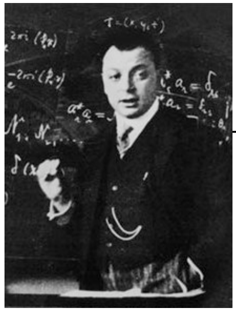

Turbulence in Magnetised Plasma

Successes, Failures, and the Known Unknowns

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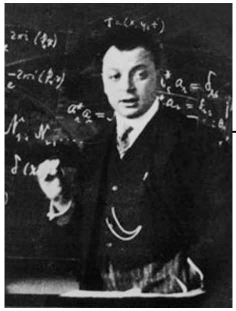
S. Cowley *(Culham)*
W. Dorland, T. Tatsuno, G. Plunk *(Maryland)*,
G. Howes *(Iowa)*, E. Quataert *(Berkeley)*, G. Hammett *(Princeton)*
T. Horbury, R. Wicks *(Imperial)*, C. Chen *(Berkeley)*, A. Mallet *(Oxford)*
M. Kunz *(Oxford)*, F. Rincon *(Toulouse)*, M. Rosin *(UCLA)*

Schekochihin *et al.*, ApJS **182**, 310 (2009)
Schekochihin *et al.*, MNRAS **405**, 291 (2010)
Rosin *et al.*, MNRAS, in press; arXiv:1002.4017



Part I. The Knowns

Schekochihin *et al*., ApJS **182**, 310 (2009)
Schekochihin *et al*., MNRAS **405**, 291 (2010)
Rosin *et al*., MNRAS, in press; arXiv:1002.4017

1. Free Energy Cascade

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\underbrace{\frac{T_{0s}\delta f_s^2}{2F_{0s}}}_{-T\delta S}+\underbrace{\frac{E^2+B^2}{8\pi}}_{\text{energy}}\right]$$

$$=\underbrace{\varepsilon}_{\text{injection}}+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\underbrace{\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}_{\text{heating}}$$

Generalised energy = free energy of the particles + fields

Kruskal & Oberman 1958
Fowler 1968
Krommes & Hu 1994
Krommes 1999
Sugama et al. 1996
Hallatschek 2004
Howes et al. 2006
Schekochihin et al. 2007
Scott 2007

PPCF **50**, 124024 (2008)

Plasma Turbulence: Analogous to Fluid, But…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

injection heating

small scales in 6D phase space

$$\partial_t\boldsymbol{u}+\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}=-\boldsymbol{\nabla}p+\nu\nabla^2\boldsymbol{u}+\boldsymbol{f}$$

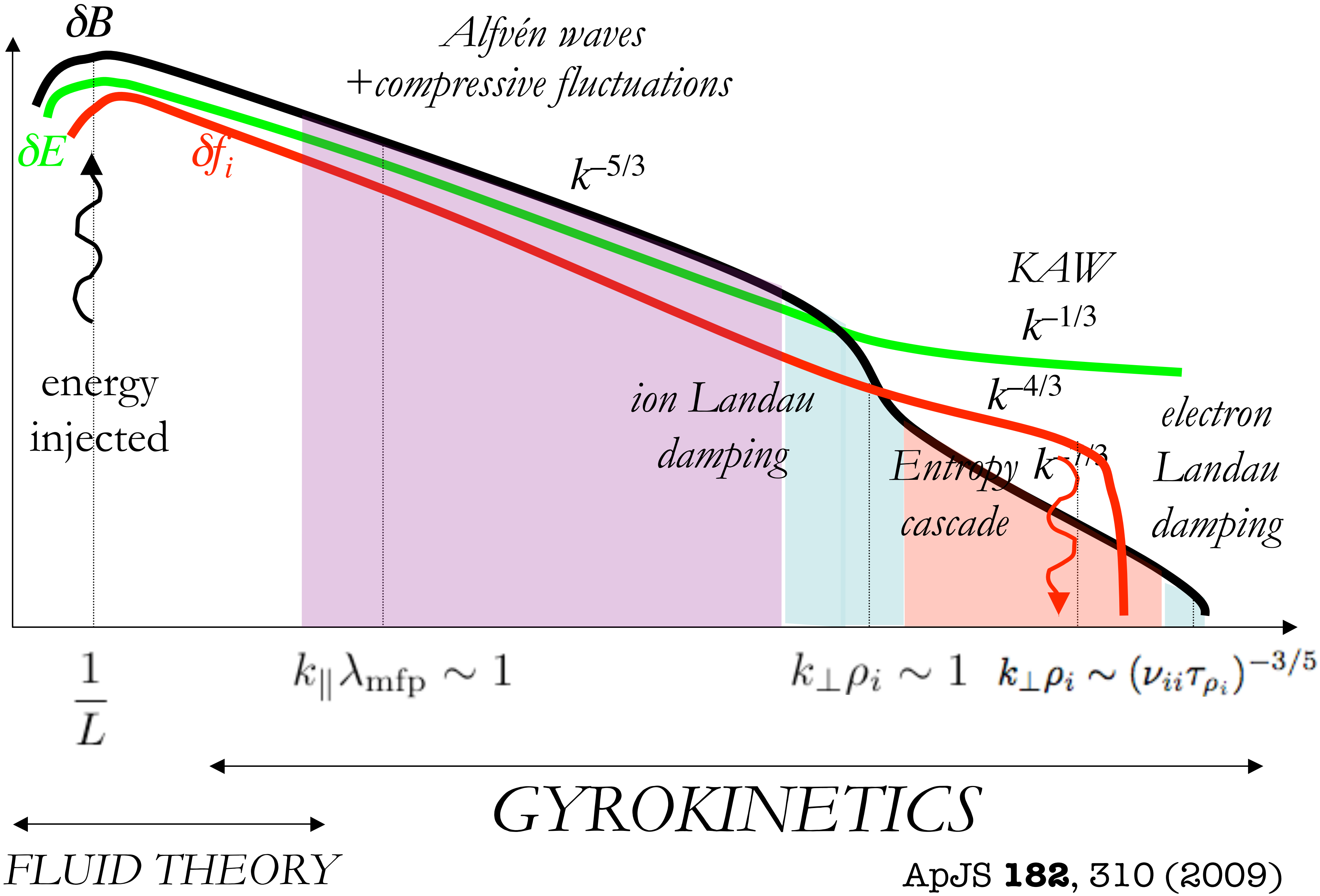
$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{u^2}{2}=\varepsilon-\nu\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}|\boldsymbol{\nabla}\boldsymbol{u}|^2$$

small scales in 3D physical space

$$\varepsilon=(1/V)\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{u}\cdot\boldsymbol{f}$$

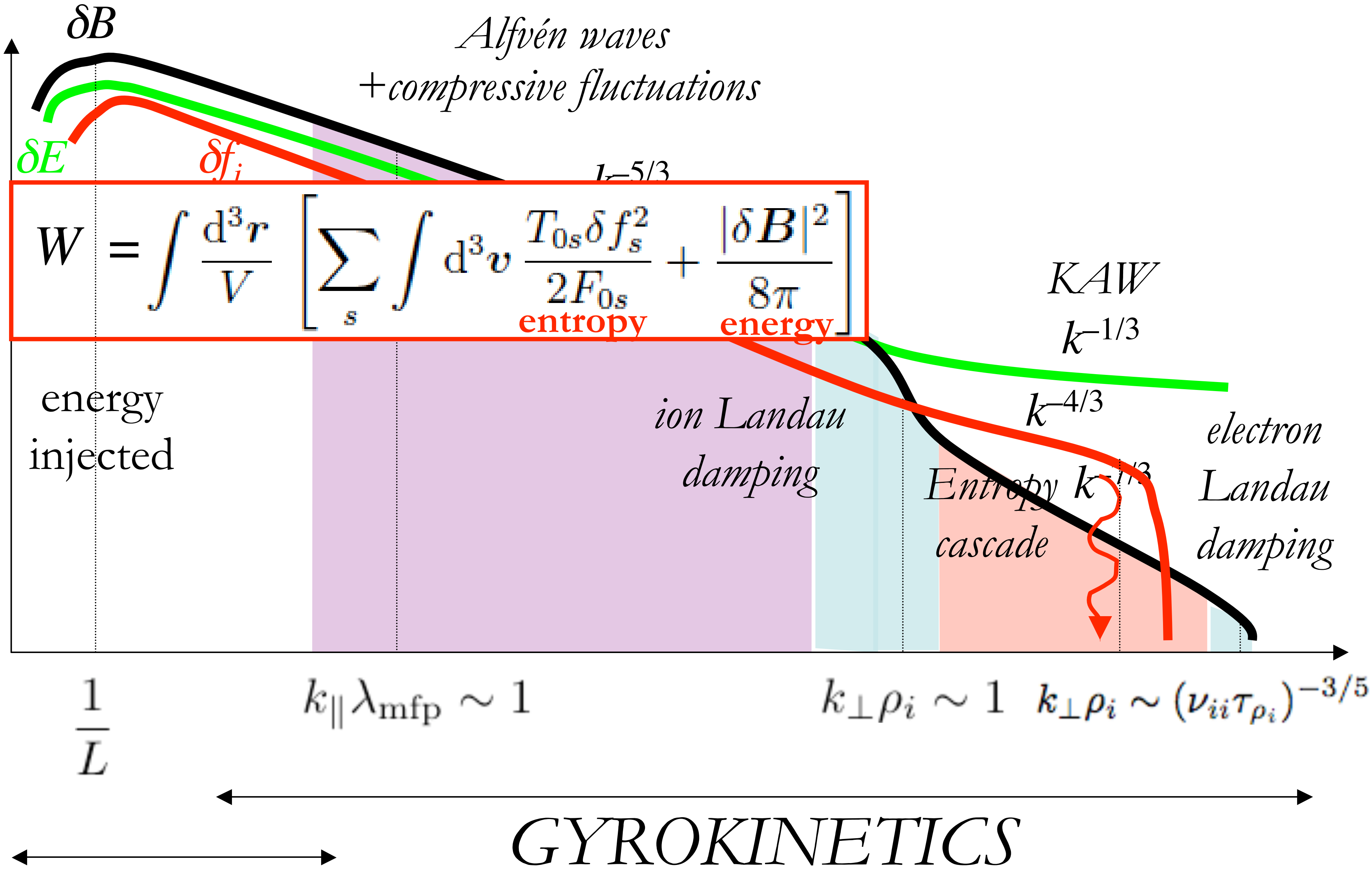
PPCF **50**, 124024 (2008)

Free Energy Cascade



ApJS **182**, 310 (2009)

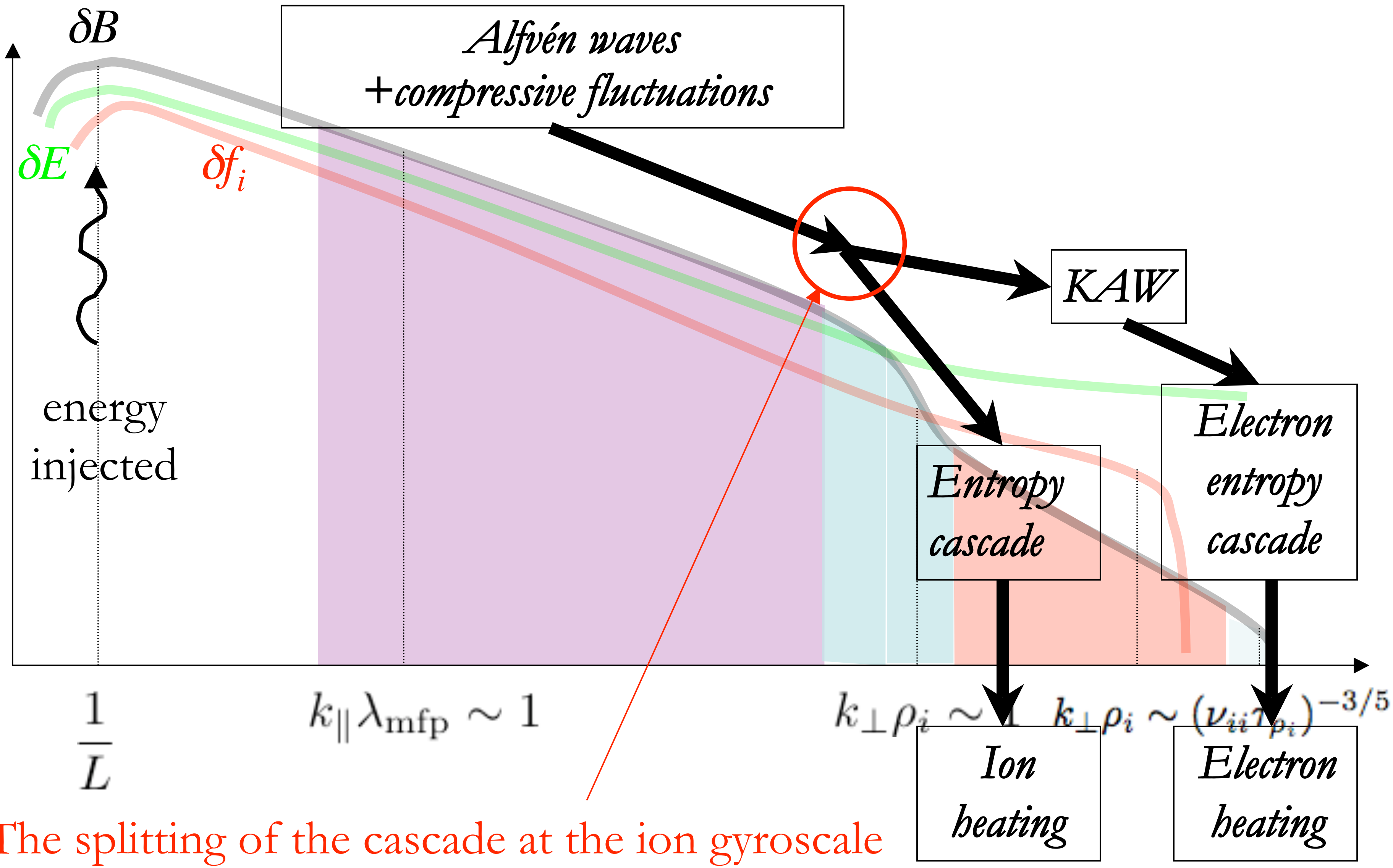
Free Energy Cascade



FLUID THEORY

ApJS **182**, 310 (2009)

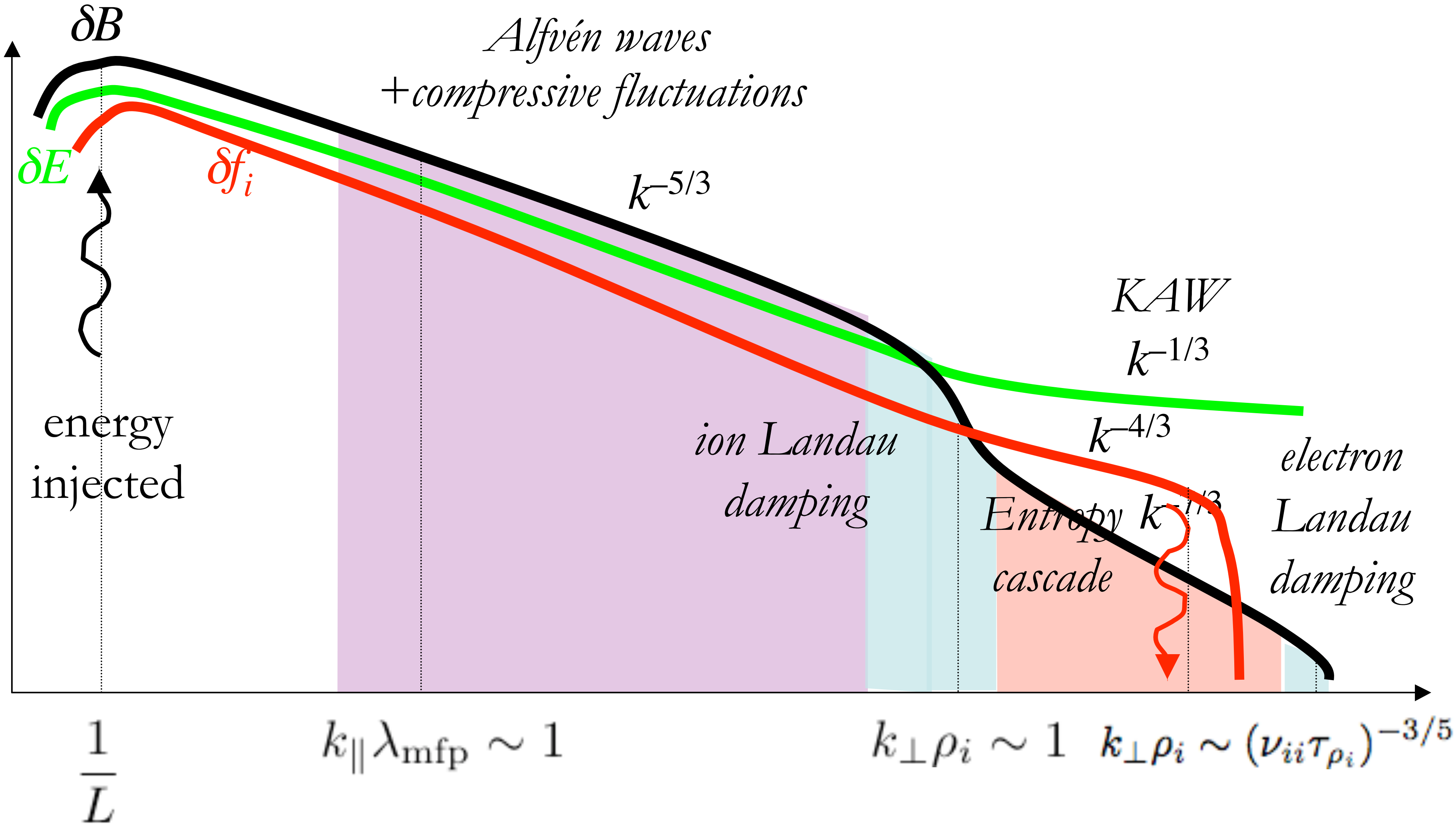
Route to Heating (Dissipation)



