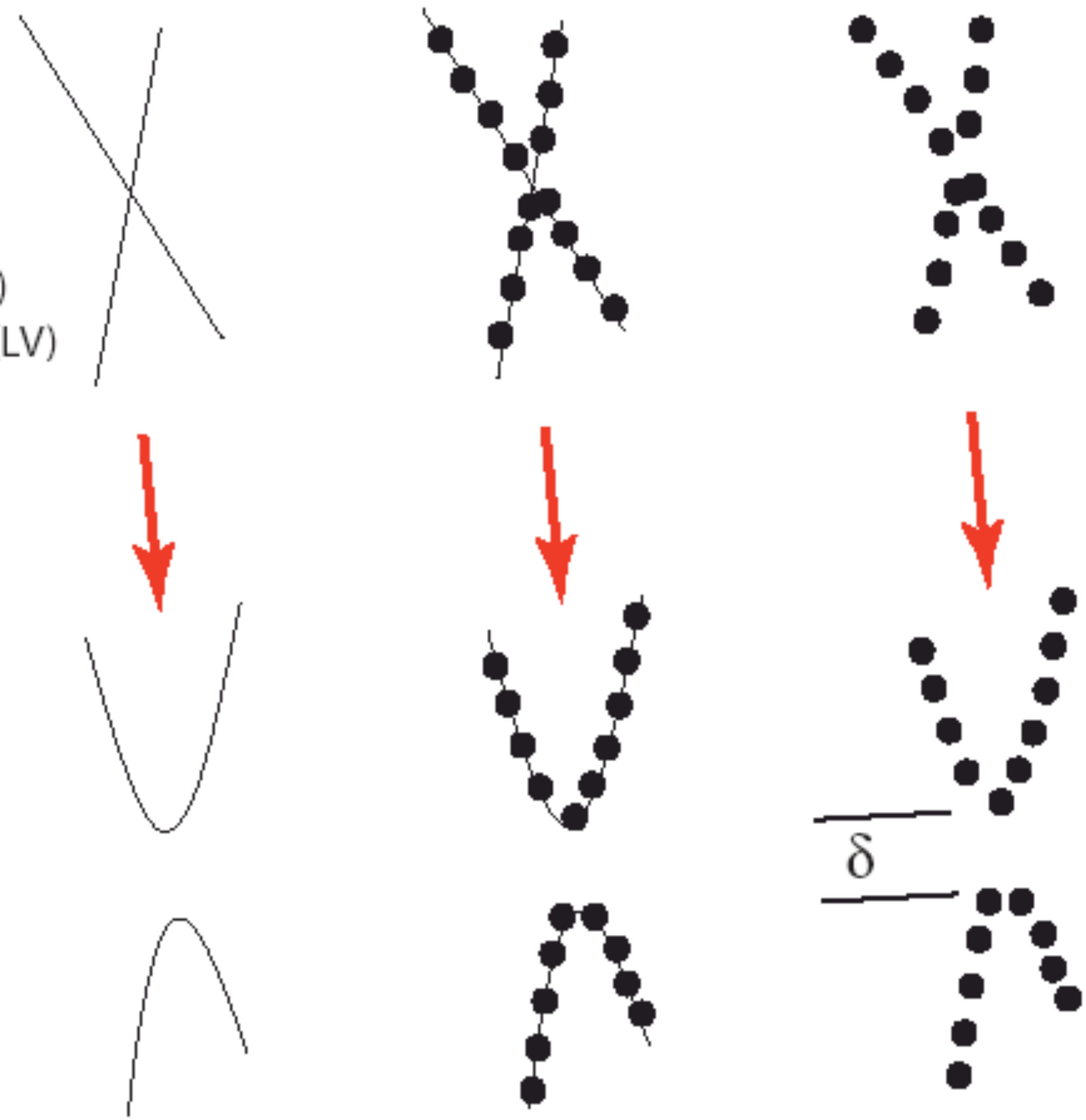


From Schwarz PRB 1985

Vortex reconnection

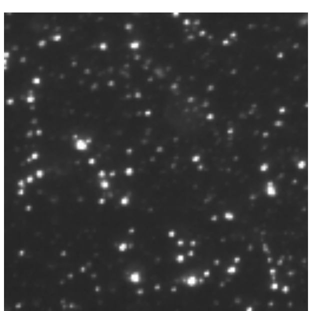
Theoretical work

Schwarz, PRB 1985 (LV)
de Waele and Aarts, PRL 1994 (LV)
Koplik and Levine, PRL 1993 (NLSE)
Tsubota and Maekawa, JPSJ 1992 (LV)
Nazarenko and West 2003 (NLSE)
Much more recent work!
Kerr PRL 2011 (NLSE)



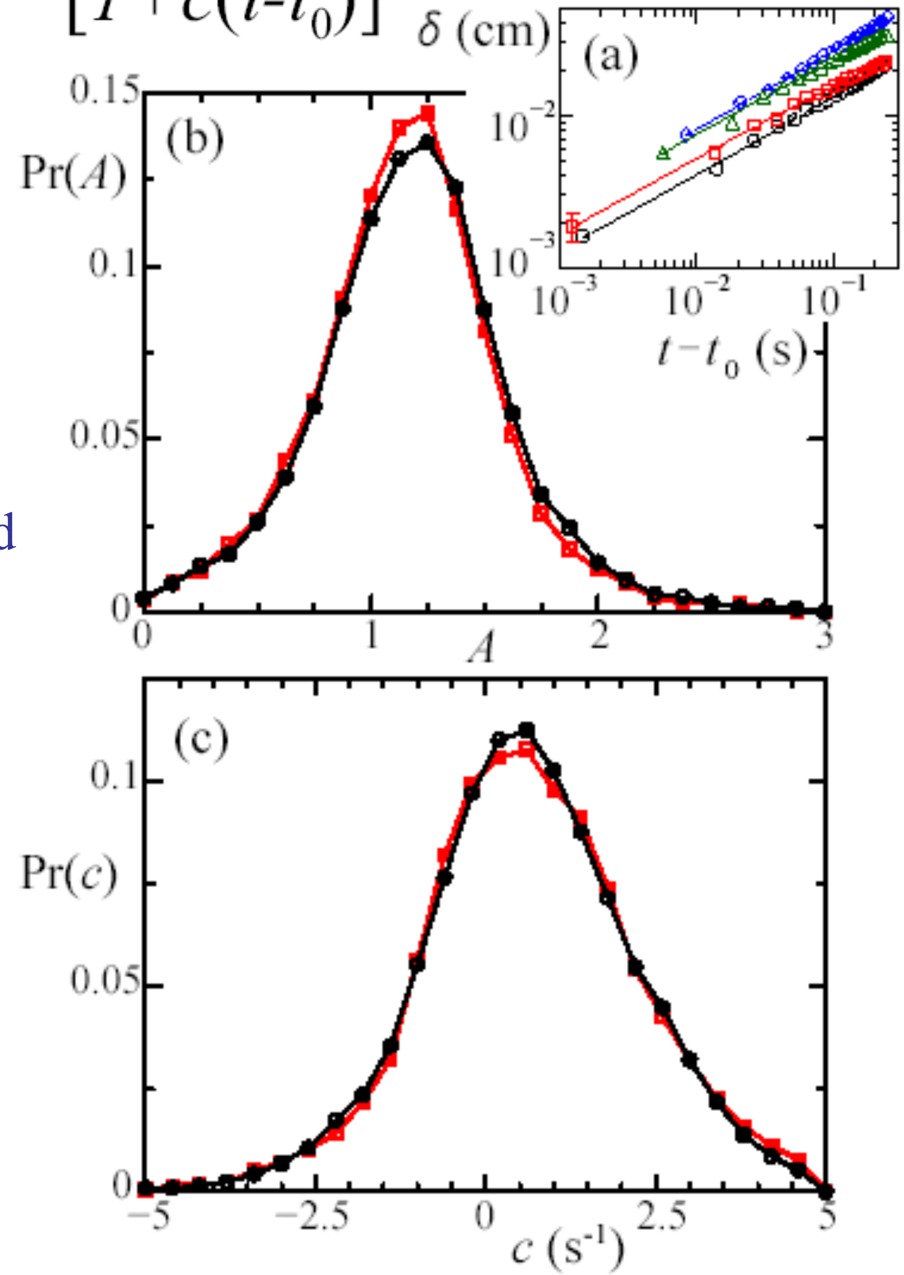
$$\delta \sim \kappa^{1/2}(t_0-t)^{1/2}$$