The splitting of the cascade at the ion gyroscale determines relative heating of the species

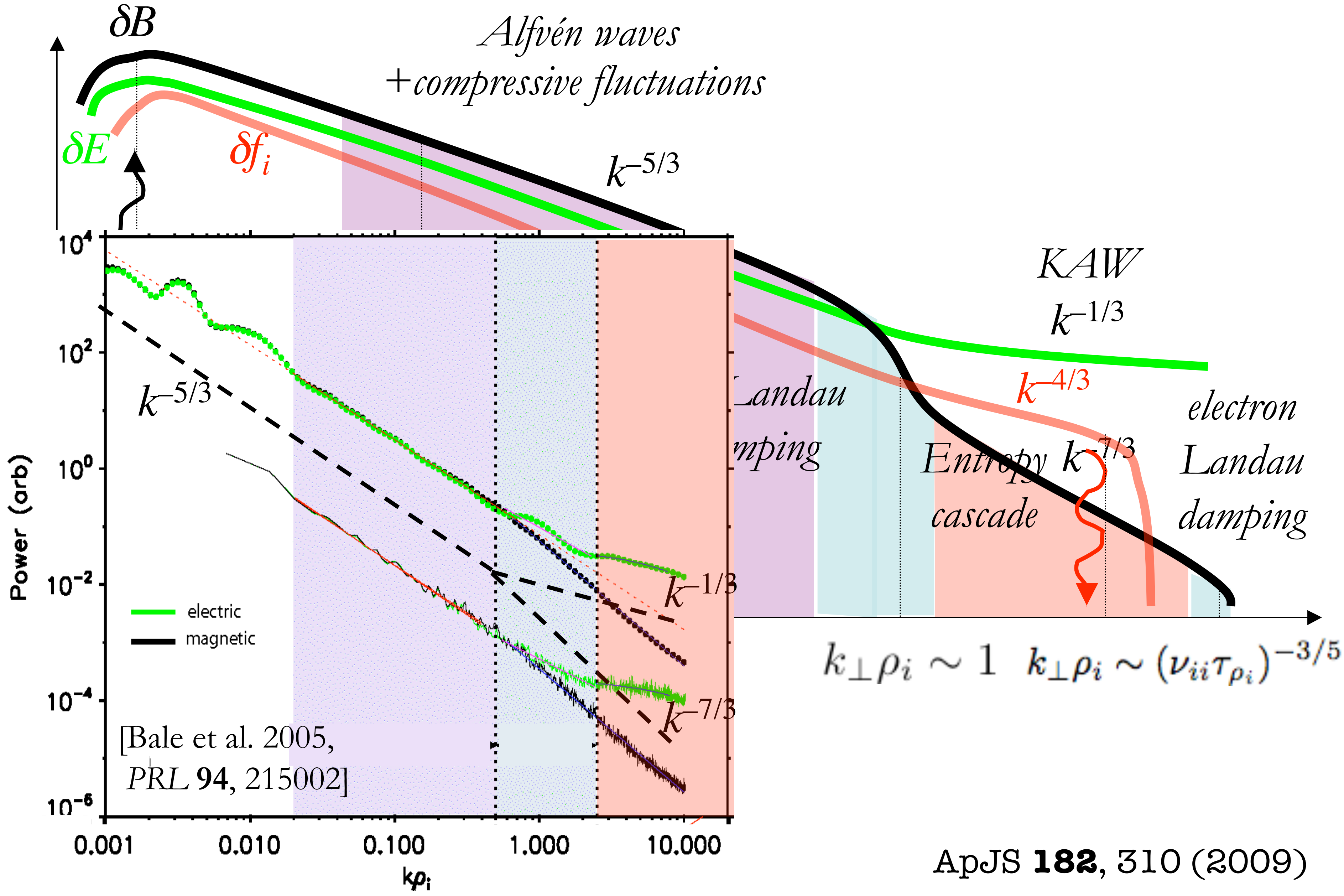
ApJS **182**, 310 (2009)

Free Energy Cascade: Solar Wind, DNS



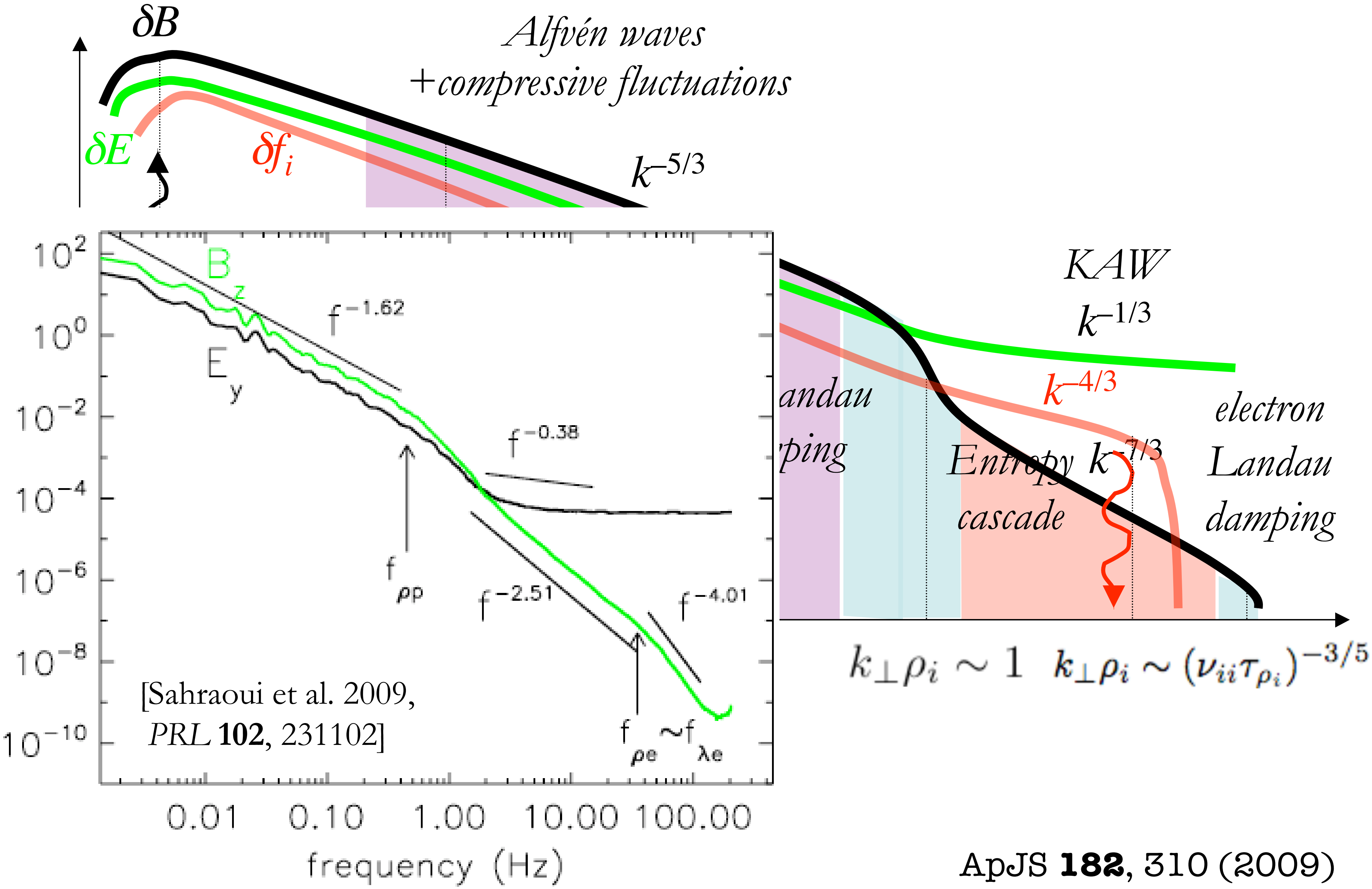
ApJS **182**, 310 (2009)

Free Energy Cascade: Solar Wind



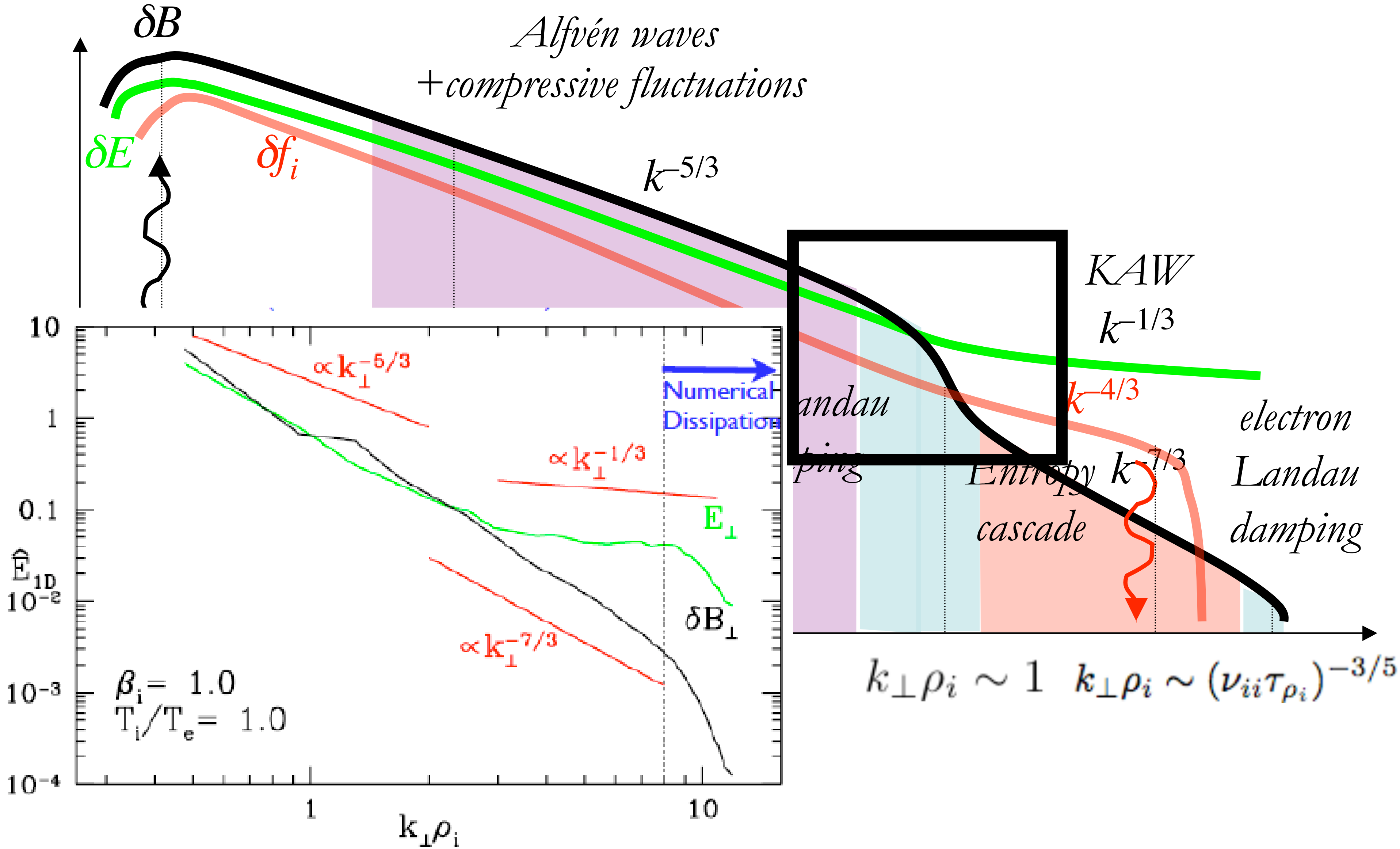
ApJS **182**, 310 (2009)

Free Energy Cascade: Solar Wind



ApJS **182**, 310 (2009)

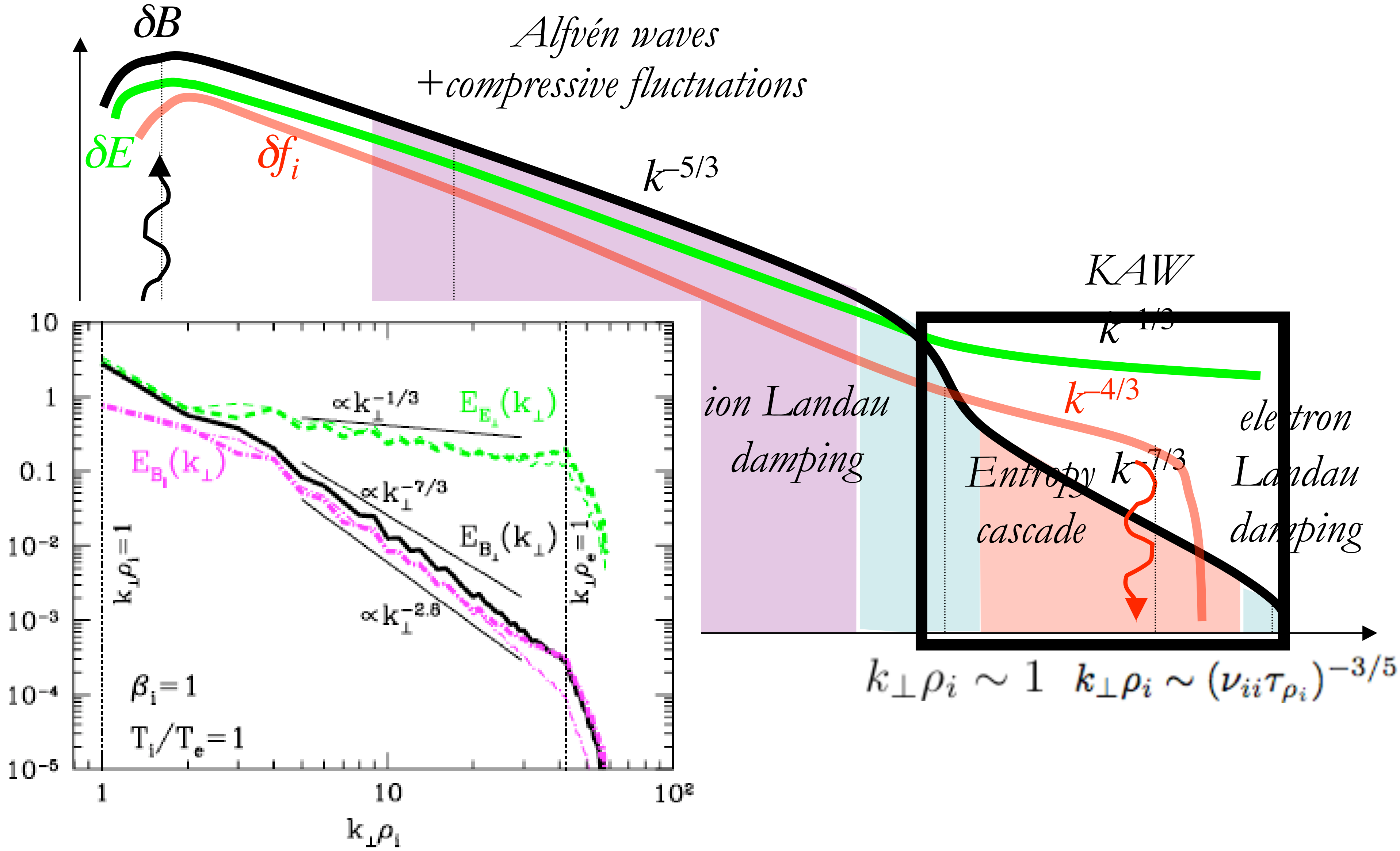
GK Cascade: 3D DNS (by G. Howes)



[Howes et al. 2008, *PRL* **100**, 065004]

ApJS **182**, 310 (2009)

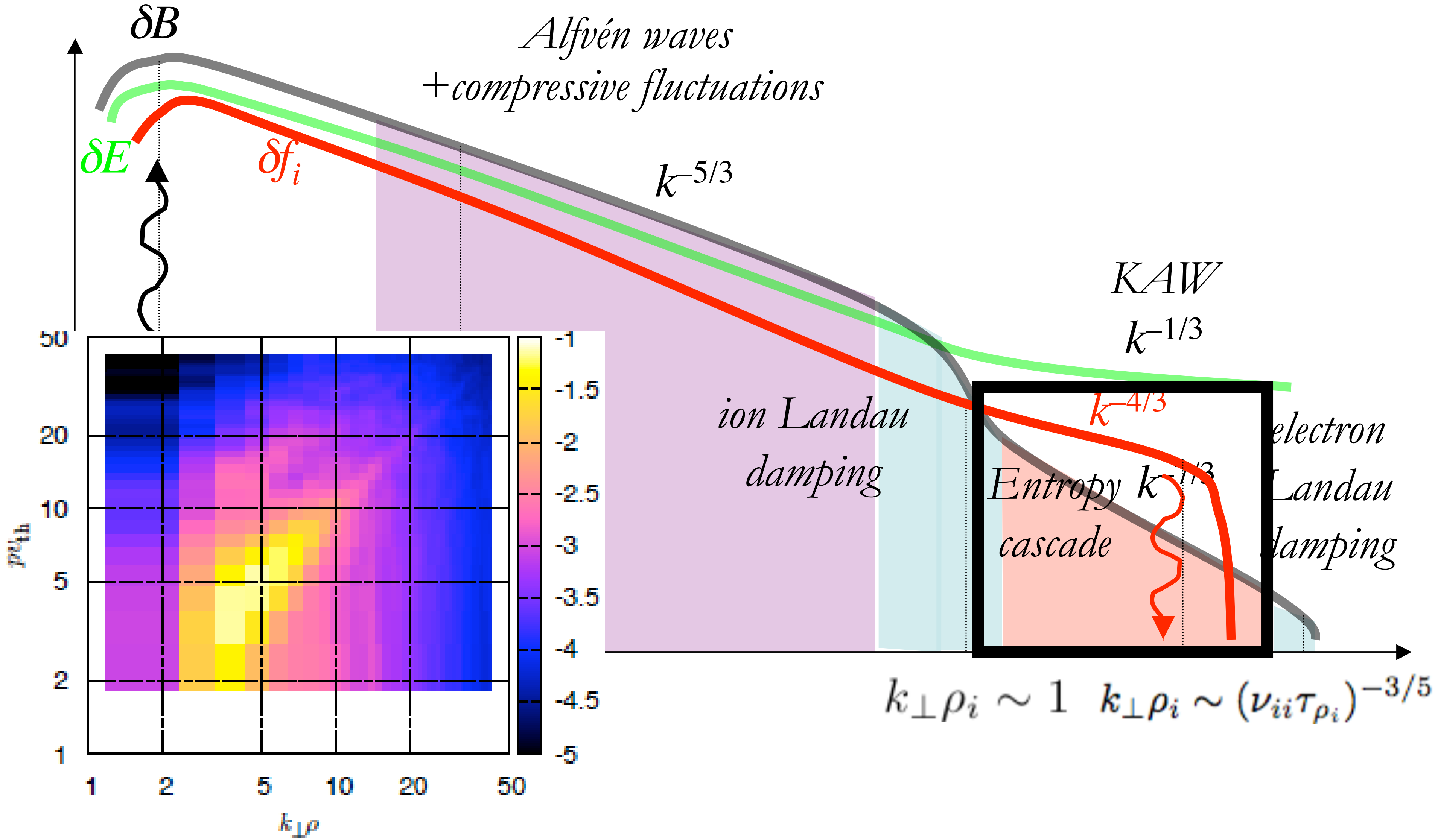
GK Cascade: 3D DNS (by G. Howes)



[Howes et al. 2011, submitted]

ApJS **182**, 310 (2009)

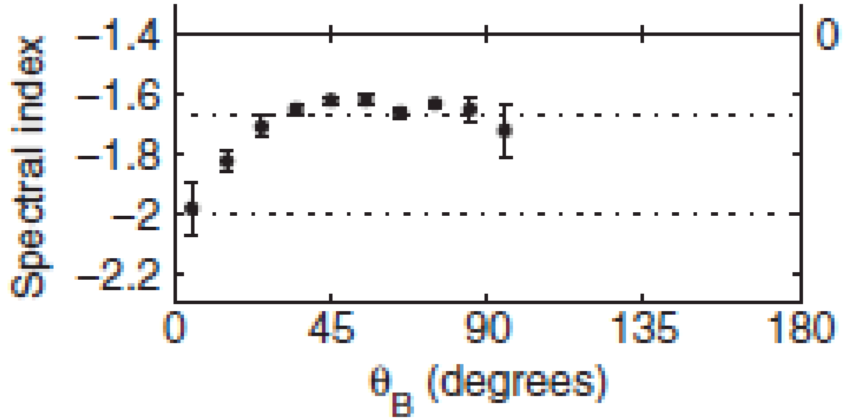
GK Cascade: 2D DNS (by T. Tatsuno)



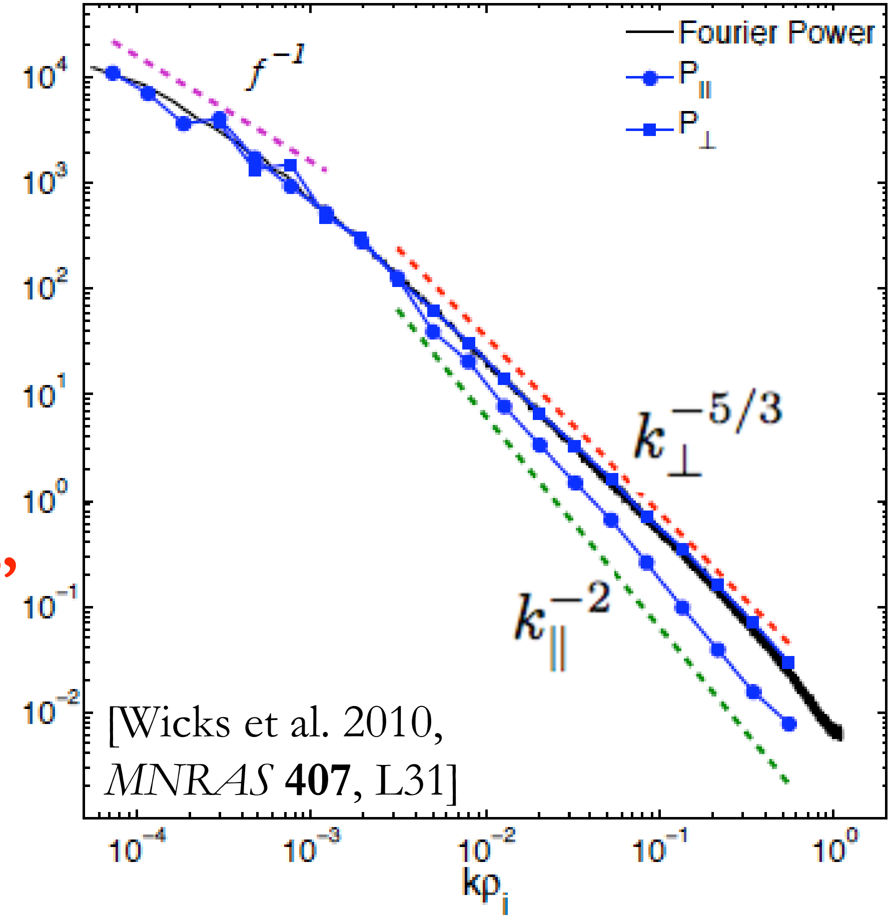
[Tatsuno et al. 2009, *PRL* **103**, 015003;
more detail in arXiv:1003.3933]

ApJS **182**, 310 (2009)

2. Anisotropy at All Scales: Inertial Range



[Horbury et al. 2008, *PRL* **101**, 175005]



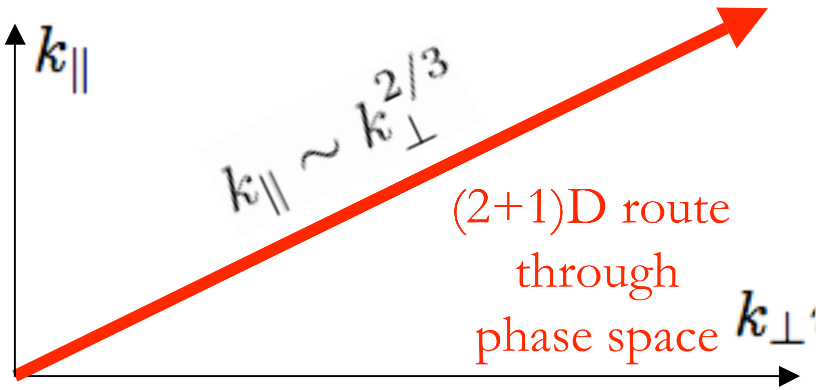
[Wicks et al. 2010, *MNRAS* **407**, L31]

$\omega_{\text{linear}} \sim \omega_{\text{nonlinear}}$

"critical balance"

Alfvénic (MHD) turbulence:

$k_{\|} v_A \sim k_{\perp} u_{\perp}$ [Goldreich & Sridhar 1995]



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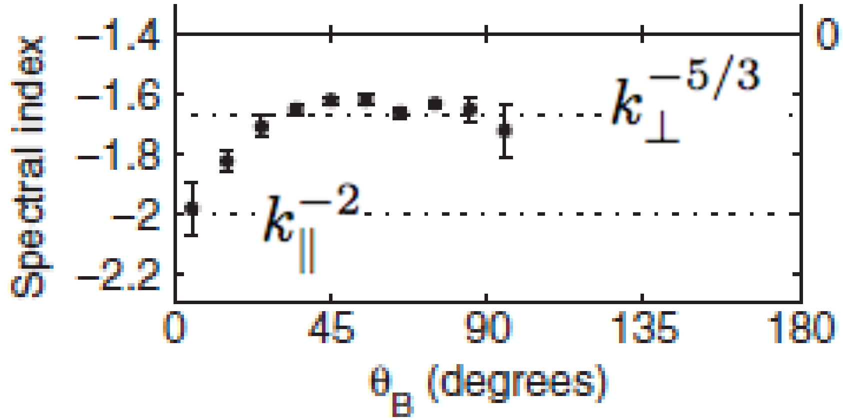

[Horbury et al. 2008, *PRL* **101**, 175005]

$\omega_{\text{linear}} \sim \omega_{\text{nonlinear}}$ **"critical balance"**

Alfvénic (MHD) turbulence:

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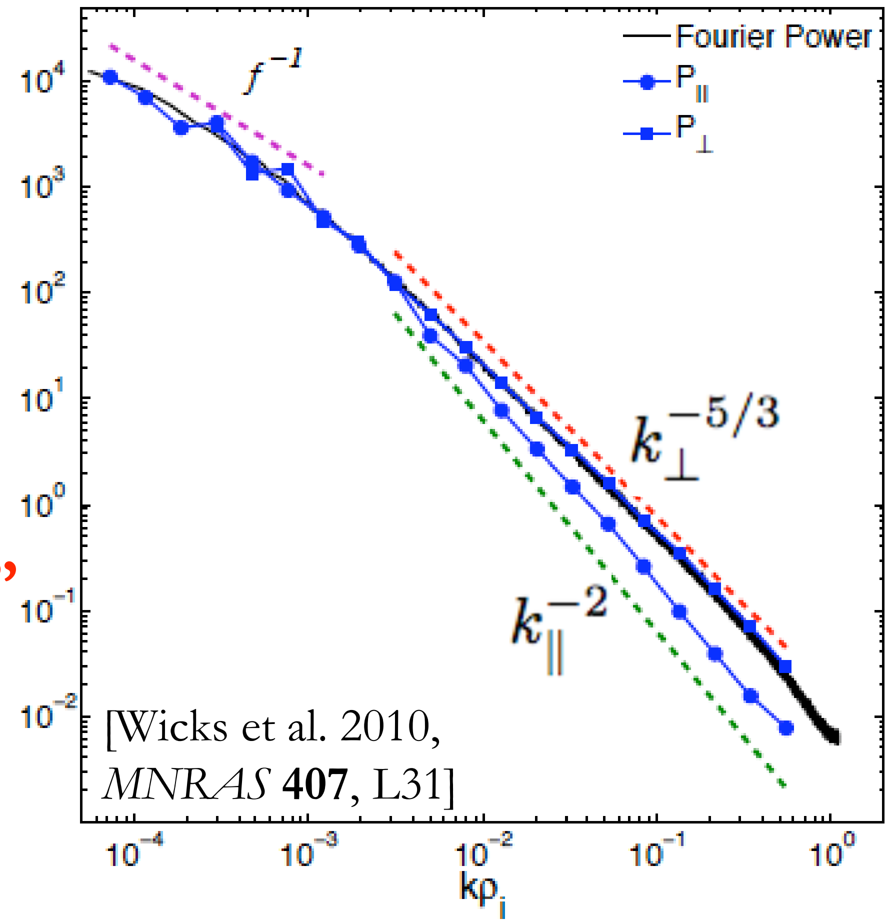
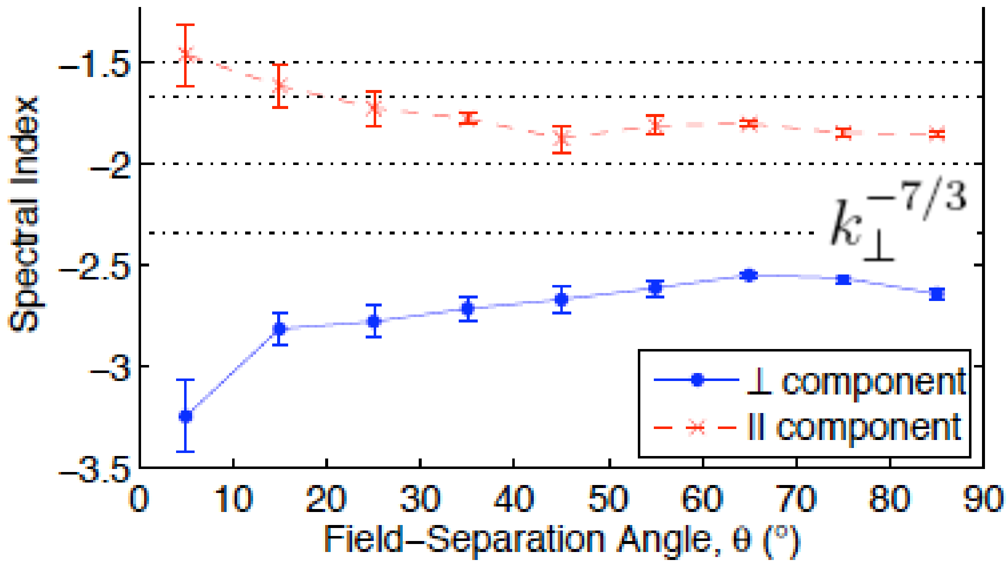
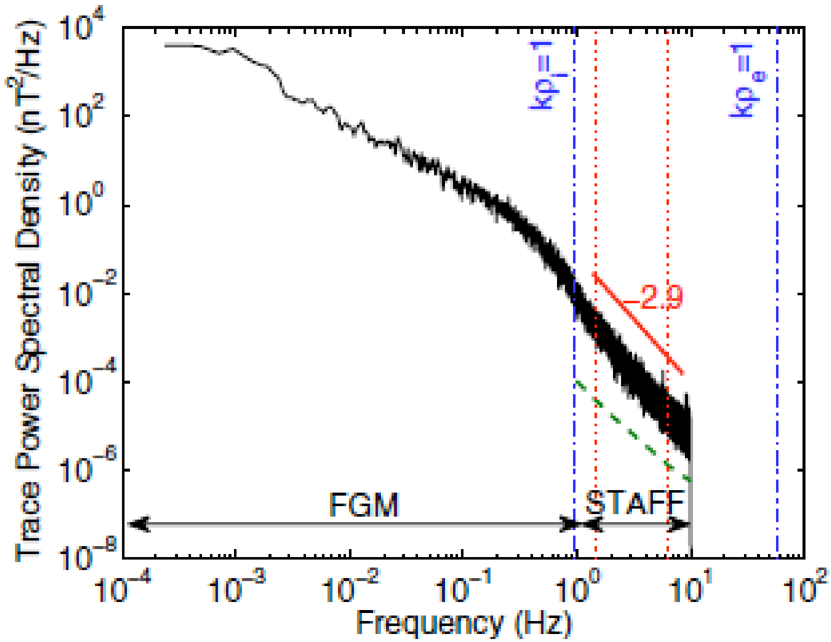


This can be argued to be a **universal feature** of anisotropic wave turbulence and works! E.g.,

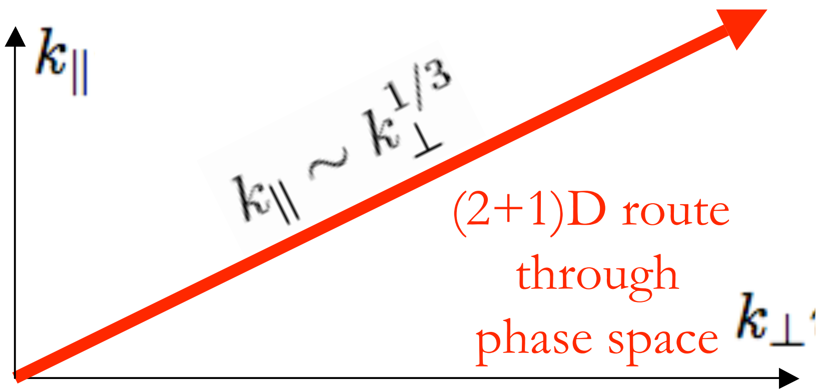
- KAW turbulence [Cho & Lazarian 2004, *ApJ* **615**, L41]
- Rotating hydro turbulence [Nazarenko & Schekochihin 2011, *JFM*; arXiv:0904.3488]
- ITG turbulence in tokamaks [Barnes, Parra & Schekochihin 2011, in preparation]

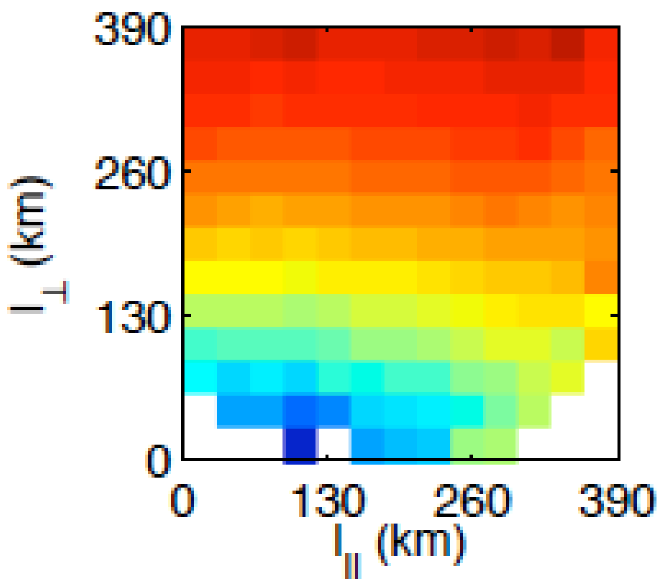
2. Anisotropy at All Scales: Sub-Larmor Rang





[Chen et al. 2010, *PRL* **104**, 255002]





3. Plasma Microinstabilities: Origin

First adiabatic invariant $\mu = \frac{mv_\perp^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$

holds already for $B > 10^{-18}$ G

Changes in field strength ⇔ pressure anisotropy

$$\sum_{\text{particles}} \mu = \frac{p_\perp}{B} = \text{const}$$

3. Plasma Microinstabilities: Origin

First adiabatic invariant $\mu = \frac{mv_\perp^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$

holds already for $B > 10^{-18}$ G

Changes in field strength ⇔ pressure anisotropy

$$\frac{d\Delta}{dt} \sim \frac{1}{B}\frac{dB}{dt} - \nu_{ii}\Delta \qquad \Delta \equiv \frac{p_\perp - p_\parallel}{p_\perp}$$

ignore evolution of $p_\parallel$ — change in B drives anisotropy — anisotropy relaxed by collisions