$$\delta \sim \kappa^{1/2}(t-t_0)^{1/2}$$



Pre-reconnection: $\delta(t) = A[\kappa(t_0-t)]^{1/2}[1+c(t_0-t)]$

Post-reconnection: $\delta(t) = A[\kappa(t-t_0)]^{1/2}[1+c(t-t_0)]$



Only small pre- and post- differences

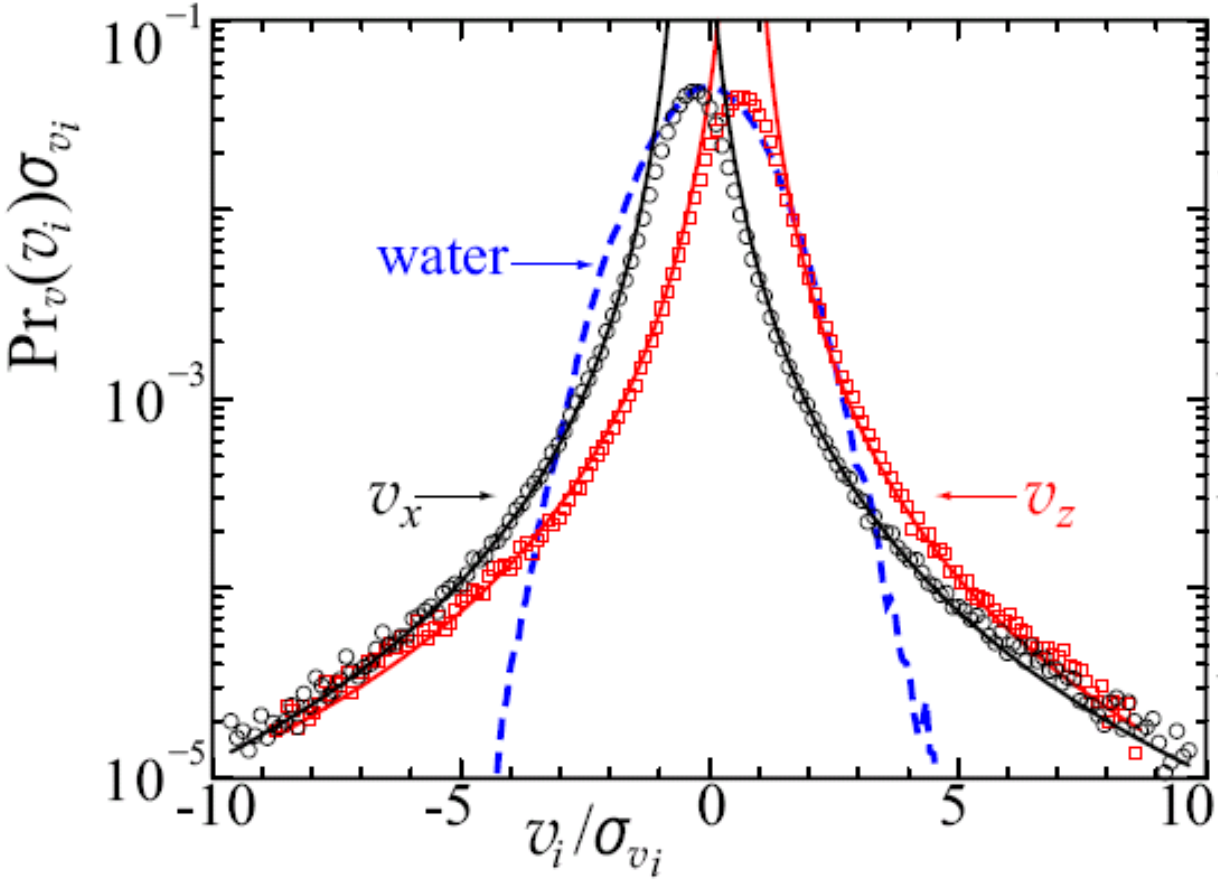
c may represent the affect of local strains

reconnection and ring collapse represented

NEARLY TIME REVERSIBLE!

M.S. Paoletti, M.E. Fisher, and D.P. Lathrop, "Reconnection dynamics for quantized vortices," Physica D (2010)

Velocity Statistics



$$\delta = A\,\kappa^{1/2}\,(t-t_o)^{1/2}$$

$$\mathbf{v}(t) \sim \kappa^{1/2}\,|t-t_0|^{-1/2}$$

$$P(\mathbf{v})d\mathbf{v} = P(t)dt$$

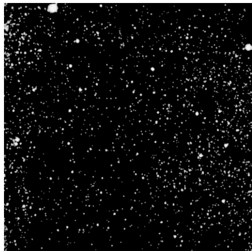
$$P(\mathbf{v}) \sim \left|\frac{dt}{d\mathbf{v}}\right|$$

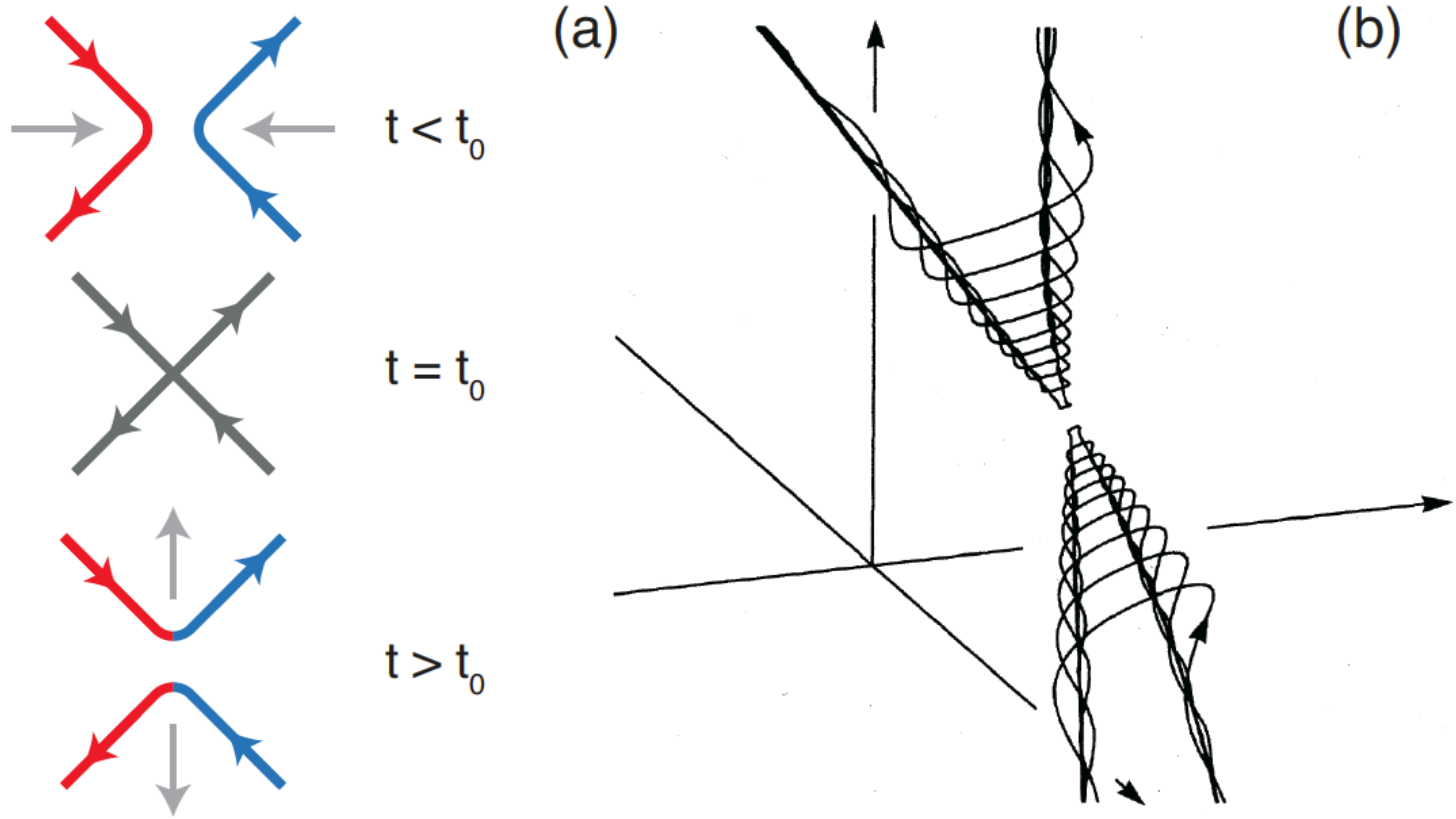
$$P(\mathbf{v}) \sim \left|\frac{1}{\mathbf{v}^3}\right|$$

Reconnection produces predictable power-law velocity tails quite distinct from classical turbulence

M.S. Paoletti, M.E. Fisher, K.R. Sreenivasan, and D.P. Lathrop, “Velocity statistics distinguish quantum from classical turbulence,” Phys. Rev. Lett. (2008)

Bagaley and Barenghi, PRE (2011).




(a)
(b)
t < t0
t = t0
t > t0

Vortex Filament Models

- Local Induction Approximation (LIA)

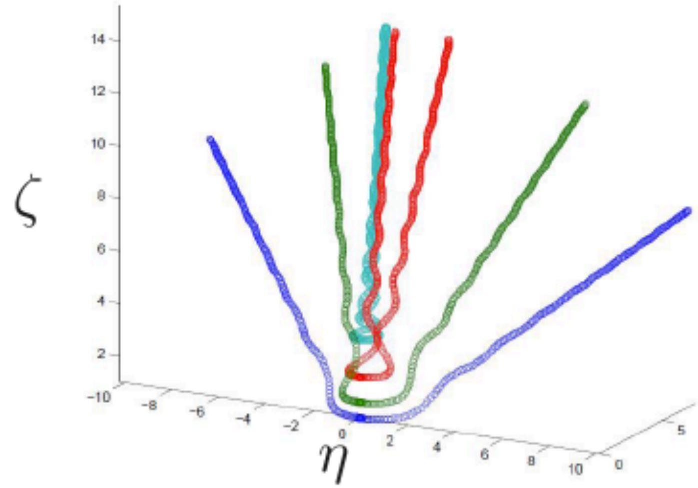
$$\frac{\partial \vec{s}(\sigma,t)}{\partial t} = \beta \frac{\partial \vec{s}(\sigma,t)}{\partial \sigma} \times \frac{\partial^2 \vec{s}(\sigma,t)}{\partial \sigma^2} + \alpha(T) \frac{\partial^2 \vec{s}(\sigma,t)}{\partial \sigma^2}$$

- LIA has one-parameter family of self-similar solutions in dimensionless similarity coordinates

- Adopt dimensionless similarity coordinates

$$\eta = (x - x_0)/\sqrt{\kappa(t-t_0)}$$
$$\zeta = (z - z_0)/\sqrt{\kappa(t-t_0)}$$

LIA Curves vs Vortex Angle

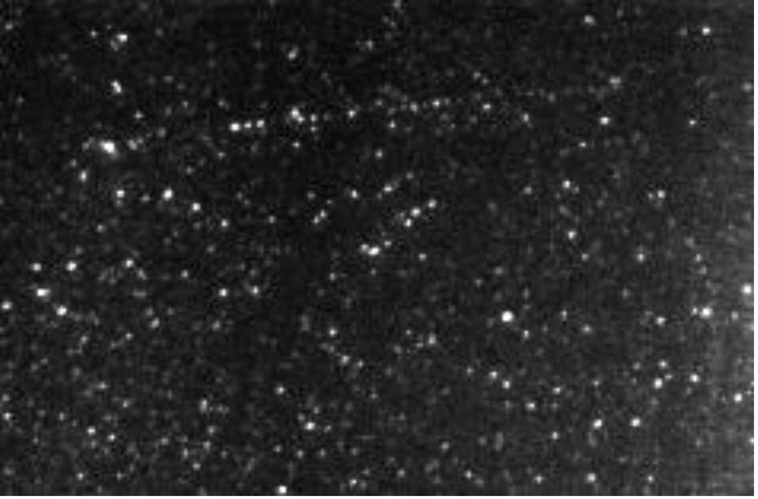


Eur. J. Mech. B - Fluids 19 (2000) 361–378

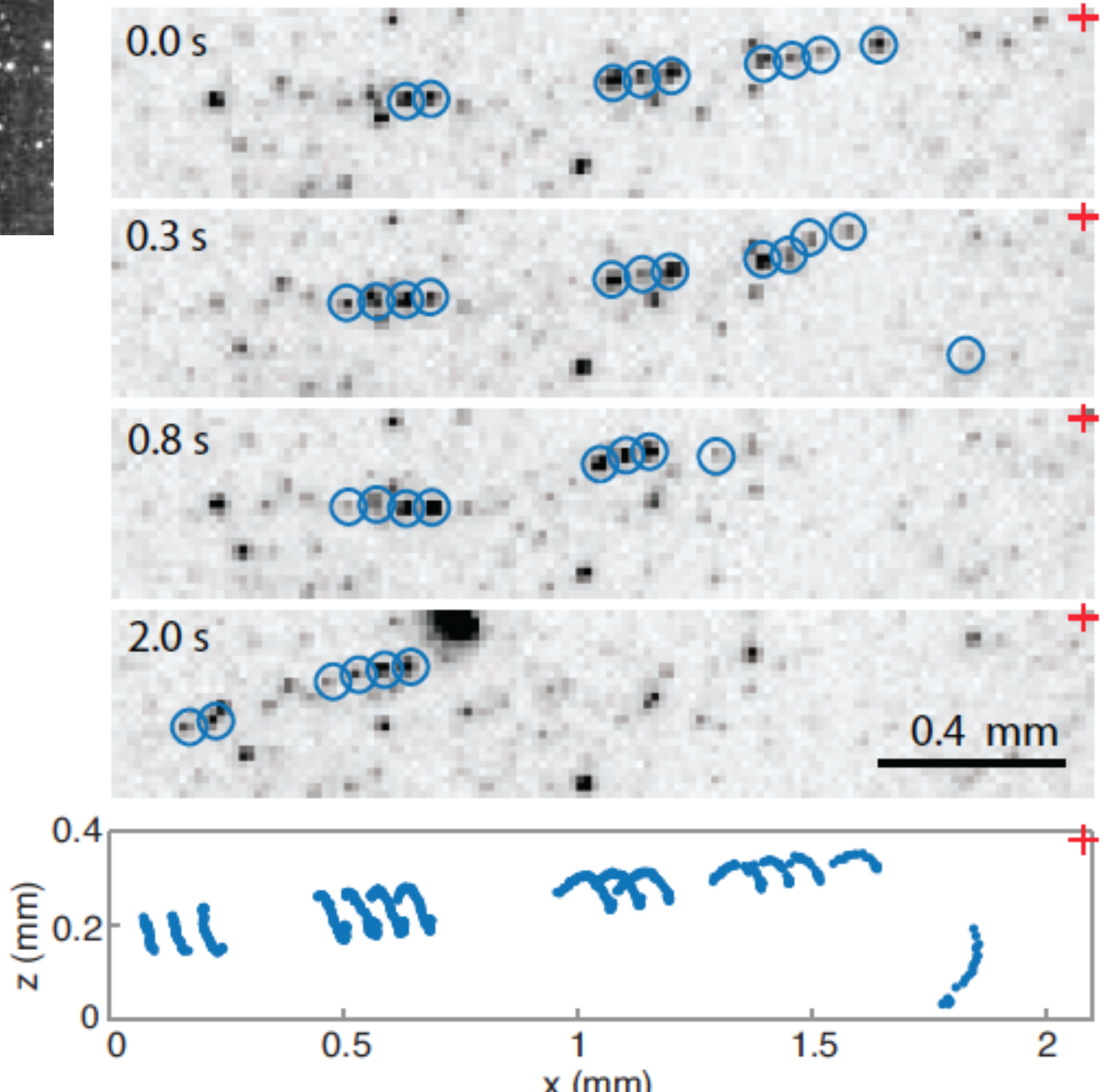
S0997-7546(00)00123-0/FLA

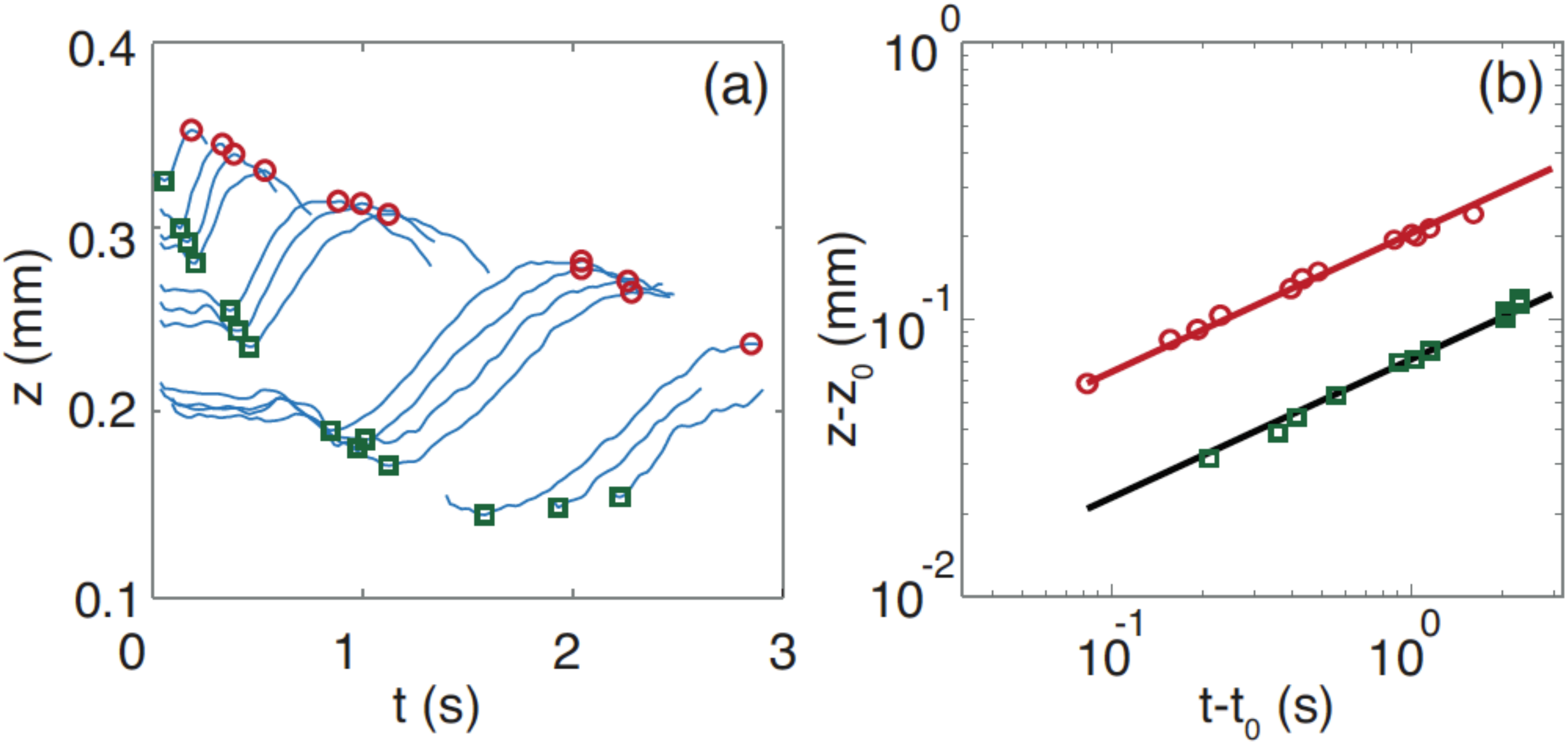
Evolution of quantum vortices following reconnection