[Schekochihin *et al.*, *ApJ* **629**, 139 (2005)]

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$$\frac{d\Delta}{dt} \sim \frac{1}{B}\frac{dB}{dt} - \nu_{ii}\Delta$$

ignore evolution of $p_{\|}$ — change in B drives anisotropy — anisotropy relaxed by collisions

$$\Delta \sim \frac{1}{\nu_{ii}}\frac{d\ln B}{dt} = \frac{\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u}}{\nu_{ii}}$$

because $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$

and so $\frac{1}{B}\frac{dB}{dt} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u}$

[Schekochihin *et al.*, *ApJ* **629**, 139 (2005)]

3. Plasma Microinstabilities: Taxonomy

First adiabatic invariant $\mu = \dfrac{mv_\perp^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$

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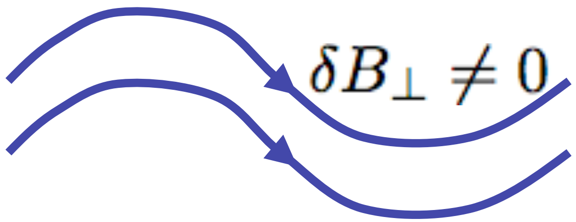
Changes in field strength ⇔ pressure anisotropy

$$\frac{d\Delta}{dt} \sim \frac{1}{B}\frac{dB}{dt} - \nu_{ii}\Delta \;\Longrightarrow\; \Delta \sim \frac{1}{\nu_{ii}}\frac{d\ln B}{dt} = \frac{\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u}}{\nu_{ii}}$$

Magnetic field decreases: Δ<0

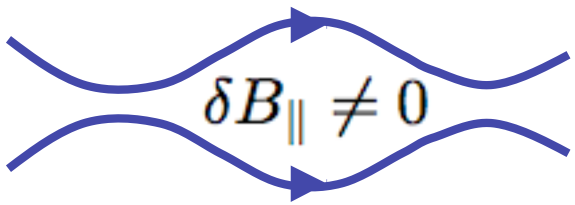
FIREHOSE: $\omega^2 = \dfrac{k_\parallel^2 v_{\mathrm{th}i}^2}{2}\left(\Delta + \dfrac{2}{\beta_i}\right)$



destabilised Alfvén wave

Magnetic field increases: Δ>0

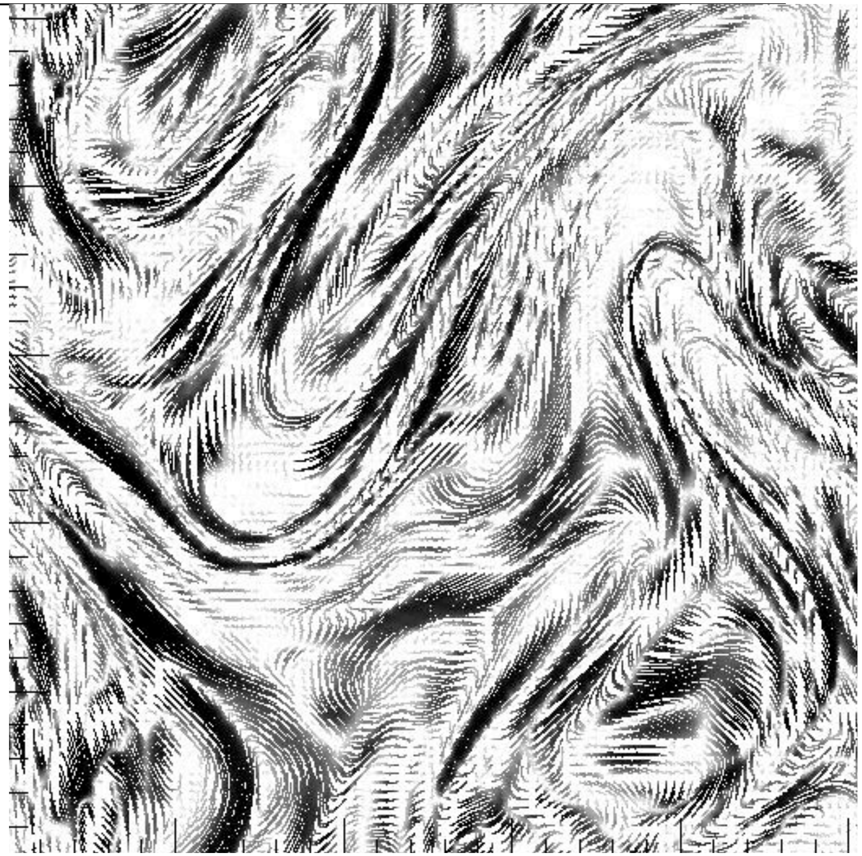
MIRROR: $\gamma = \dfrac{|k_\parallel| v_{\mathrm{th}i}}{\sqrt{\pi}}\left(\Delta - \dfrac{1}{\beta_i}\right)$



resonant instability

3. Plasma Microinstabilities: Where and When?

Typical structure of magnetic fields generated by turbulence (MHD simulations with Pm >> 1 by A. B. Iskakov & AAS) for details see Schekochihin *et al.* 2004, *ApJ* **612**, 276



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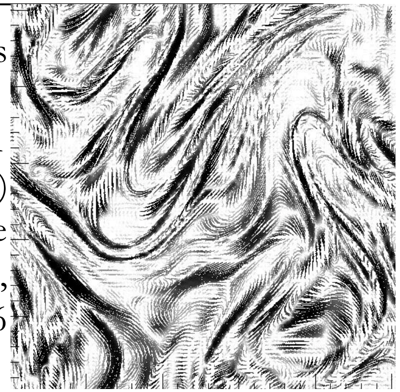

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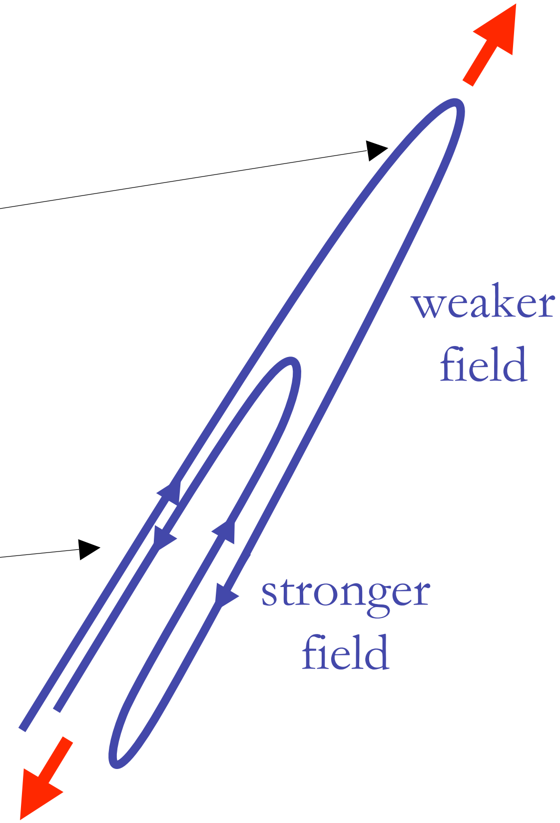
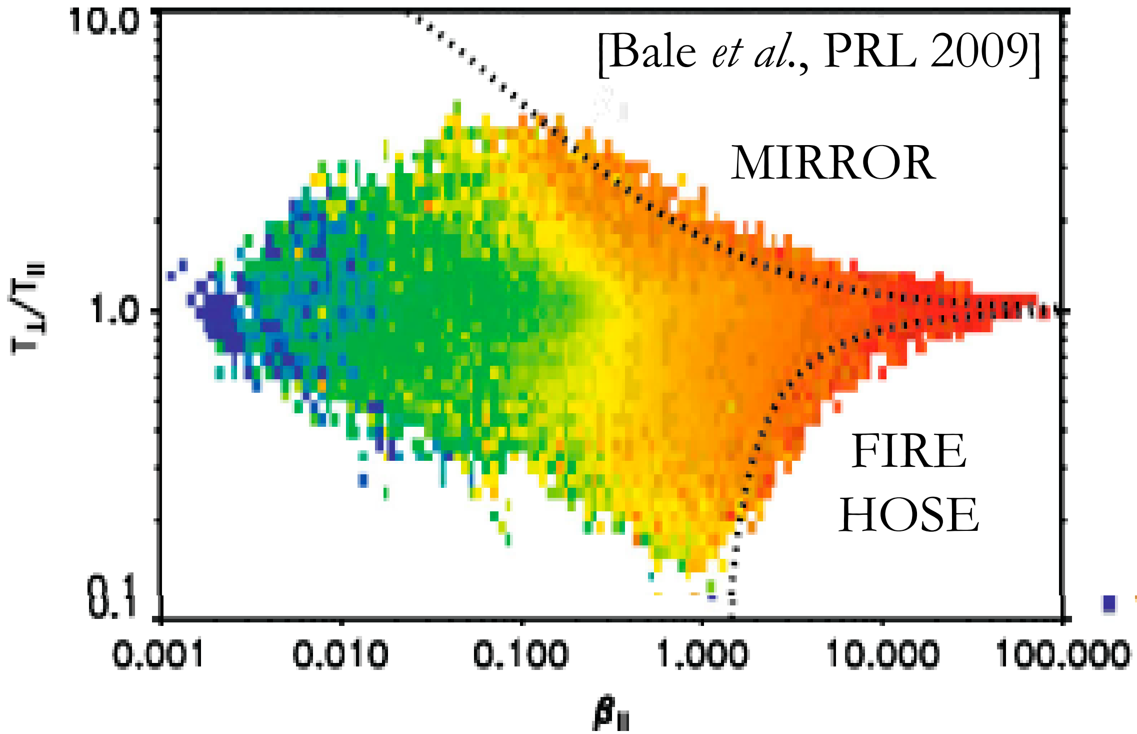


[Schekochihin *et al.*, *ApJ* **629**, 139 (2005)]

Solar Wind: Marginal



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FIREHOSE: $\omega^2 = \frac{k_\parallel^2 v_{\mathrm{th}i}^2}{2}\left(\Delta + \frac{2}{\beta_i}\right)$

Magnetic field increases: Δ>0

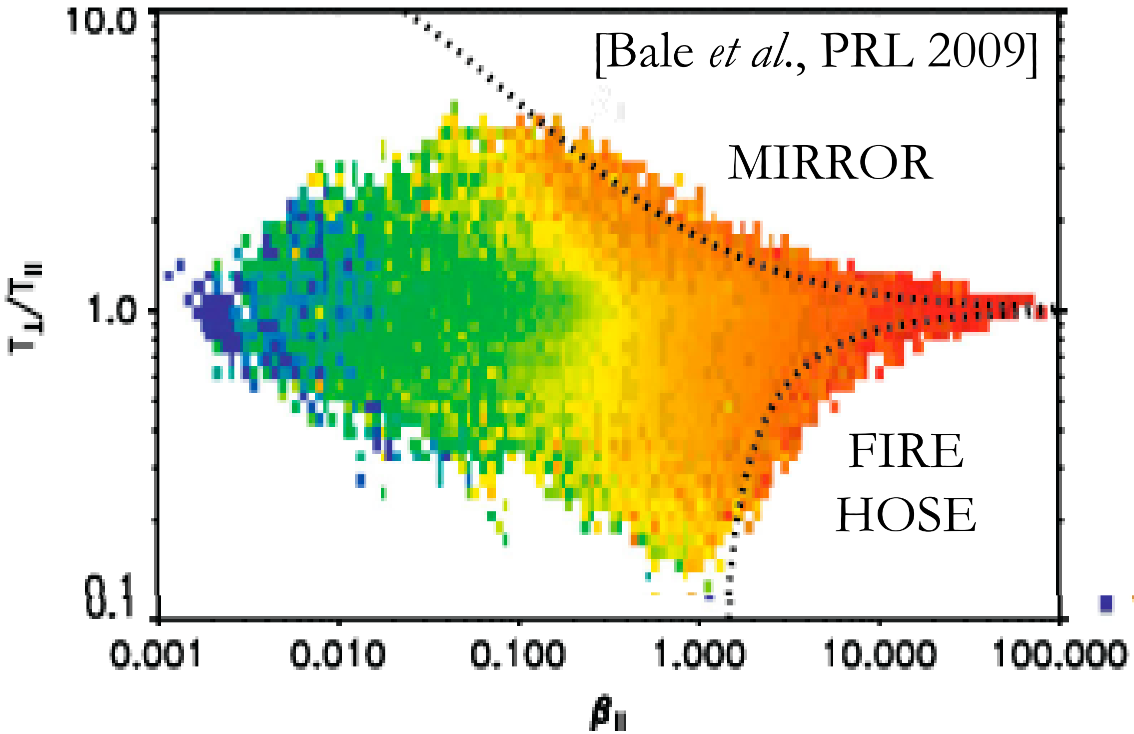
MIRROR: $\gamma = \frac{|k_\parallel| v_{\mathrm{th}i}}{\sqrt{\pi}}\left(\Delta - \frac{1}{\beta_i}\right)$

Plasma is in the marginal state with respect to plasma microinstabilities

How to Model The Marginal State?

To leapfrog having to do
an honest microphysical job,
simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$



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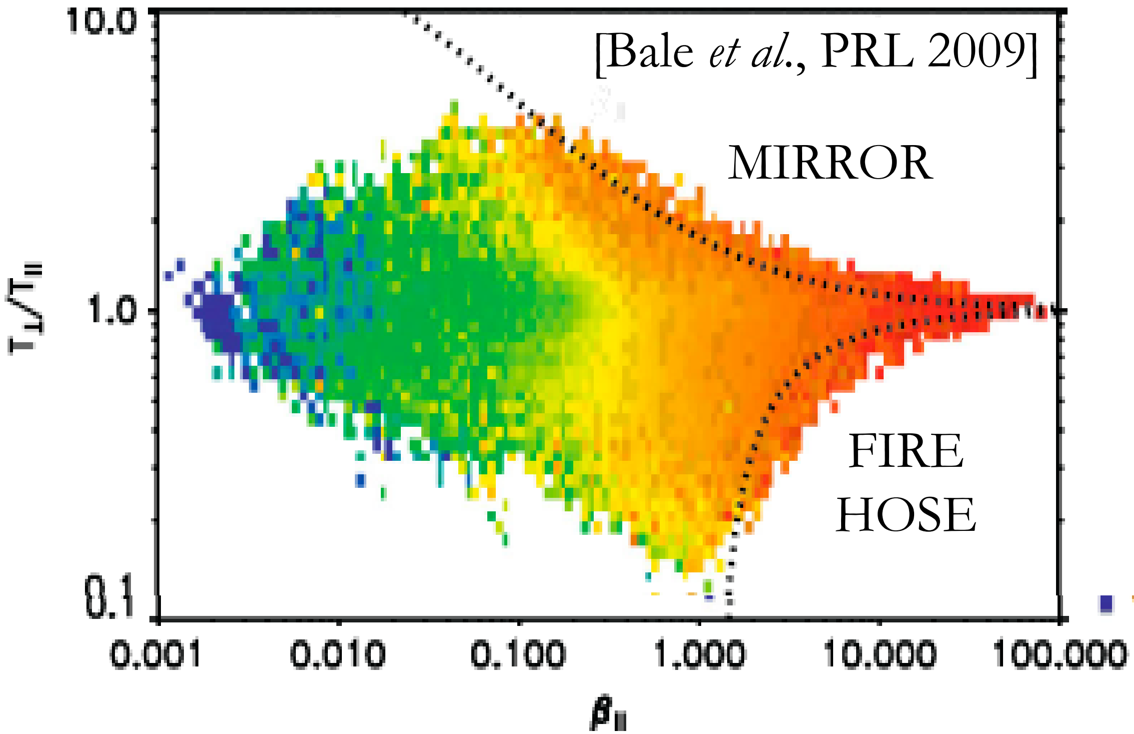
[Kunz *et al.*, *MNRAS* **410**, 2446 (2011)]

A Microphysical Dilemma

To leapfrog having to do an honest microphysical job, simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$



How is this achieved?

- Enhanced particle scattering isotropises pressure
  AND/OR
- Magnetic field structure and evolution modified to offset change

Why This Is An Important Question

Oxford Physics

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an honest microphysical job,
simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$

$$\frac{d\Delta}{dt} \sim \boxed{\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u}} - \boxed{\nu_{ii}\Delta}$$

How is this achieved?

- Enhanced particle scattering isotropises pressure → Model by limiting Δ (more collisionality → **less** viscosity) [Sharma et al. 2006; Schekochihin & Cowley 2006]

AND/OR

- Magnetic field structure and evolution modified to offset change → Model by limiting rate of strain (in a sense, **more** viscosity) [Kunz et al. 2011]

Why This Is An Important Question

Oxford Physics

To leapfrog having to do an honest microphysical job, simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$

I believe this is going to be hard to justify because microinstabilities are not sufficiently close to the Larmor scale, so can't have much scattering

How is this achieved?

- Enhanced particle scattering isotropises pressure

AND/OR

- Magnetic field structure and evolution modified to offset change

Model by limiting Δ (more collisionality → **less** viscosity) [Sharma et al. 2006; Schekochihin & Cowley 2006]

Model by limiting rate of strain (in a sense, **more** viscosity) [Kunz et al. 2011]

Nonlinear Firehose

Principle of nonlinear evolution: *firehose fluctuations cancel on average the change in the mean field to keep anisotropy at marginal level*

$$\Delta \sim \frac{1}{\nu_{ii}} \frac{1}{B} \frac{dB}{dt} \sim \frac{1}{\nu_{ii}} \left(- \left| \frac{d \ln B_0}{dt} \right| + \frac{1}{2} \frac{d}{dt} \frac{\overline{|\delta \mathbf{B}_\perp|^2}}{B_0^2} \right) \to -\frac{2}{\beta_i}$$

macroscale field — microscale fluctuations

Schekochihin *et al.*, *PRL* **100**, 081301 (2008)
Rosin *et al.*, arXiv:1002.4017 (2010)

How is this achieved?