Tomasz Lipniacki *



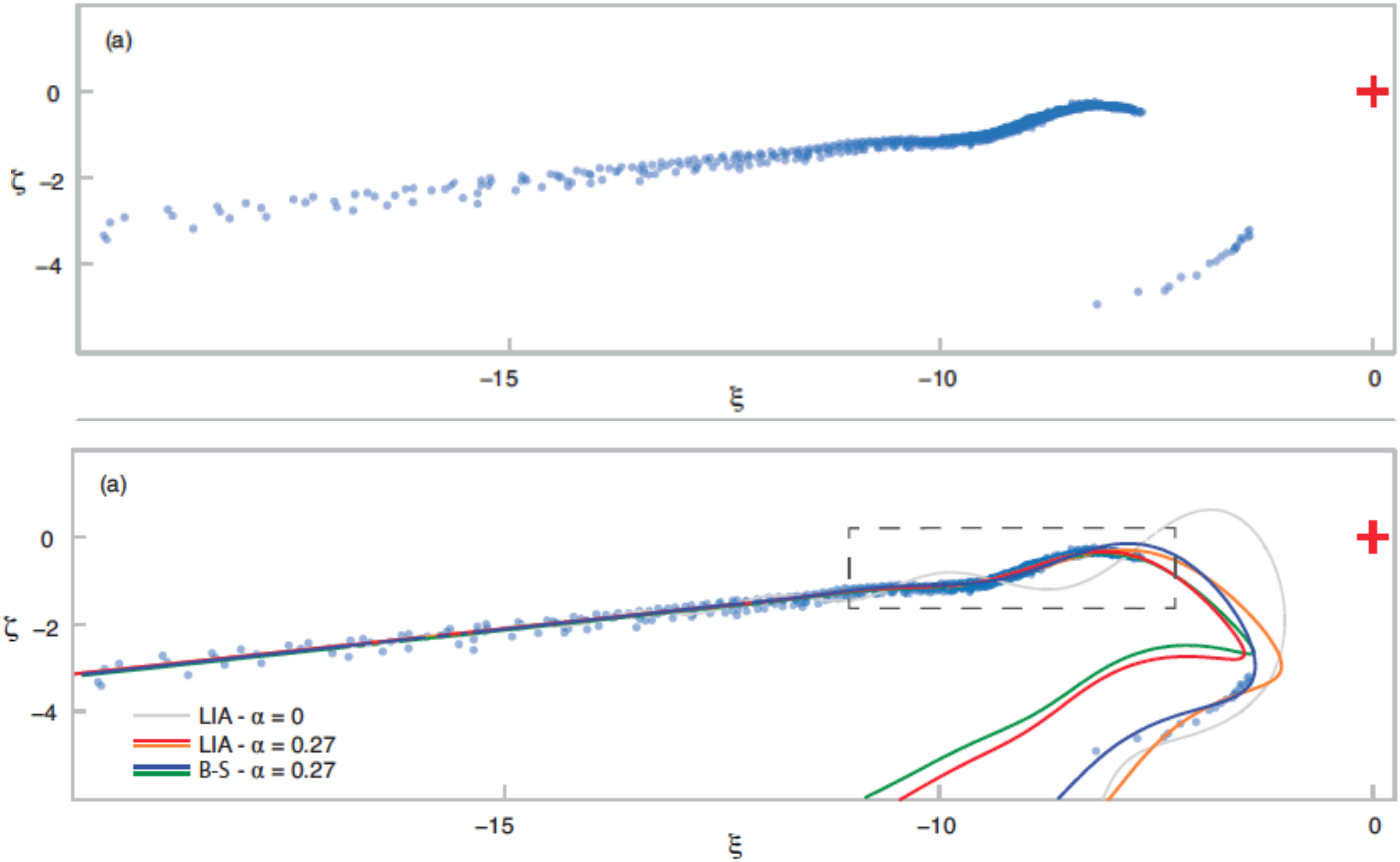
Kelvin waves!





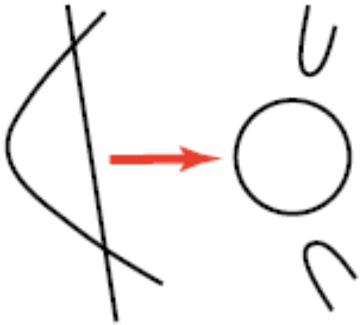
$$\zeta = (z - z_o)/\sqrt{\kappa(t - to)}$$

$$\xi = (x - xo)/\sqrt{\kappa(t - to)}$$

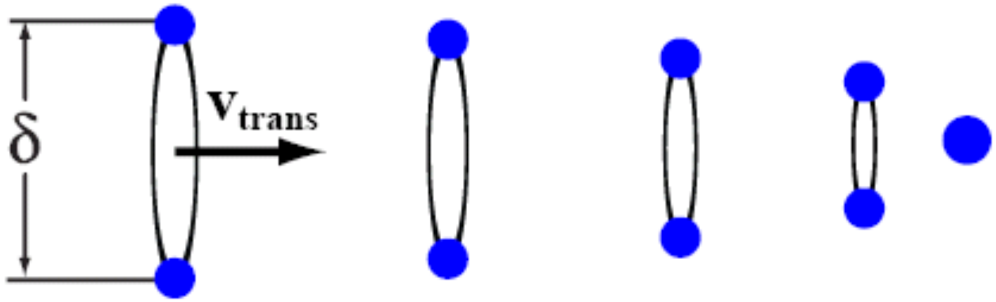


Collapse in similarity coordinates and a comparison with theoretical models

Quantized Vortex Rings



Reconnection can produce vortex rings



Pair of particles on collapsing rings look like reconnection backwards in time with additional transverse velocity

Fixed points of the nonlinear Schrodinger equation

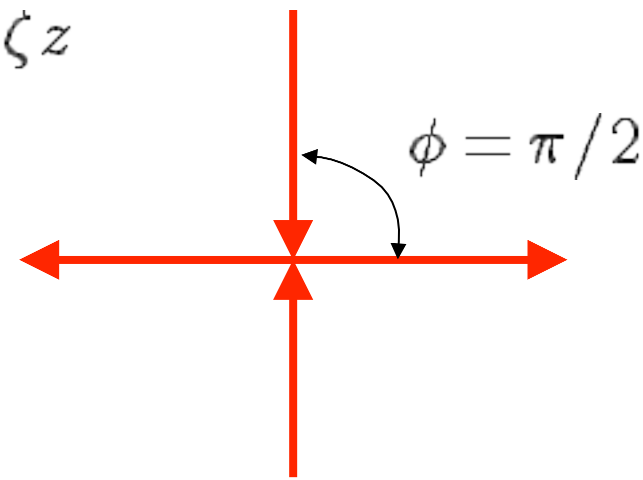
$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + g\,|\Psi|^2\,\Psi - \mu\Psi$$

$$\Psi_0 = \sqrt(\mu/g)$$

$$\Psi_1 = f(r)e^{i\theta}$$

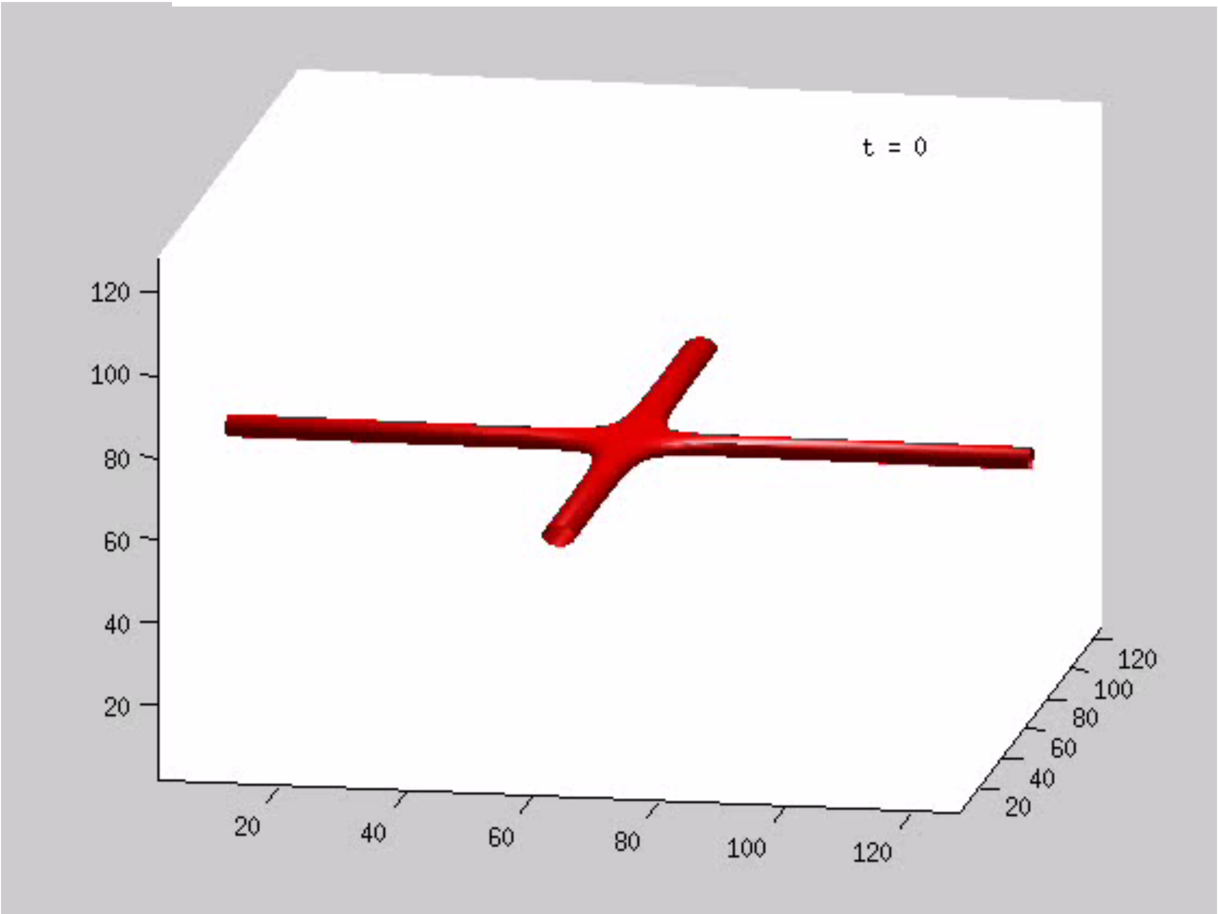
New fixed point. Near origin:

$$\Psi_2 = xy + i\zeta z$$



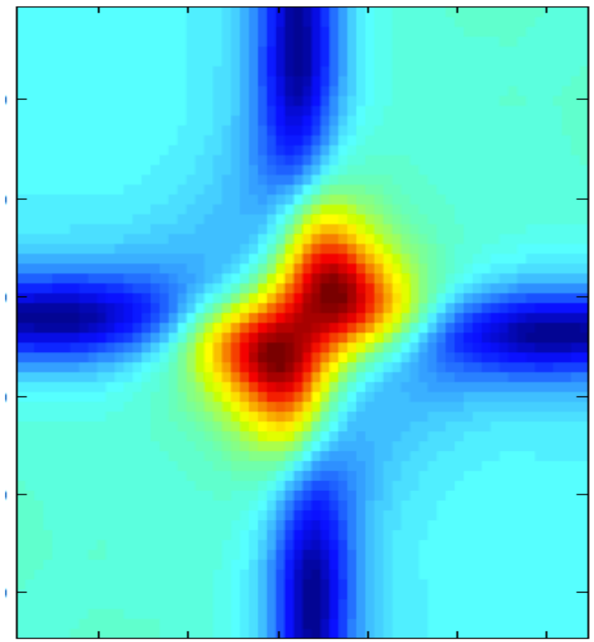
New fixed point is a saddle

David P. Meichle, Cecilia Rorai, Michael E. Fisher, and D. P. Lathrop
Phys. Rev. B **86**, 014509 – Published 10 July 2012