- Enhanced particle scattering isotropises pressure
  AND/OR
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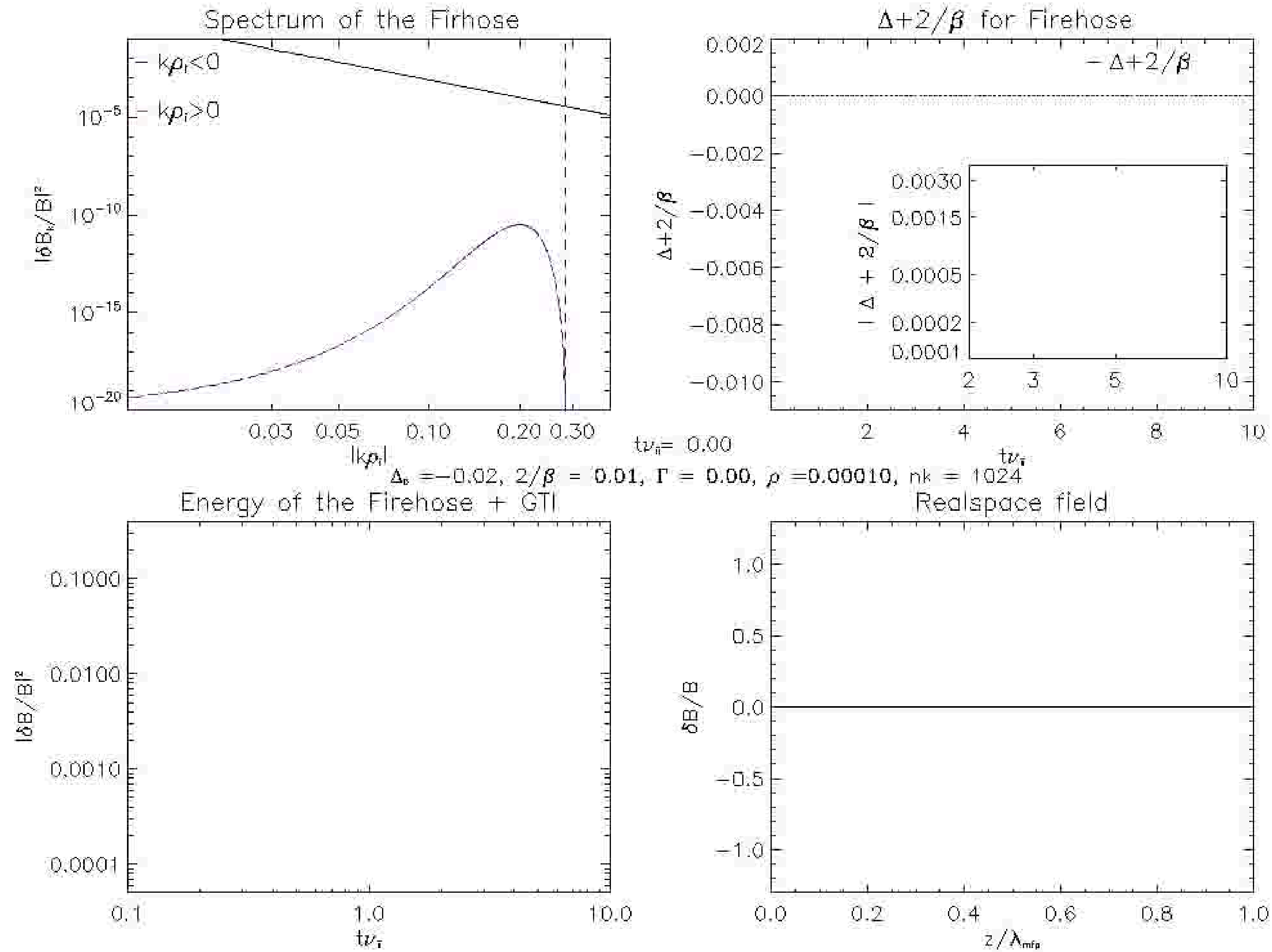
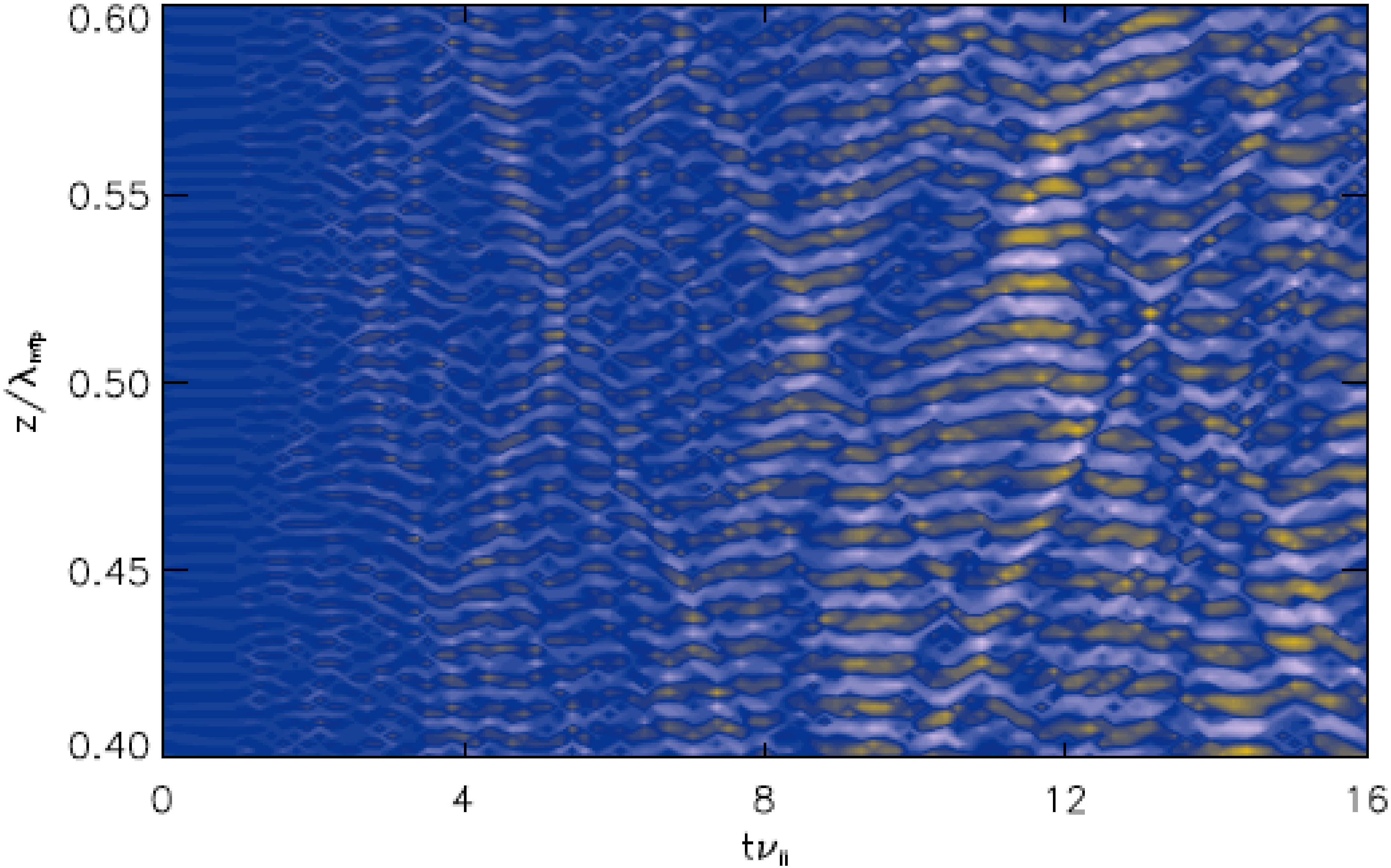
Nonlinear Firehose



[Rosin *et al.*, arXiv:1002.4017 (2010)]

Nonlinear Firehose



[Rosin *et al.*, arXiv:1002.4017 (2010)]

Gyrothermal Instability (GTI)

Heat fluxes also drive fast microphysical instabilities

[Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)]

Gyrothermal Instability: Equations

Heat fluxes also drive fast microphysical instabilities

- Keep the gyroviscous terms in the "Braginskii" stress (this is valid even without collisions and is necessary to get the fastest growing mode for the firehose)
- Keep pressure anisotropies and parallel ion heat fluxes

$$mn\frac{d\boldsymbol{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\boldsymbol{bb}\left(p_\perp - p_\parallel + \frac{B^2}{4\pi}\right) - \mathsf{G}\right]$$

$$\mathsf{G} = \frac{1}{4\Omega}[\boldsymbol{b}\times\mathsf{S}\cdot(\mathsf{I}+3\boldsymbol{bb}) - (\mathsf{I}+3\boldsymbol{bb})\cdot\mathsf{S}\times\boldsymbol{b}] + \frac{1}{\Omega}[\boldsymbol{b}(\boldsymbol{\sigma}\times\boldsymbol{b}) + (\boldsymbol{\sigma}\times\boldsymbol{b})\boldsymbol{b}]$$

$$\mathsf{S} = (p_\perp\nabla\boldsymbol{u} + \nabla\boldsymbol{q}_\perp) + (p_\perp\nabla\boldsymbol{u} + \nabla\boldsymbol{q}_\perp)^T$$

$$\boldsymbol{\sigma} = (p_\perp - p_\parallel)\left(\frac{d\boldsymbol{b}}{dt} + \boldsymbol{b}\cdot\nabla\boldsymbol{u}\right) + (3q_\perp - q_\parallel)\boldsymbol{b}\cdot\nabla\boldsymbol{b}$$

- Consider just $\boldsymbol{k}_\perp = 0$ (Alfvénically polarised parallel-propagating modes – they decouple and can be calculated without knowing pressures or heat fluxes)

[Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)]

Gyrothermal Instability: Linear Theory

Instability criterion:

$$\Lambda \equiv \Gamma_T^2 - \frac{(1-\delta)^2}{2}\left(\Delta + \frac{2}{\beta}\right) > 0$$

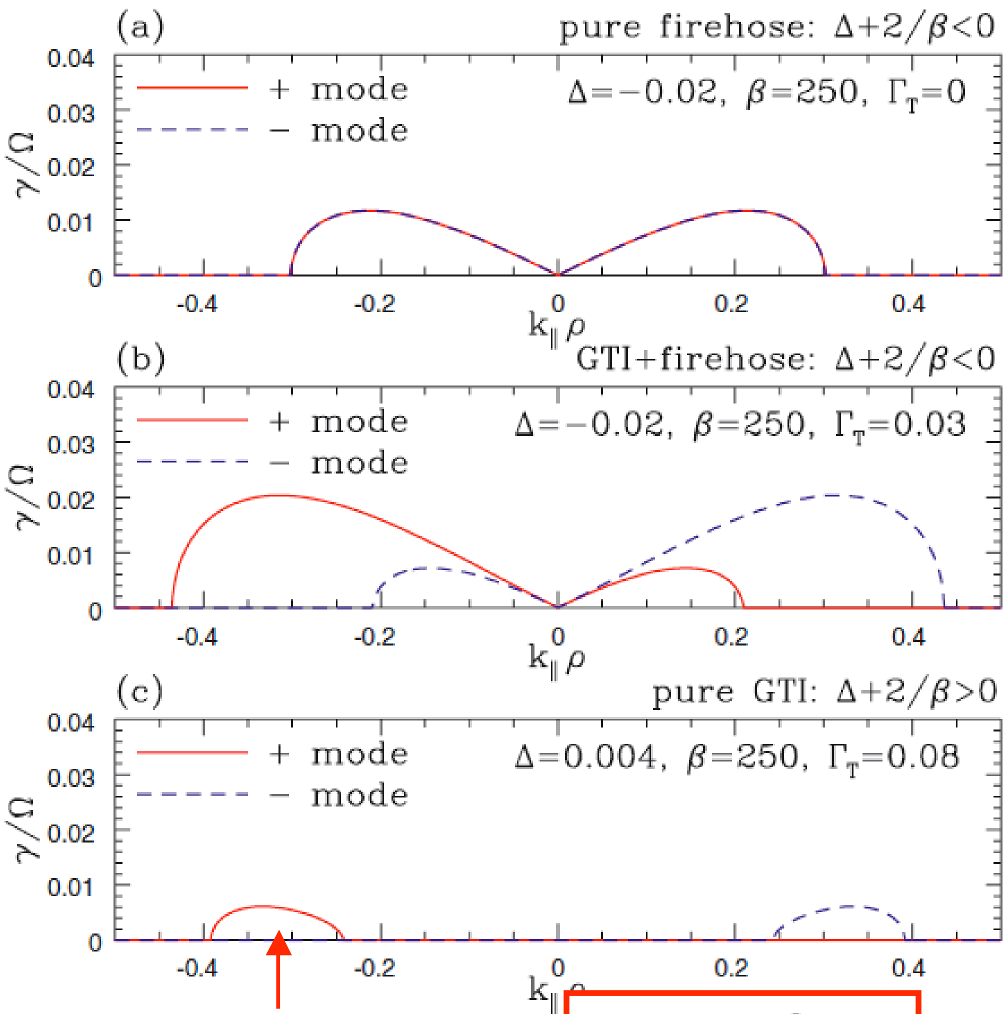
$$\Delta = \frac{p_{\perp i} - p_{\| i} + p_{\perp e} - p_{\| e}}{p_{\| i}}$$

$$\delta = \frac{p_{\perp i} - p_{\| i} - (p_{\perp e} - p_{\| e})}{p_{\| i}} - \frac{2}{\beta}$$

$$\Gamma_T = \frac{2q_{\perp i} - q_{\| i}}{p_{\| i} v_{\rm th}}$$

In the collisional limit,

$$q_\perp = \frac{1}{3} q_\| = -\frac{1}{2} n \frac{v_{\rm th}^2}{\nu} \boldsymbol{b} \cdot \nabla T$$



Preferred scale in marginal state:

$$k_\| \rho_i \sim \frac{\lambda_{\rm mfp}}{l_T}$$

[Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)]

Gyrothermal Instability: Linear Theory

Instability criterion:

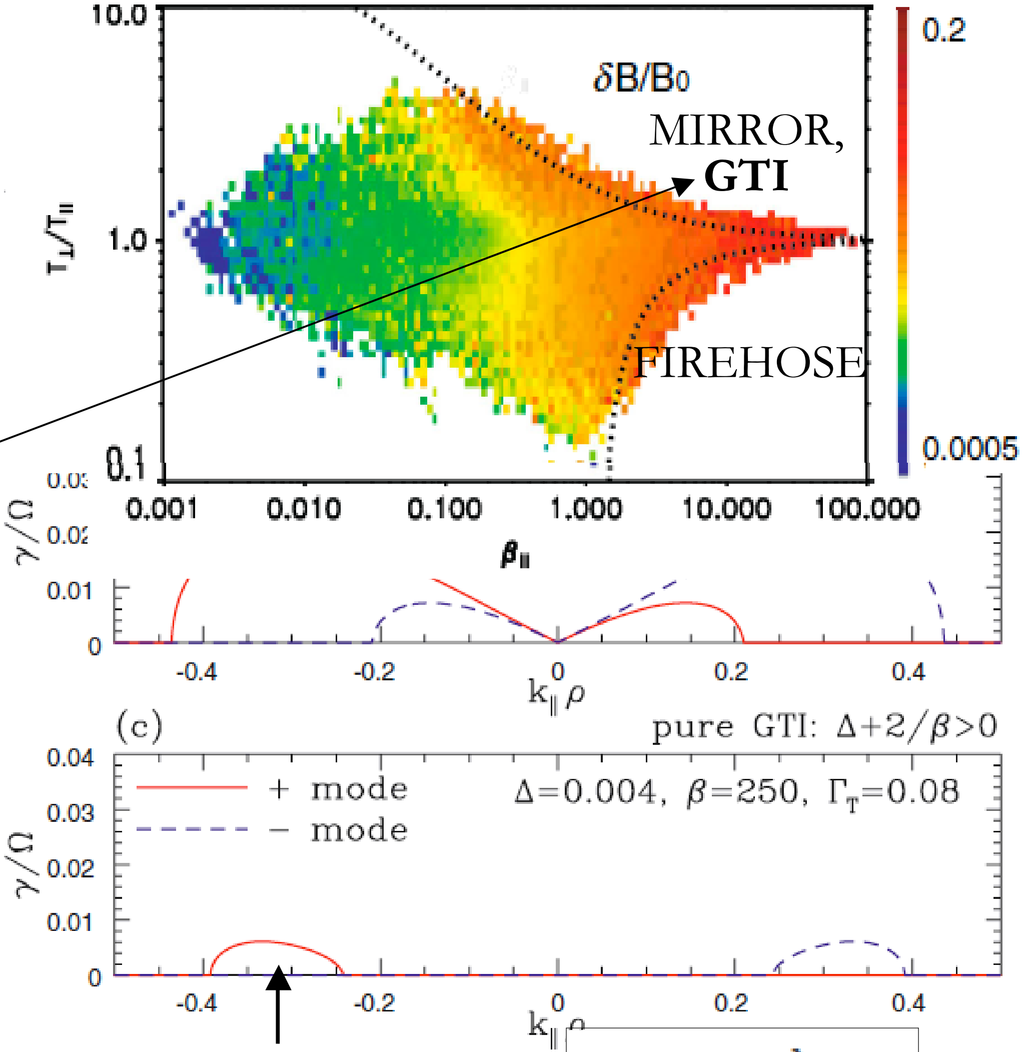
$$\Lambda \equiv \Gamma_T^2 - \frac{(1-\delta)^2}{2}\left(\Delta + \frac{2}{\beta}\right) > 0$$

So, Alfvénically polarised perturbations can be unstable at $\Delta > 0$!

$$\Gamma_T = \frac{2q_{\perp i} - q_{\parallel i}}{p_{\parallel i} v_{\rm th}}$$

In the collisional limit,

$$q_\perp = \frac{1}{3} q_\parallel = -\frac{1}{2} n \frac{v_{\rm th}^2}{\nu} \boldsymbol{b}\cdot\nabla T$$



Preferred scale in marginal state:

$$k_\parallel \rho_i \sim \frac{\lambda_{\rm mfp}}{l_T}$$

[Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)]

Gyrothermal Instability: Nonlinear Theory

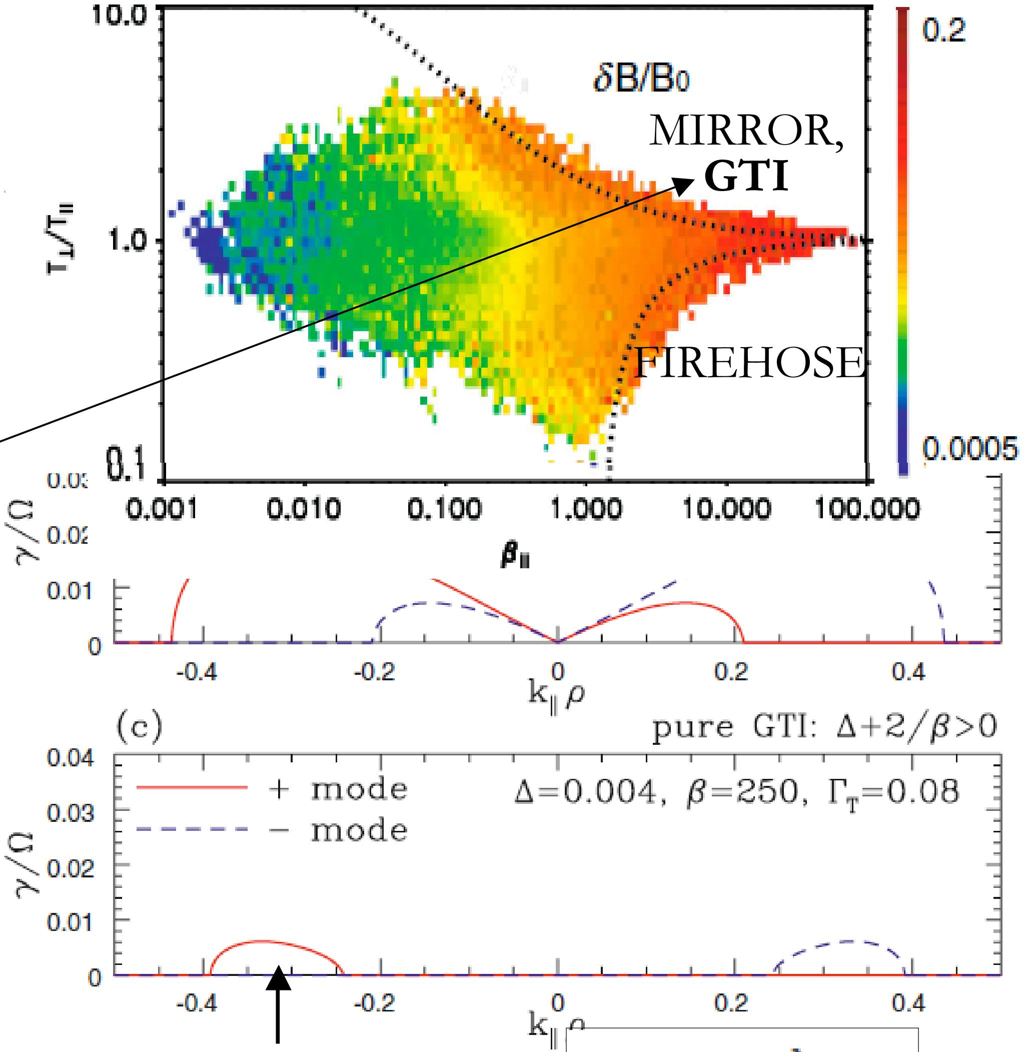
Instability criterion:

$$\Lambda \equiv \Gamma_T^2 - \frac{(1-\delta)^2}{2}\left(\Delta + \frac{2}{\beta}\right) > 0$$

So, Alfvénically polarised perturbations can be unstable at $\Delta>0$!

GTI saturates by the same mechanism as the firehose: magnetic fluctuations adjusting (increasing) Δ

[It might actually destabilise mirror — no idea what then]

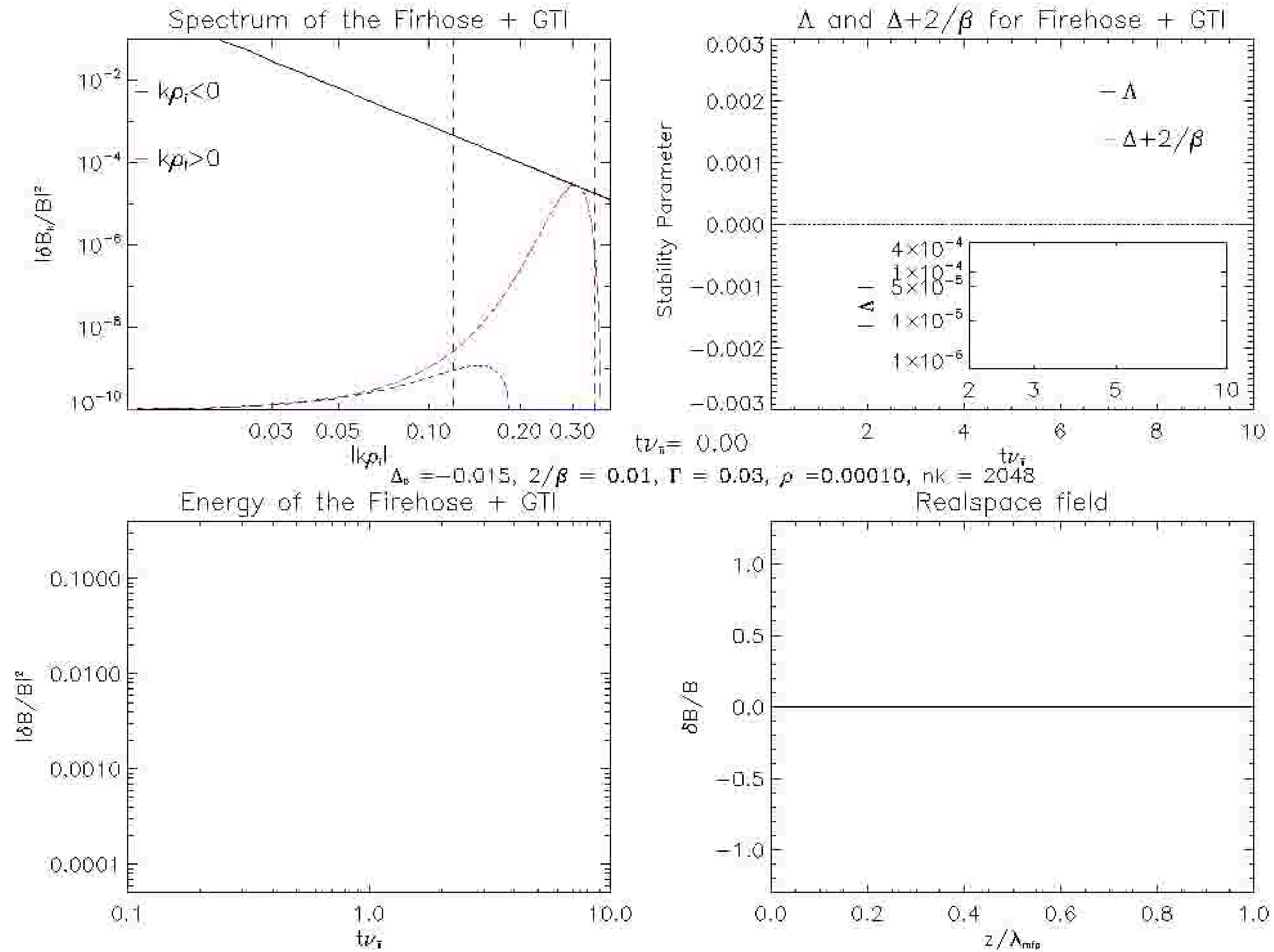


Preferred scale in marginal state:

$$k_\parallel \rho_i \sim \frac{\lambda_{\rm mfp}}{l_T}$$

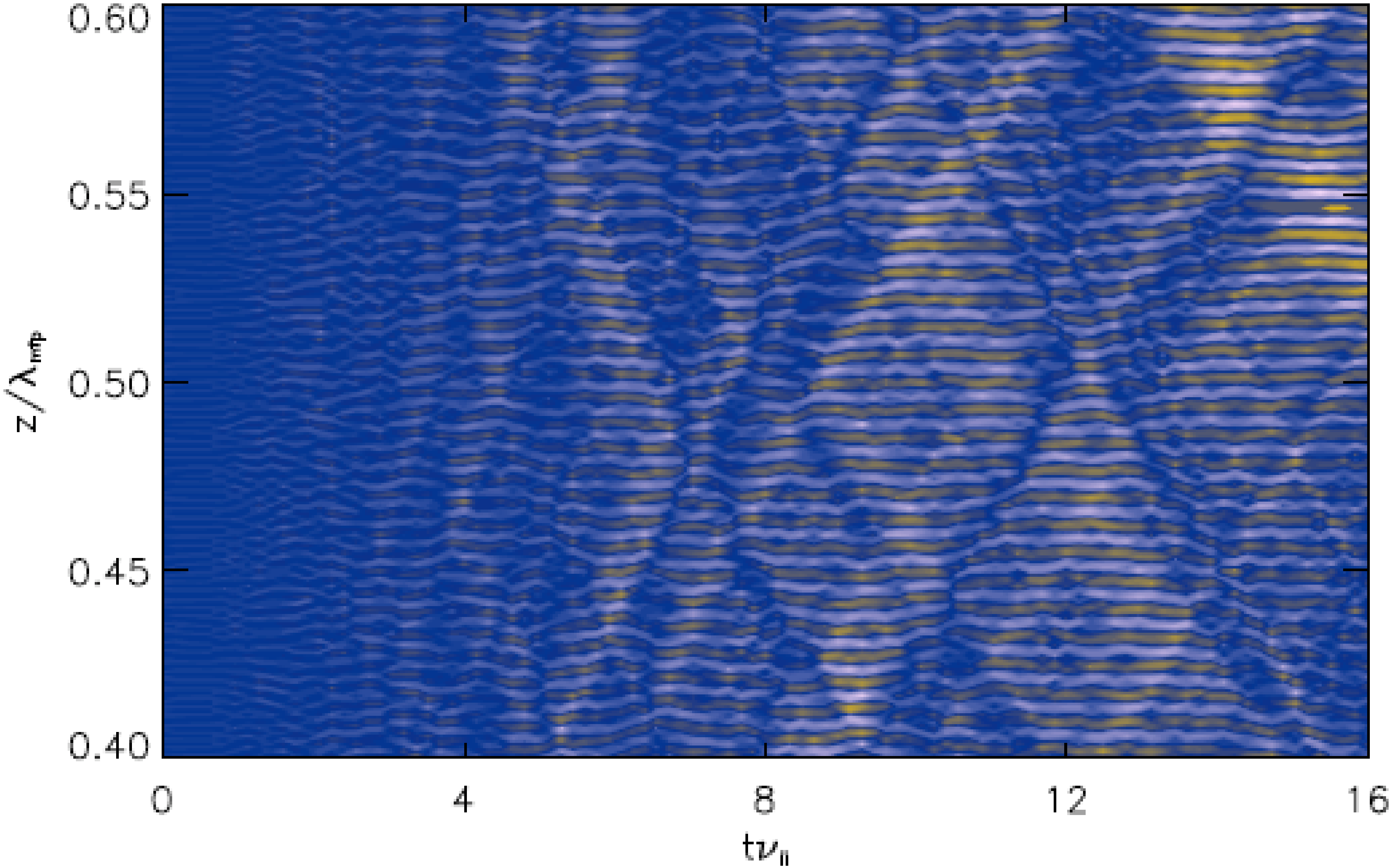
[Rosin *et al.*, arXiv:1002.4017 (2010)]

Nonlinear GTI



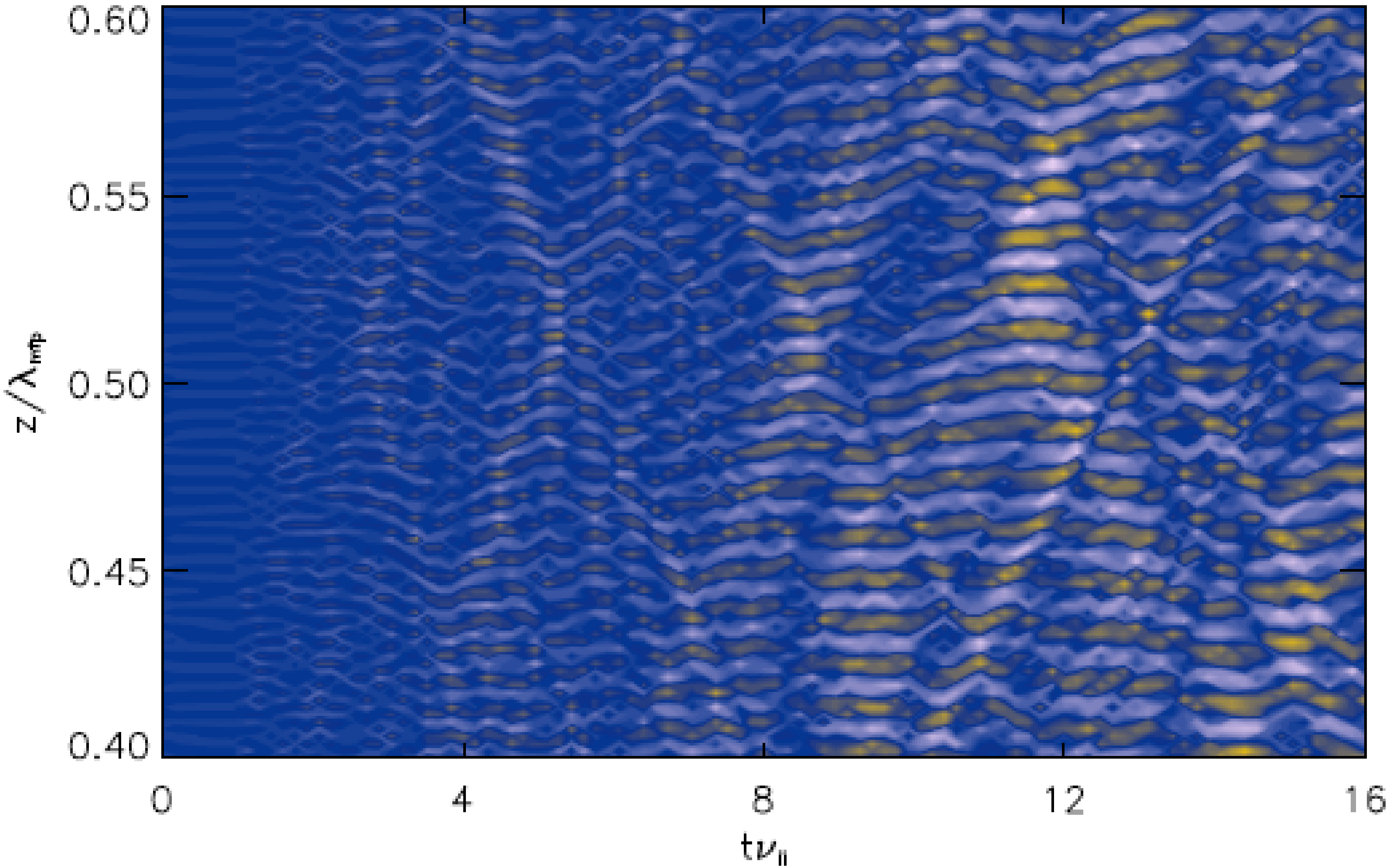
[Rosin *et al.*, arXiv:1002.4017 (2010)]

Nonlinear GTI

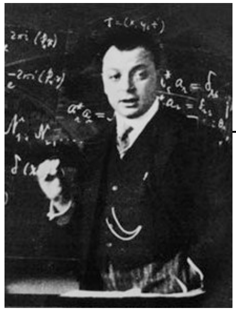


[Rosin *et al.*, arXiv:1002.4017 (2010)]

[Cf. Nonlinear Firehose]



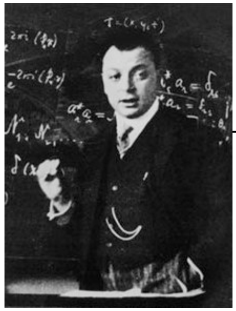
[Rosin *et al.*, arXiv:1002.4017 (2010)]



Part I. The Knowns

1. **Kinetic turbulence is a generalised (free) energy cascade in phase space towards collisional scales.**
   The free energy cascade splits into various channels:
   AW + *compressive* above ion gyroscale ("inertial range")
   KAW + *entropy cascade* belowion gyroscale ("dissipation range")
2. **Turbulence is anisotropic at all scales**
   Scaling theories based on the **critical balance** conjecture give results that seem broadly to be consistent with SW evidence and GK simulations
3. **Plasma is marginal to microinstabilities** (firehose, mirror etc. driven so by spontaneous generation of pressure anisotropies)