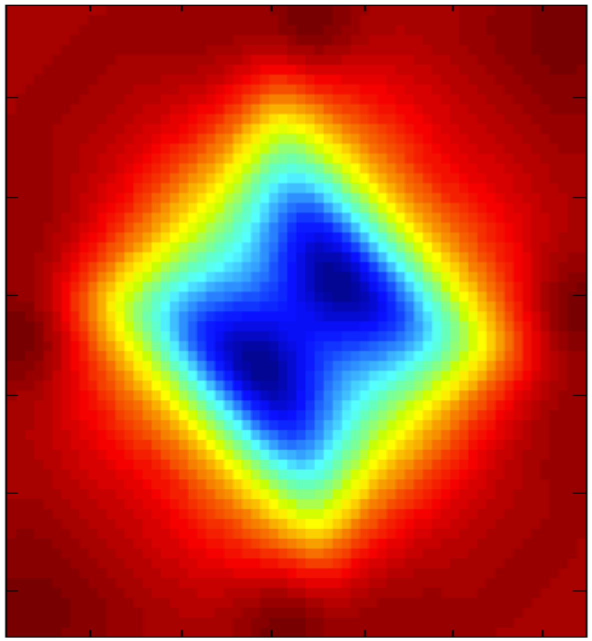


Eigenvector for saddle directions away from Ψ_2

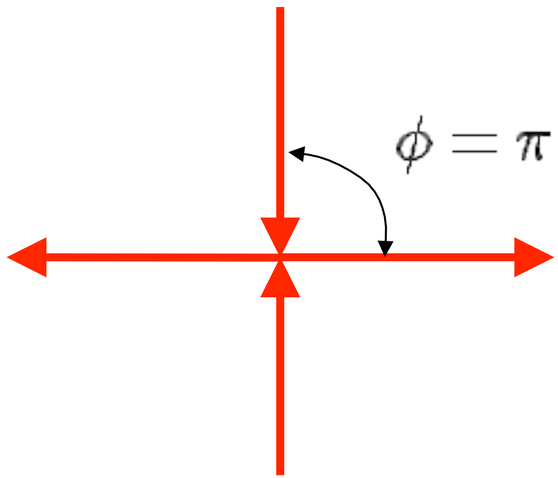
Real part

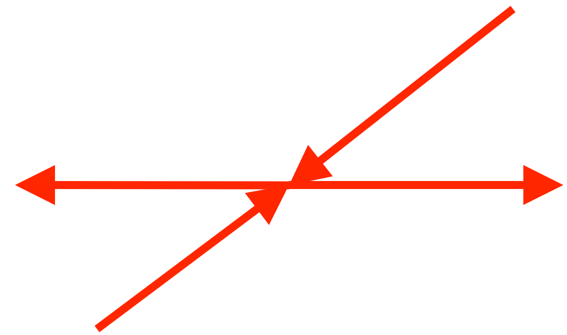


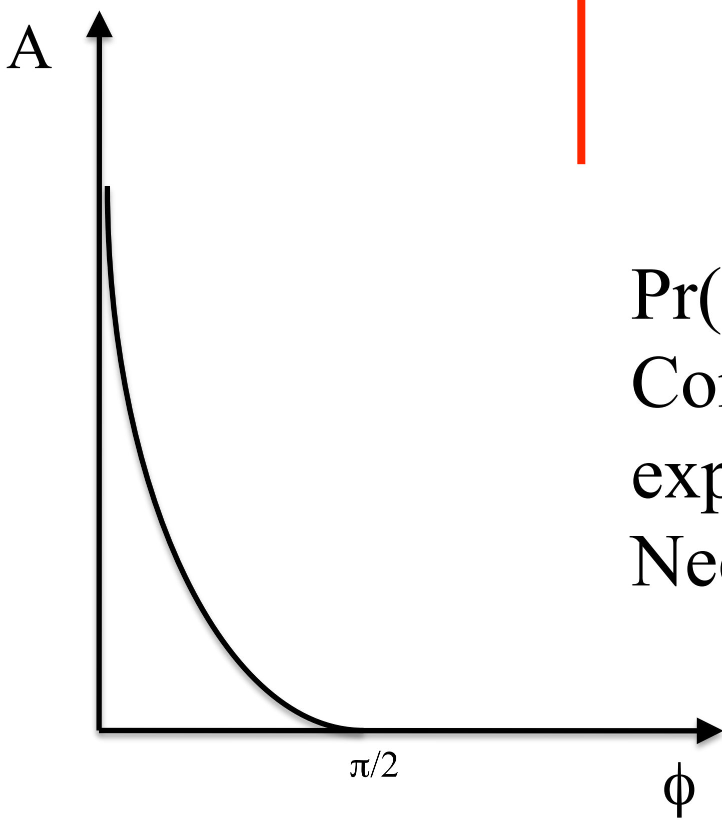
Imaginary part



Prefactor depends on reconnection geometry



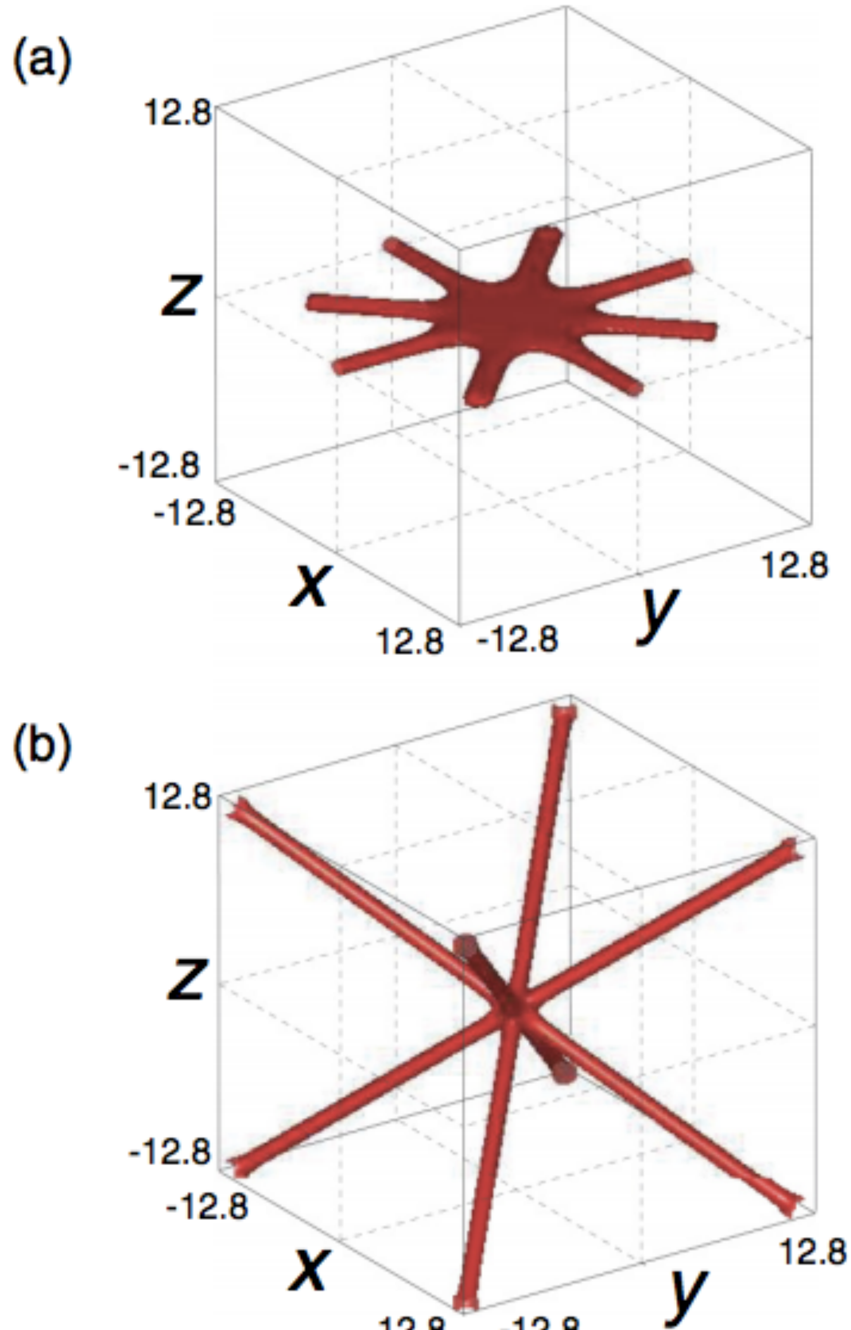




$Pr(A) = Pr(\phi)\ |d\phi/dA|$

Connection possible between experiment and theory

Need $A(\phi)$!