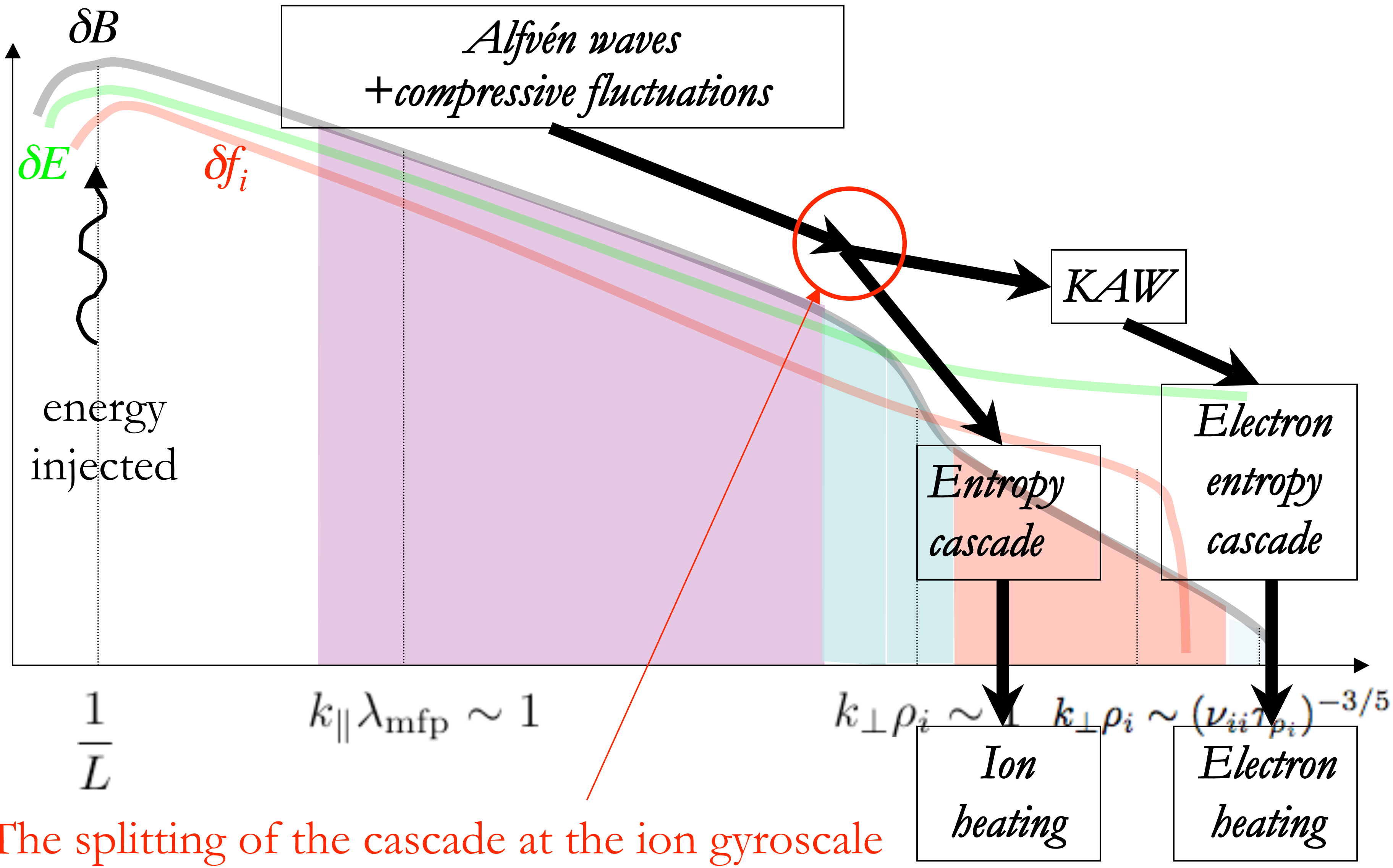
Schekochihin *et al.*, ApJS **182**, 310 (2009)
Schekochihin *et al.*, MNRAS **405**, 291 (2010)
Rosin *et al.*, MNRAS, in press; arXiv:1002.4017



Part II. The Known Unkonwns

Schekochihin *et al.*, ApJS **182**, 310 (2009)
Schekochihin *et al.*, MNRAS **405**, 291 (2010)
Rosin *et al.*, MNRAS, in press; arXiv:1002.4017

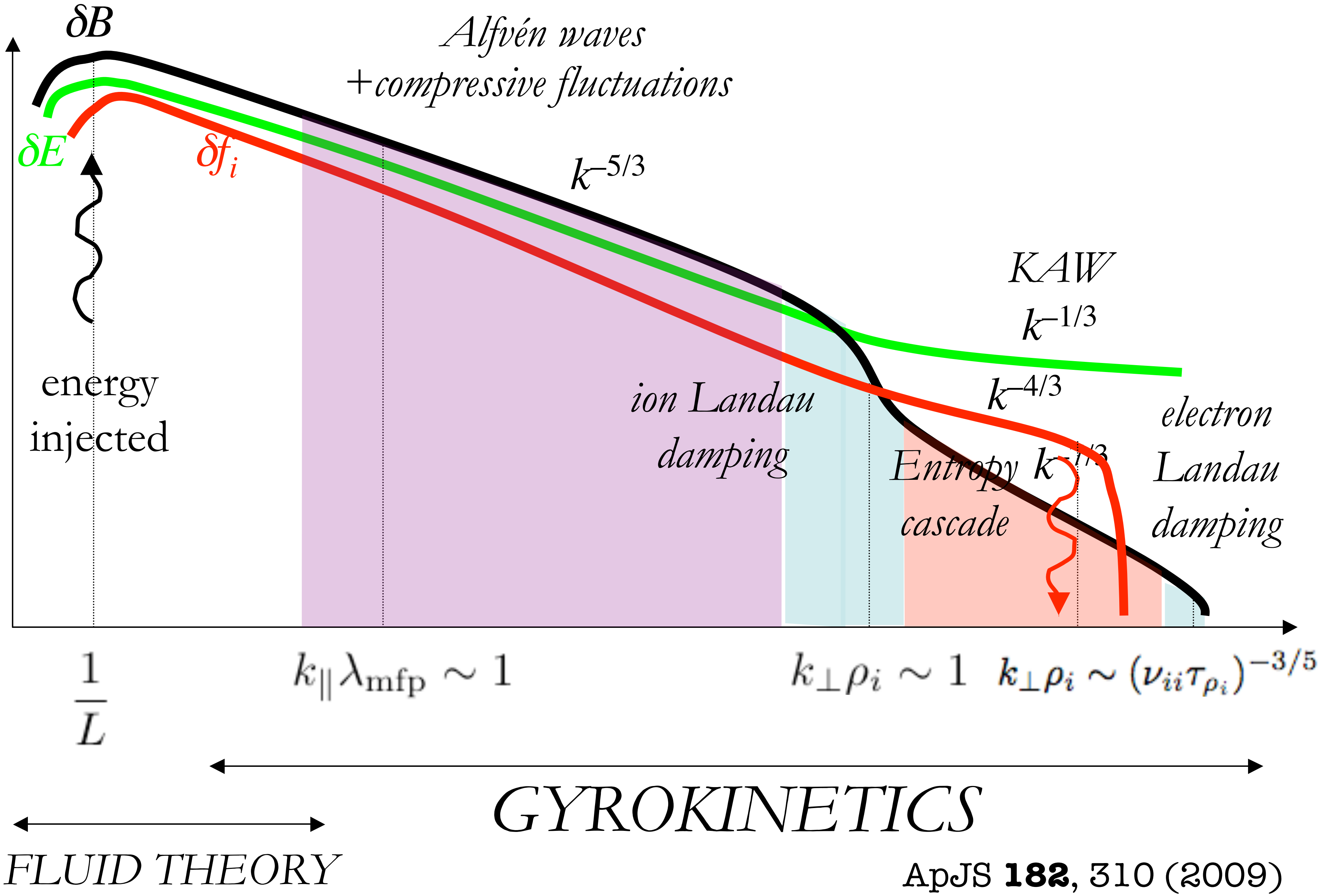
1. Ion vs. Electron Heating



The splitting of the cascade at the ion gyroscale determines relative heating of the species

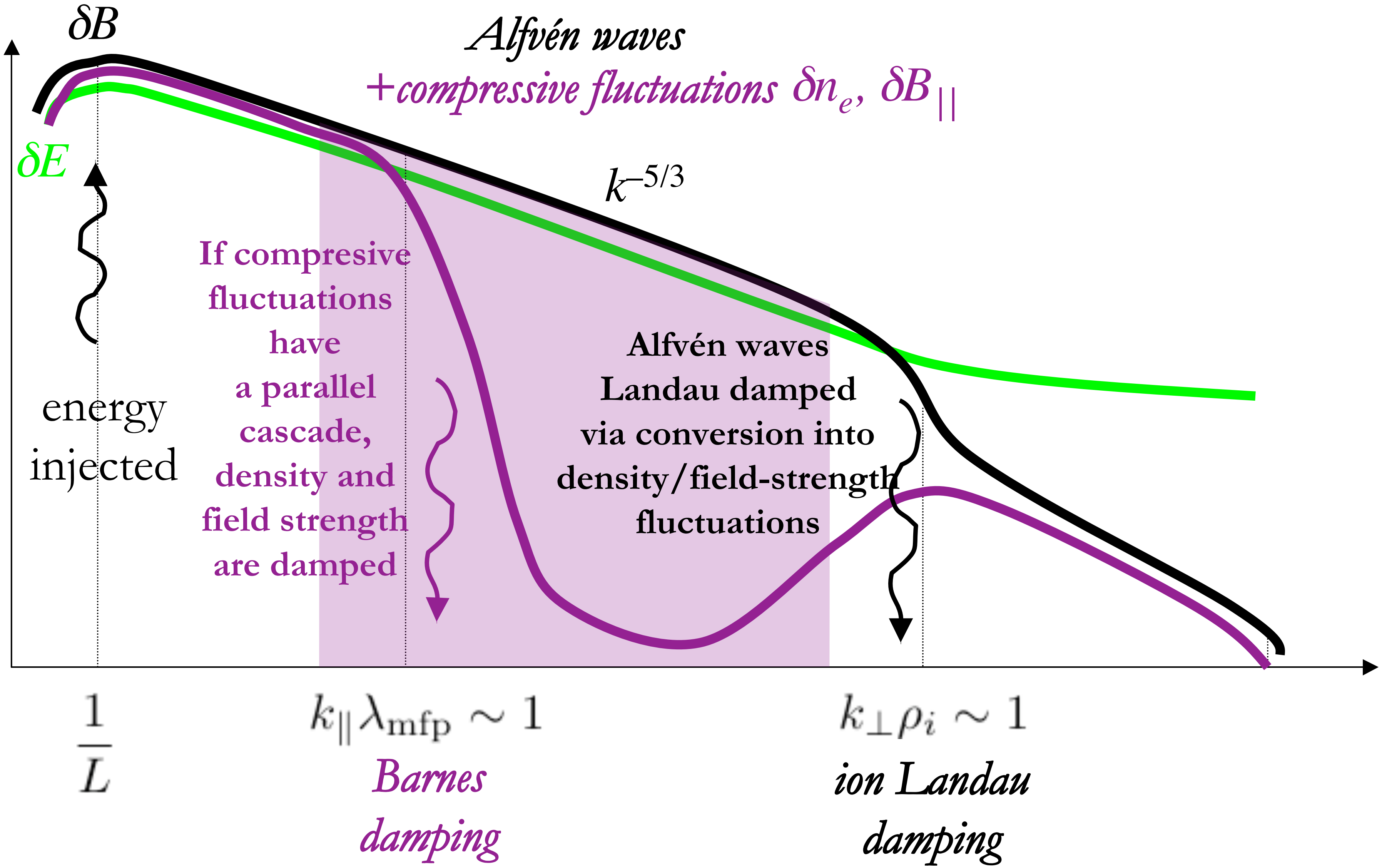
ApJS **182**, 310 (2009)

Free Energy Cascade



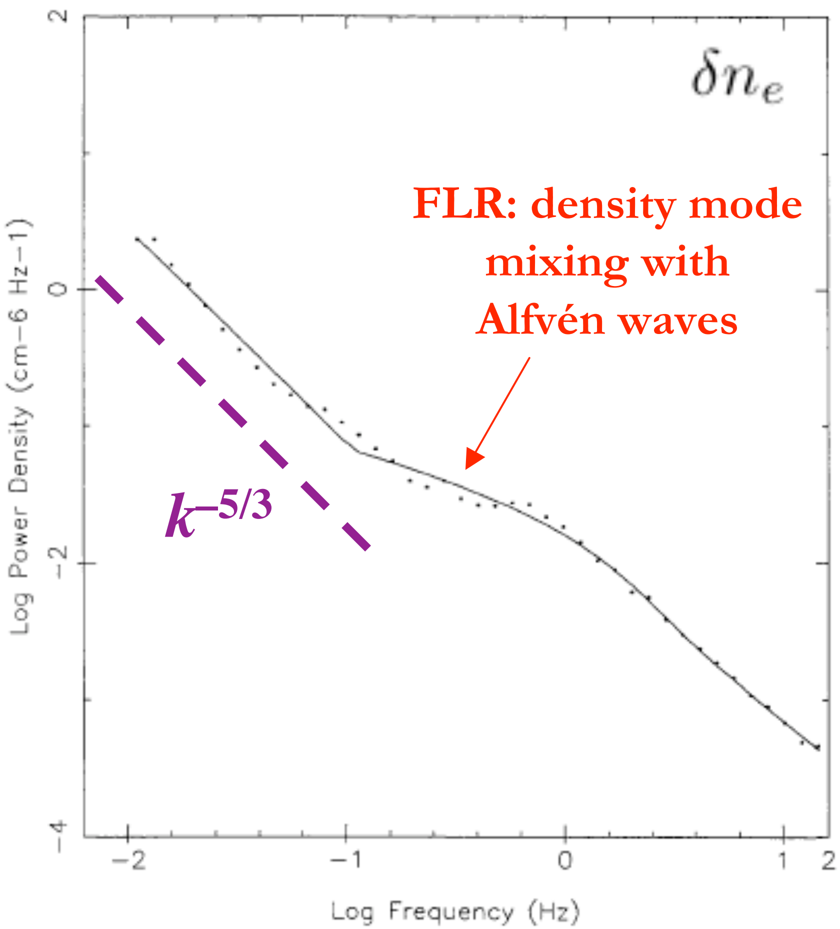
ApJS **182**, 310 (2009)

2. Compressive Fluctuations

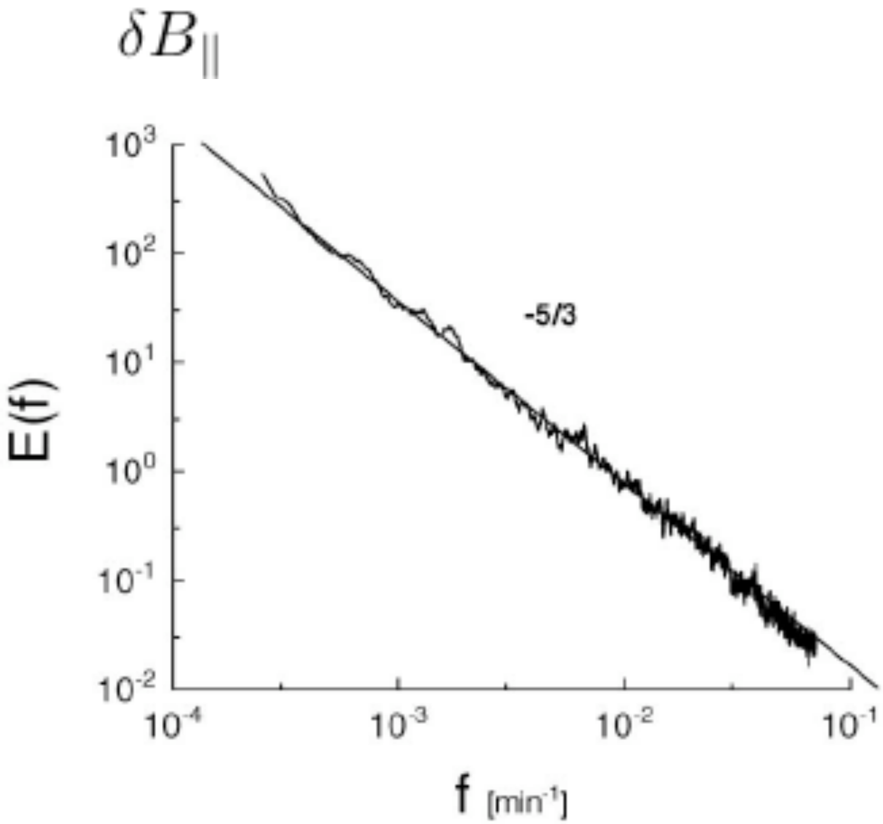


ApJS **182**, 310 (2009)

SW: Compressive Fluctuations Undamped?



Density fluctuations in the solar wind at ~1 AU (31 Aug. 1981)
[Celnikier, Muschietti & Goldman 1987, *A&A* **181**, 138]



Spectrum of magnetic-field strength in the solar wind at ~1 AU (1998)
[Bershadskii & Sreenivasan 2004, *PRL* **93**, 064501]

Compressive Fluctuations are Passive-Kinetic

δn_e and $\delta B_\|$ require kinetic description: our expansion gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\|}{B_0} f_{0i}\right) + v_\| \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} \left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0.$$

$$\frac{\delta n_e}{n_{0e}} = \frac{1}{n_{0i}} \int \mathrm{d}^3\boldsymbol{v}\, \delta f_i$$

$$\frac{\delta B_\|}{B_0} = -\frac{\beta_i}{2}\frac{1}{n_{0i}} \int \mathrm{d}^3\boldsymbol{v} \left(1 + \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\right) \delta f_i$$

Maxwellian equilibrium

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{u}_\perp \cdot \boldsymbol{\nabla}_\perp = \frac{\partial}{\partial t} + \{\Phi, \cdots\}$$

$$\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial z} + \frac{\delta \boldsymbol{B}_\perp}{B_0} \cdot \boldsymbol{\nabla}_\perp = \frac{\partial}{\partial z} + \frac{1}{v_A}\{\Psi, \cdots\}$$

Density and field-strength fluctuations are passively mixed by Alfvén waves

ApJS **182**, 310 (2009)

Compressive Fluctuations are Passive-Kinetic

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$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\|}{B_0} f_{0i}\right) + v_\| \hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0$$

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In the Lagrangian frame of the Alfvén waves…

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{u}_\perp\cdot\boldsymbol{\nabla}_\perp \to \frac{\partial}{\partial t}$$

$$\hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla} = \frac{\partial}{\partial z} + \frac{\delta\boldsymbol{B}_\perp}{B_0}\cdot\boldsymbol{\nabla}_\perp \to \frac{\partial}{\partial l_0}$$

ApJS **182**, 310 (2009)

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$$\frac{\partial}{\partial t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\|}{B_0} f_{0i}\right) + v_\| \frac{\partial}{\partial l_0}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0.$$

equation is linear!