(a)
12.8
z
-12.8
-12.8
x
12.8
-12.8
y
12.8
(b)
12.8
z
-12.8
-12.8
x
12.8
-12.8
y
12.8

Dissertations: complex.umd.edu
Youtube channel: n3umh

Bewley, Lathrop, and Sreenivasan Nature 2006
Bewley, Sreenivasan, and Lathrop, Exp. in Fluids 2008
Paoletti, Fiorito, Sreenivasan, and Lathrop, J. Phys. Soc. Japan 2008
Bewley, Paoletti, Sreenivasan, and Lathrop, Proc. Nat. Acad. Sci. 2008
Paoletti, Fisher, Sreenivasan, and Lathrop, PRL 2008
Paoletti, Fisher, and Lathrop, Physica D 2008
Paoletti and Lathrop, Ann. Rev. of Cond. Matter Phys. 2011
Meichle, Rorai, Fisher, and Lathrop, PRB 2012
Fonda, Meichle, Ouellette, Hormoz, and Lathrop PNAS 2014
Meichle and Lathrop Rev. Sci. Inst. 2014

Next steps: 3-D tracking

We love fluid turbulence

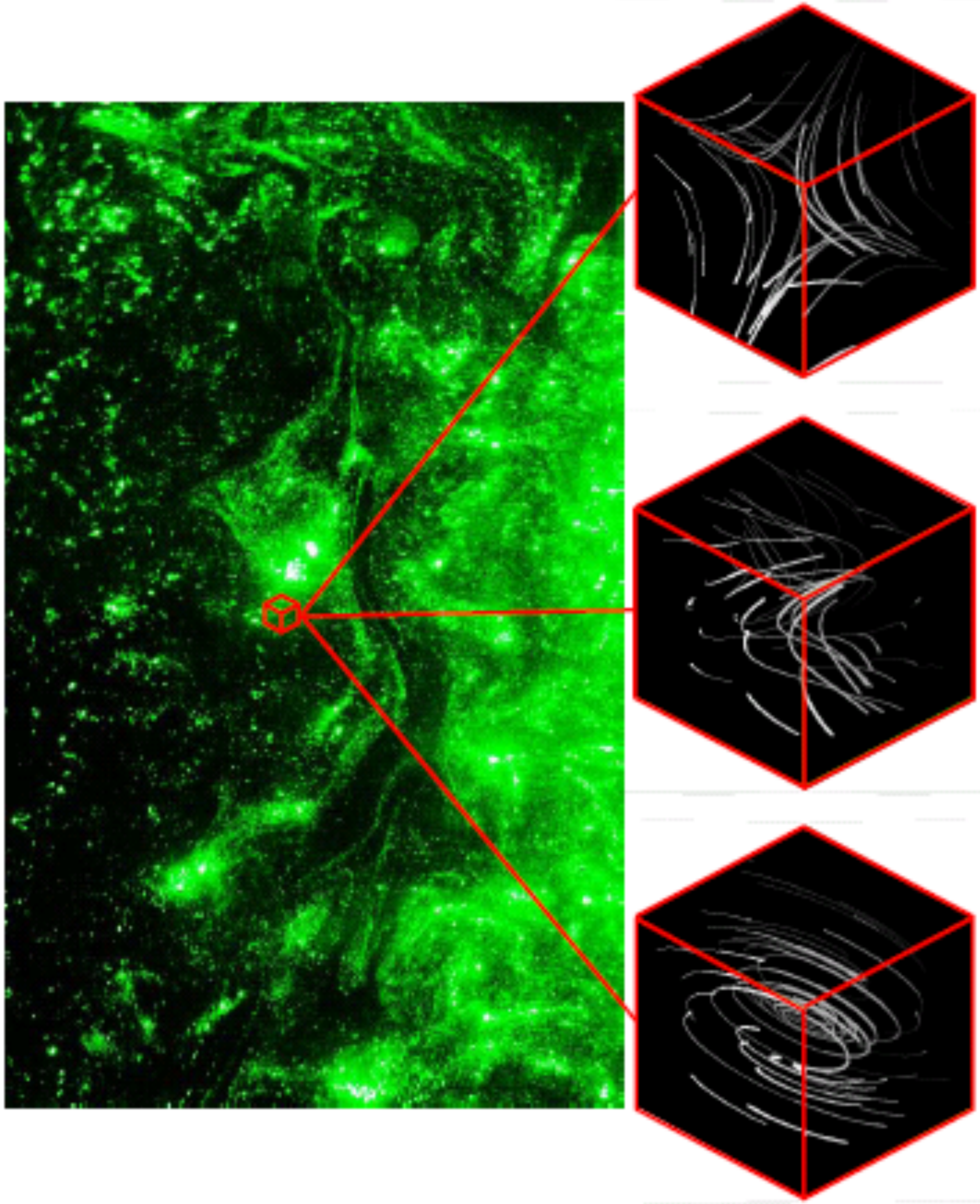
Velocity field rough $\vec{v}(\vec{x},t)$

Large range of length scales
Large range of time scales

$Pr(v_i)$ Gaussian

$E(k) \sim k^{-5/3}$ Inertial range

Known – agreed on equations of motion Navier-Stokes Eq.



Navier-Stokes flows: ?: turbulence

$Pr(v_i) \sim \exp(-v_i^2/\sigma_v^2)$

$Pr(\partial_j v_i) \sim \exp(-|v_i|/\sigma_s)$

$Pr(\epsilon)$ dissipation interesting!

$Pr(\Omega)$ enstrophy interesting!

Euler flows: ?: ?

Fixed points associated with reconnection?

New optical measurement of gradients and velocity at Kolmogorov scale

$$M = \partial u_i / \partial x_j = S + A$$

Dissipation (strain)

$$\varepsilon = \nu \|\mathbf{S}\|^2/2$$

$$\varepsilon(\vec{x}, t)$$

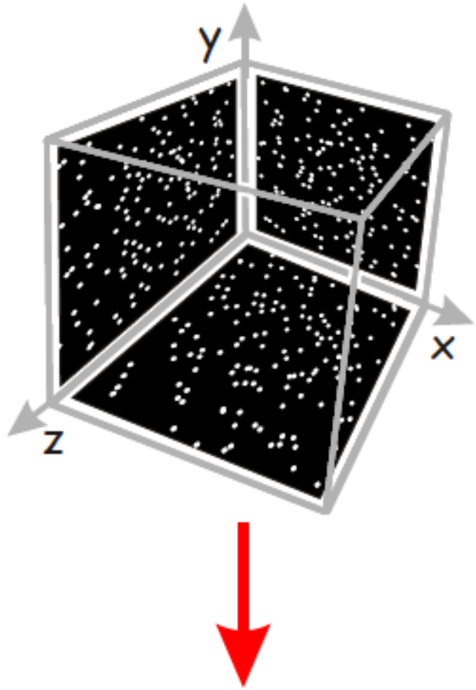
Enstrophy (rotation)

$$\Omega = \|\mathbf{A}\|^2/2$$

$$\Omega(\vec{x}, t)$$

Two interacting fields
What are the causal connections?
Oscillating grid turbulence at $R_\lambda = 54$

Measuring Gradients

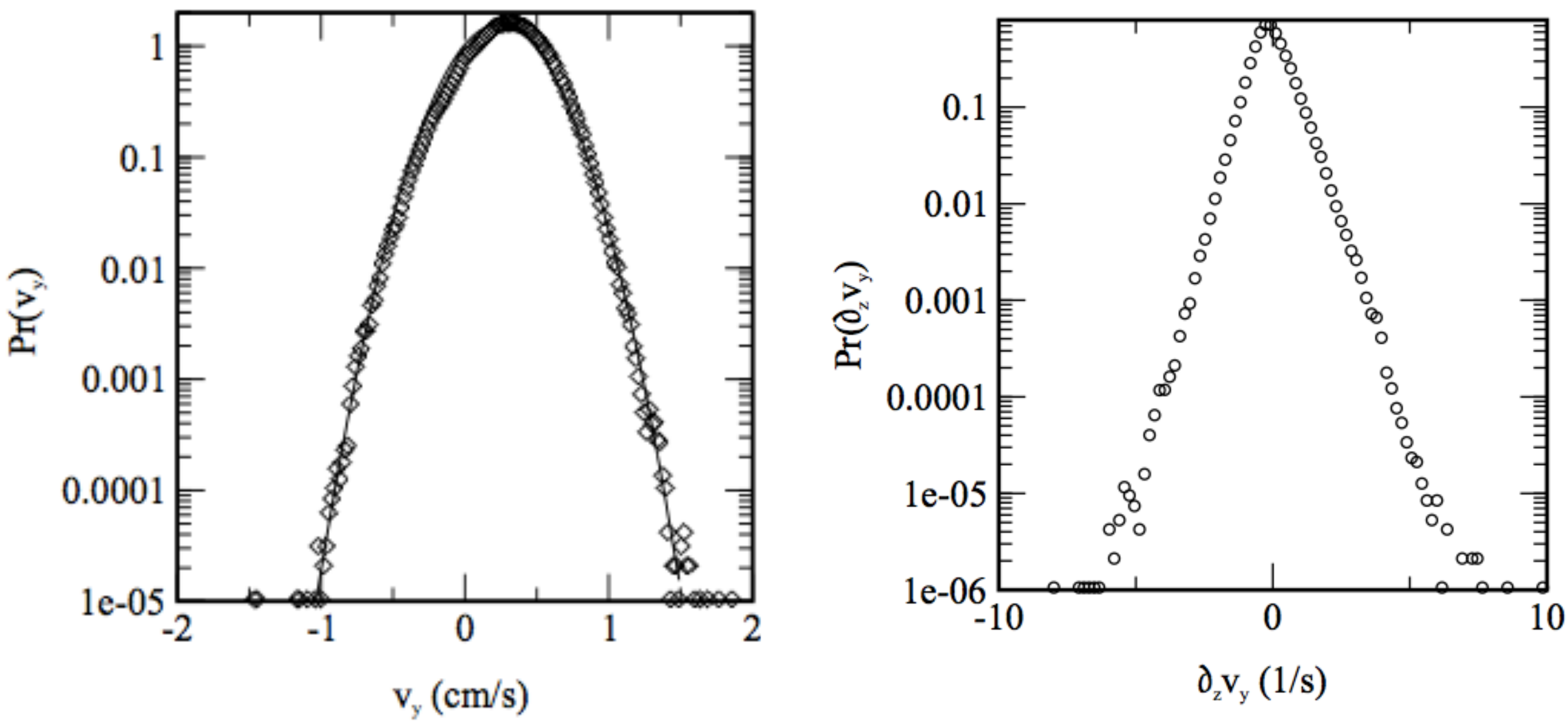


3 camera views

fit data to model

$$\vec{v} = \vec{v}_o + M \cdot \vec{x}$$

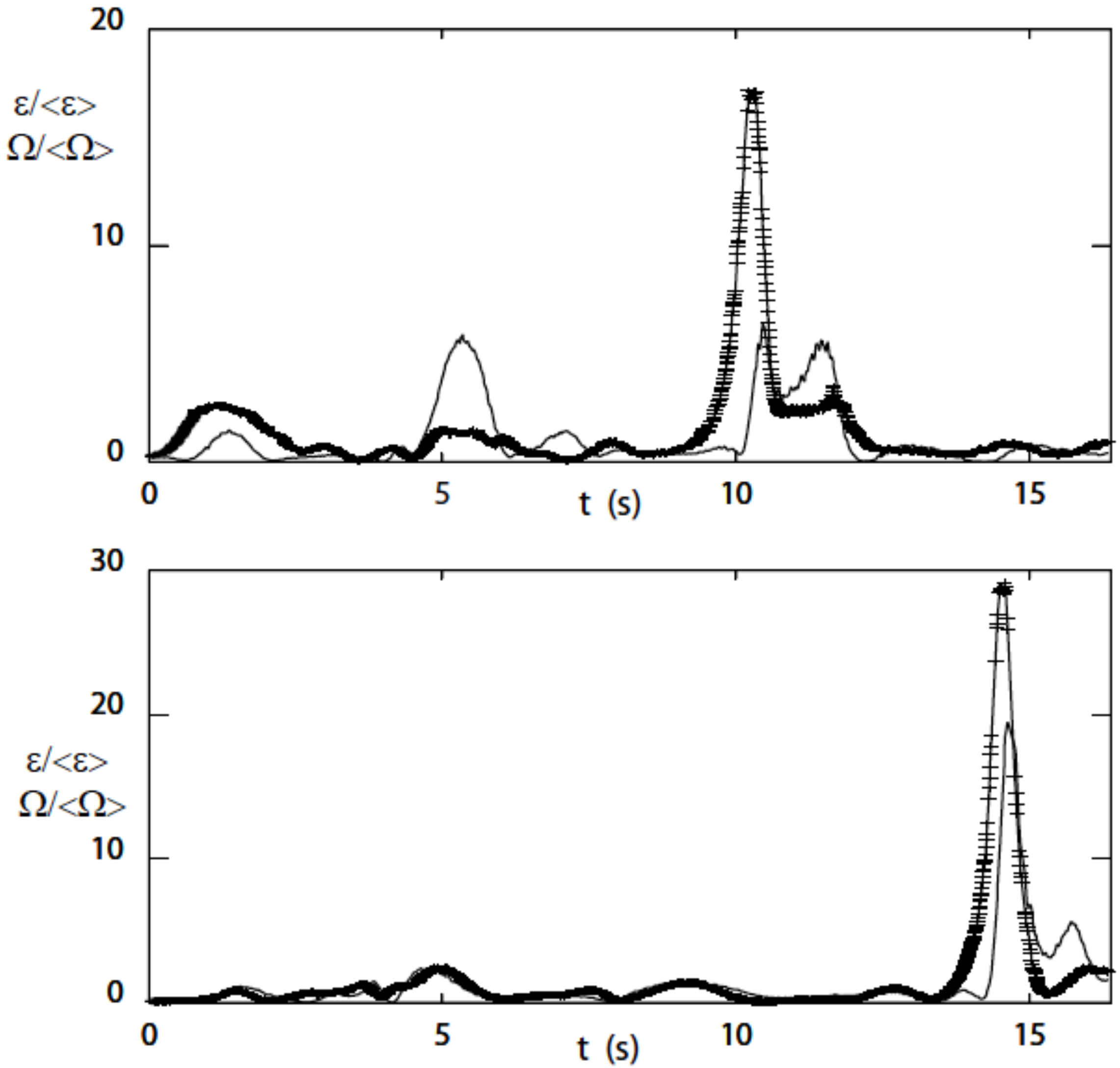
$$M = \begin{bmatrix} \partial_x V_x & \partial_y V_x & \partial_z V_x \\ \partial_x V_y & \partial_y V_y & \partial_z V_y \\ \partial_x V_z & \partial_y V_z & \partial_z V_z \end{bmatrix}$$



1
0.1
0.01
0.001
0.0001
1e-05
-2
-1
0
1
2
$Pr(v_y)$
v_y (cm/s)
0.1
0.01
0.001
0.0001
1e-05
1e-06
-10
0
10
$Pr(\partial_z v_y)$
$\partial_z v_y$ (1/s)

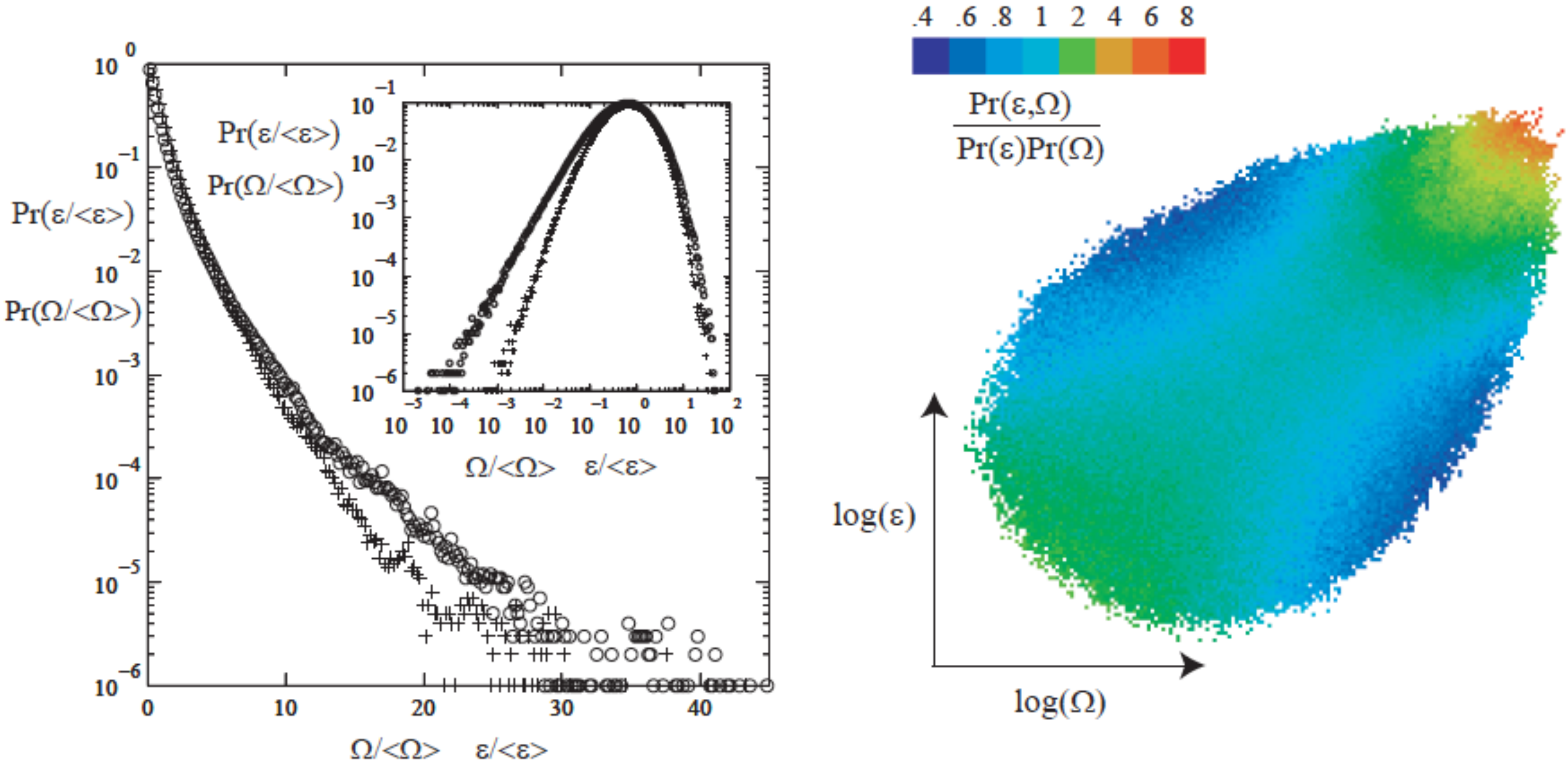
Dissipation and Enstrophy Time Series -- Intense Events

Zeff, Lanterman, McAllister, Roy, Kostelich, and Lathrop, Nature 421, 146 (2003)

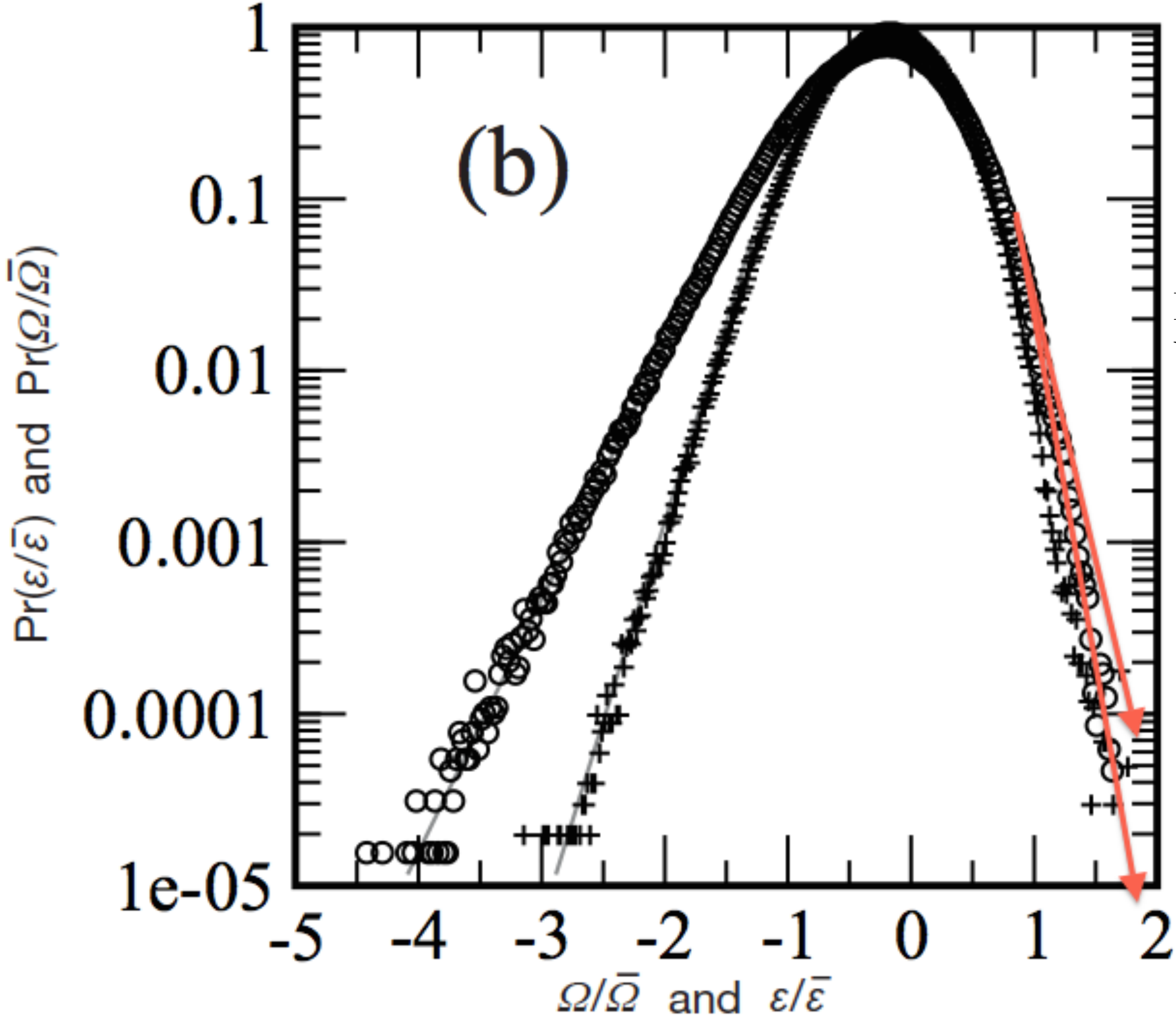


$$\varepsilon = \nu / 2 \, \| M_{ij} + M_{ji} \|_2$$

$$\Omega = 2 \, \| M_{ij} - M_{ji} \|_2 = \omega^2/2$$



Pr(ε/<ε>)
Pr(Ω/<Ω>)
Ω/<Ω> ε/<ε>
.4 .6 .8 1 2 4 6 8
Pr(ε,Ω) / Pr(ε)Pr(Ω)
log(ε)
log(Ω)

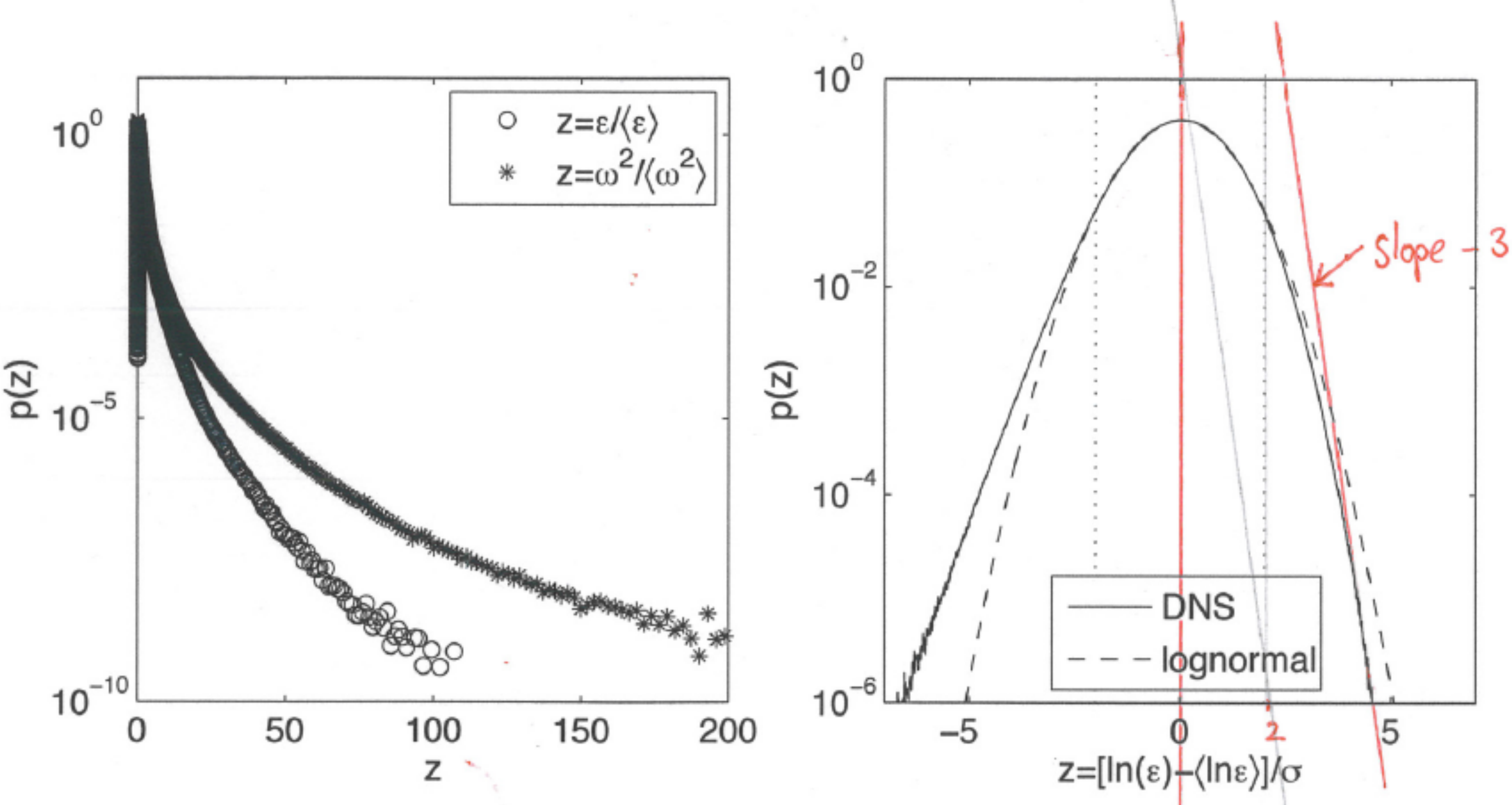


Does the Variance exist?

Hi Dan,

this is what I got for ε.

Cheers, Jörg



Euler flows: ?: ?

Fixed points associated with reconnection?

Turbulent intermittency and Euler similarity solutions

Daniel P. Lathrop

(Submitted on 20 Nov 2003)

Euler fixed points:

Stationary

Uniform velocity

Pure strain

Axisymmetric vortices $v_z(r)$ profiles

Reconnection fixed point?

$\vec{\omega} = \nabla x \vec{u}$

$\vec{u} x \vec{\omega} = \nabla \Pi$

Nontrivial Π topological constraints

Trivial Π yields $\vec{u} x \vec{\omega} = 0$

$\nabla x \vec{u} = \lambda \vec{u}$ Beltrami flows, i.e. force free fields

$\vec{u} = \vec{S} + \vec{T}$ Chandrasekhar 1954. S_2^2, T_2^2 solution similar to reconnection fixed point in GP, but oscillitory in the radial direction and helical...

Free surface flows: Topological: Wave crater collapse
Axisymmetric:
$\delta \sim (t_o - t)^{2/3}$ length scales
$v \sim (t_o - t)^{-1/3}$ velocity scales
$Pr(v) \sim |v|^{-4}$ velocity probability distribution

Similarity solutions appear to exist
2-D like axisymmetric!

Quantum fluid flows: Topological: Vortex reconnection
$\delta \sim (t_o - t)^{1/2}$ length scales
$v \sim (t_o - t)^{-1/2}$ velocity scales
$Pr(v) \sim |v|^{-3}$ velocity probability distribution in turbulence

Similarity solutions exist
Fixed points with reconnection geometry exists

Safety!

Navier-Stokes flows: ?: turbulence
$Pr(v_i) \sim \exp(-v_i^2/\sigma_v^2)$
$Pr(\partial_j v_i) \sim \exp(-|v_i|/\sigma_s)$
$Pr(\epsilon)$ dissipation interesting!
$Pr(\Omega)$ enstrophy interesting!

Euler flows: ?: ?
Fixed points associated with reconnection?

Danger!