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ApJS **182**, 310 (2009)

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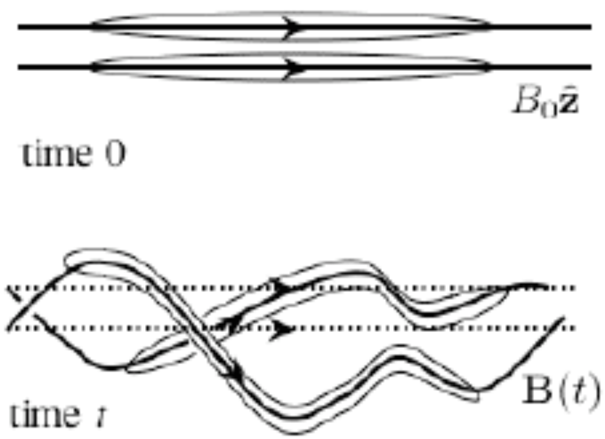

No refinement of scale along perturbed magnetic field

(but there is along the guide field, i.e. k_z grows)

ApJS **182**, 310 (2009)

Collisionless Damping

δn_e and $\delta B_\|$ require kinetic description: our expansion gives

$$\frac{\partial}{\partial t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\|}{B_0} f_{0i}\right) + v_\| \frac{\partial}{\partial l_0}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0.$$

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equation is linear!

$$\frac{\delta B_\|}{B_0} = -\frac{\beta_i}{2}\frac{1}{n_{0i}} \int \mathrm{d}^3\boldsymbol{v} \left(1 + \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\right) \delta f_i$$

For $\beta_i \sim 1$, $\gamma \sim k_{\|0} v_{\mathrm{th}i} \sim k_{\|0} v_A \ll k_\| v_A$

[Barnes 1966, *Phys. Fluids* **9**, 1483]

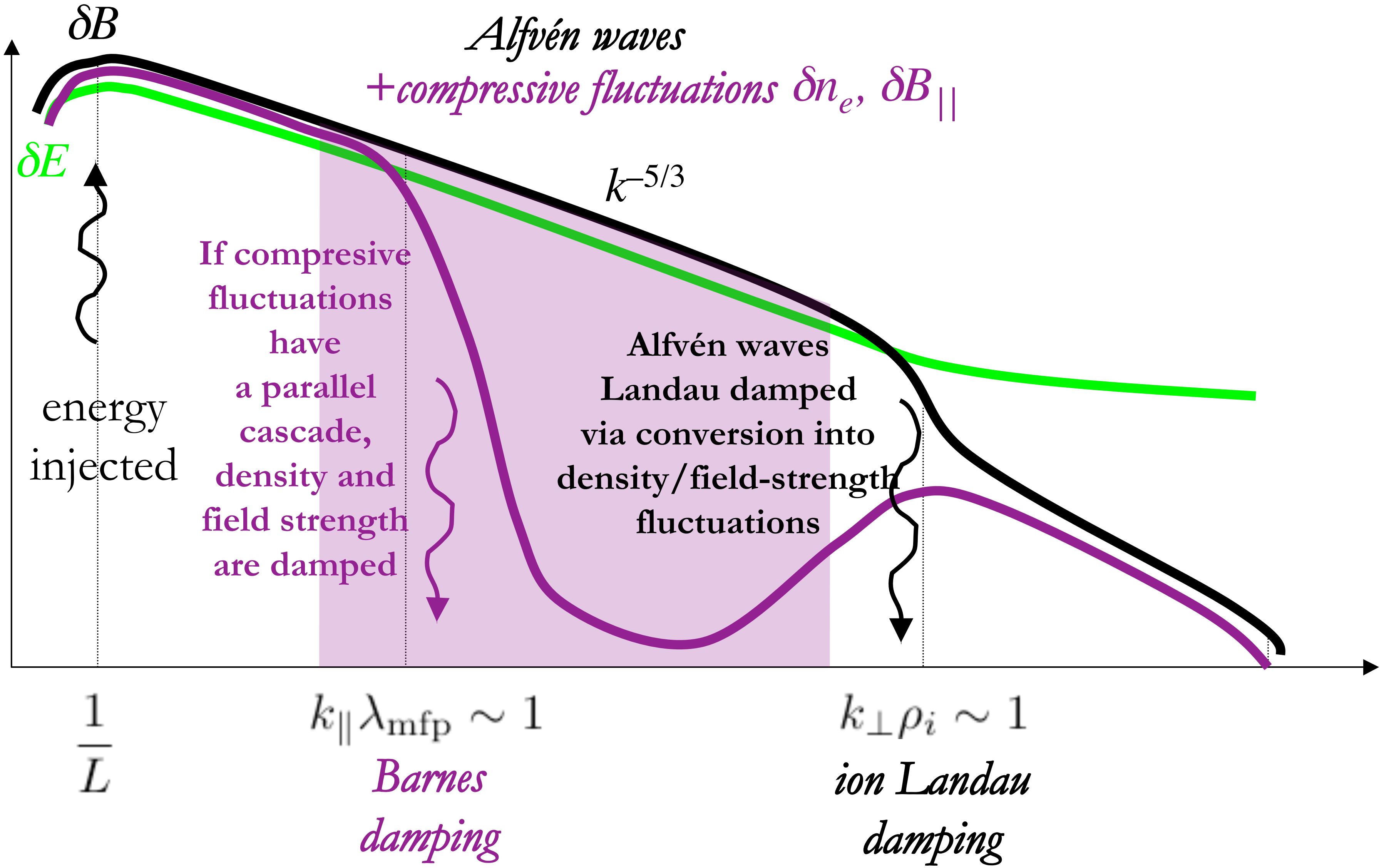
time to be cascaded in $k_\perp$ by Alfvén waves, for which $k_\| \sim k_\perp^{2/3}$

Cascades of density and field strength fluctuations are undamped above ion gyroscale

… but parallel cascade might be induced due to dissipation
[Lithwick & Goldreich 2001, *ApJ* **562**, 279]

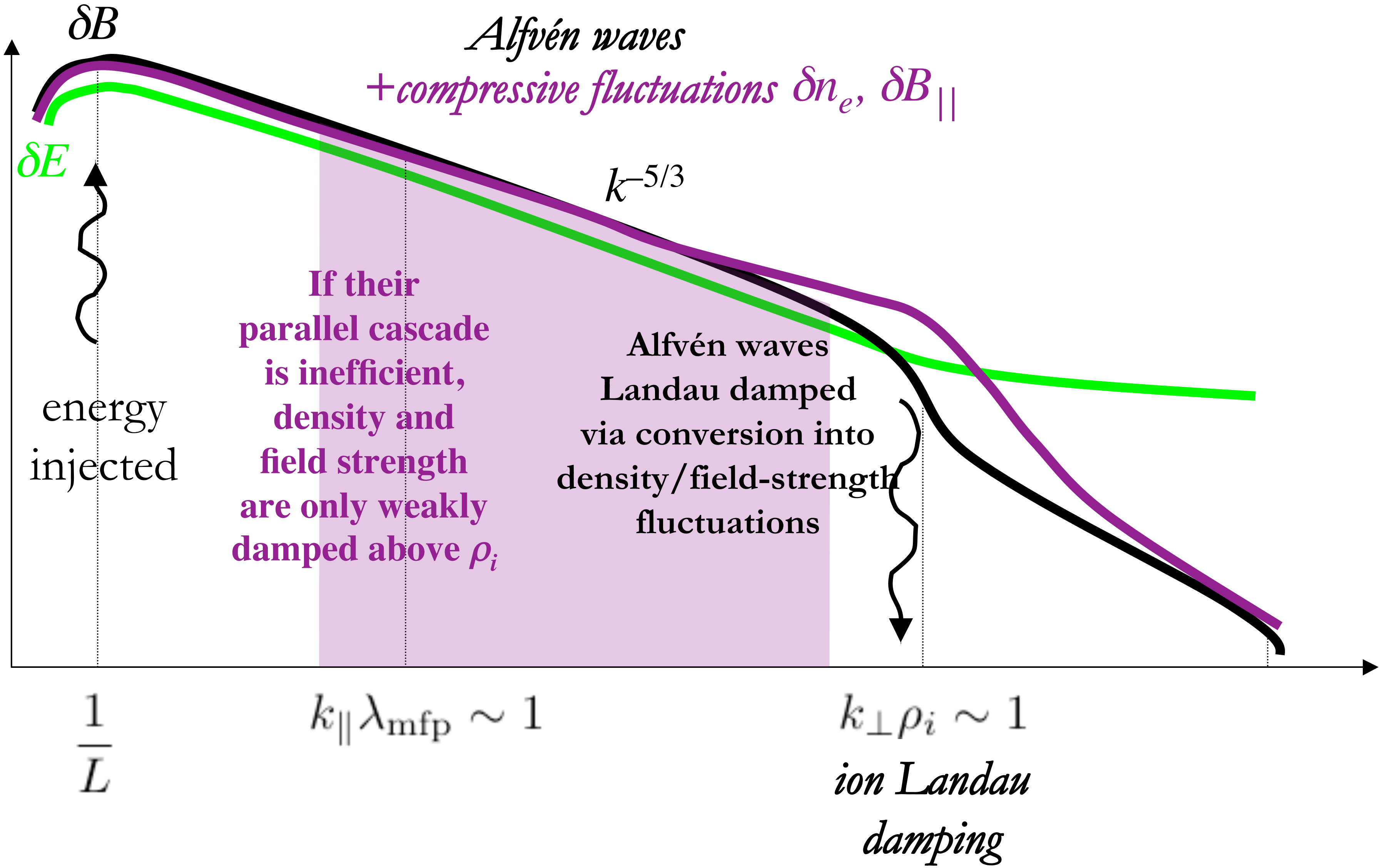
ApJS **182**, 310 (2009)

Compressive Fluctuations



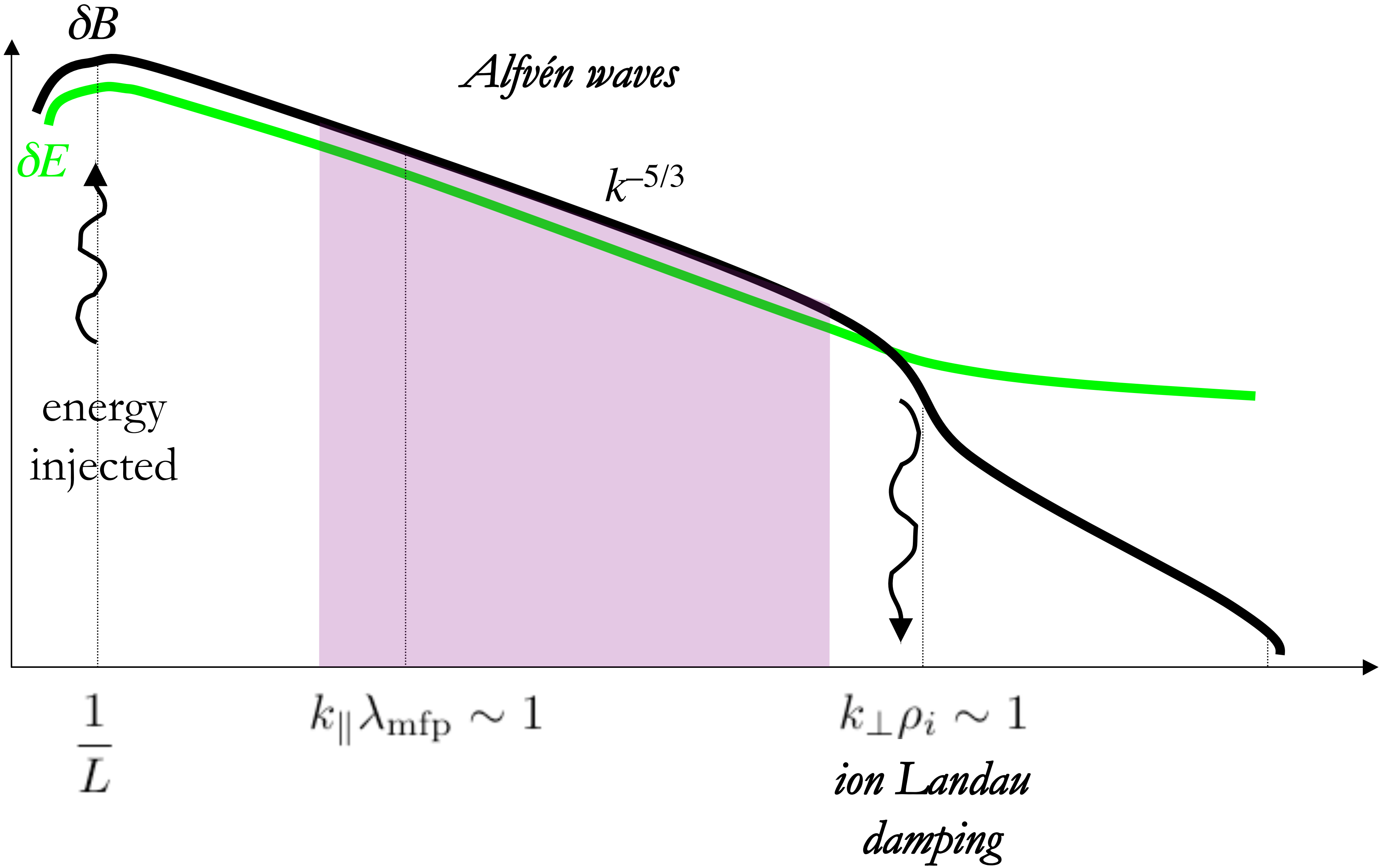
ApJS **182**, 310 (2009)

Compressive Fluctuations

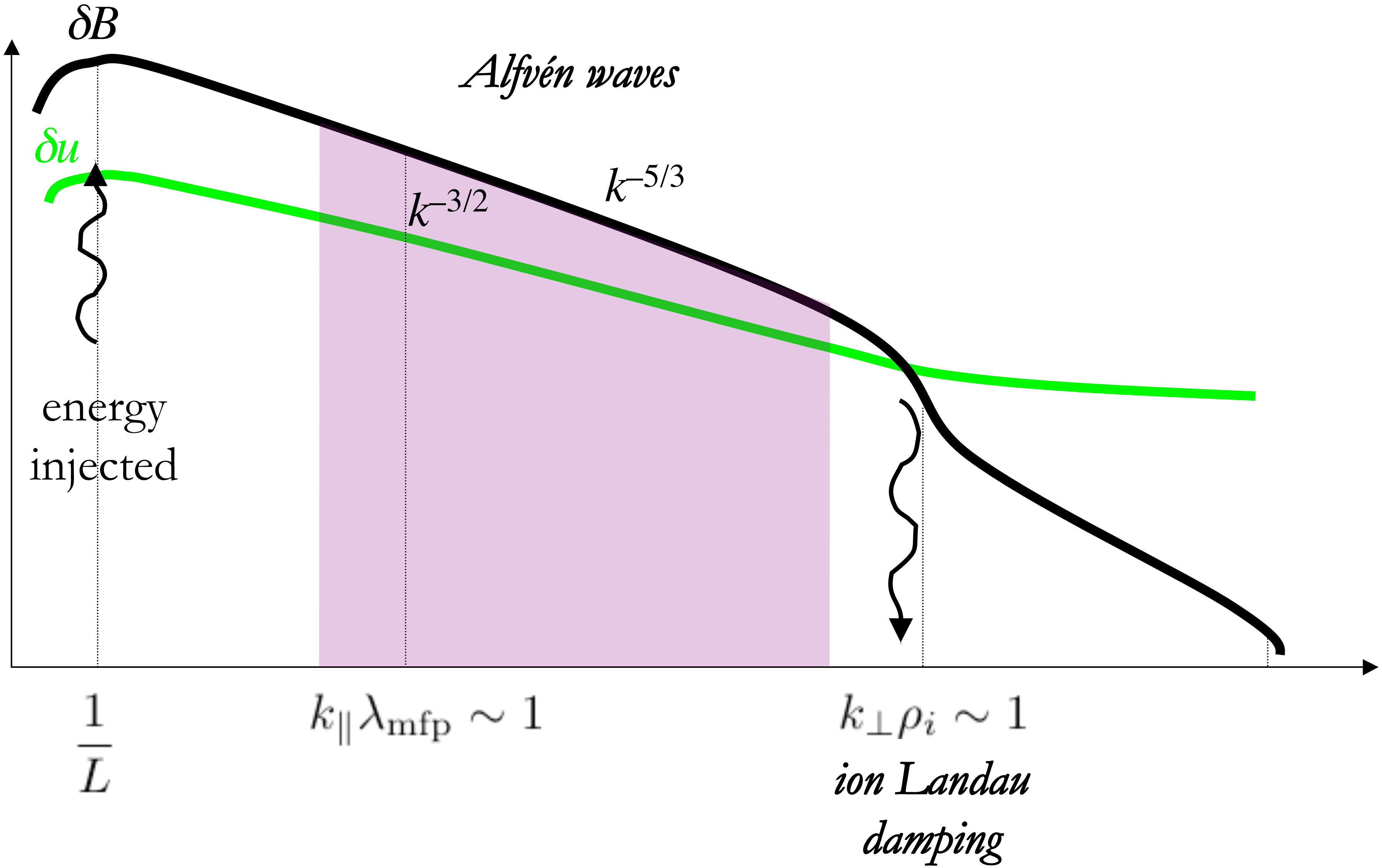


ApJS **182**, 310 (2009)

Back to Alfvén Waves…



3. The 5/3 and the 3/2



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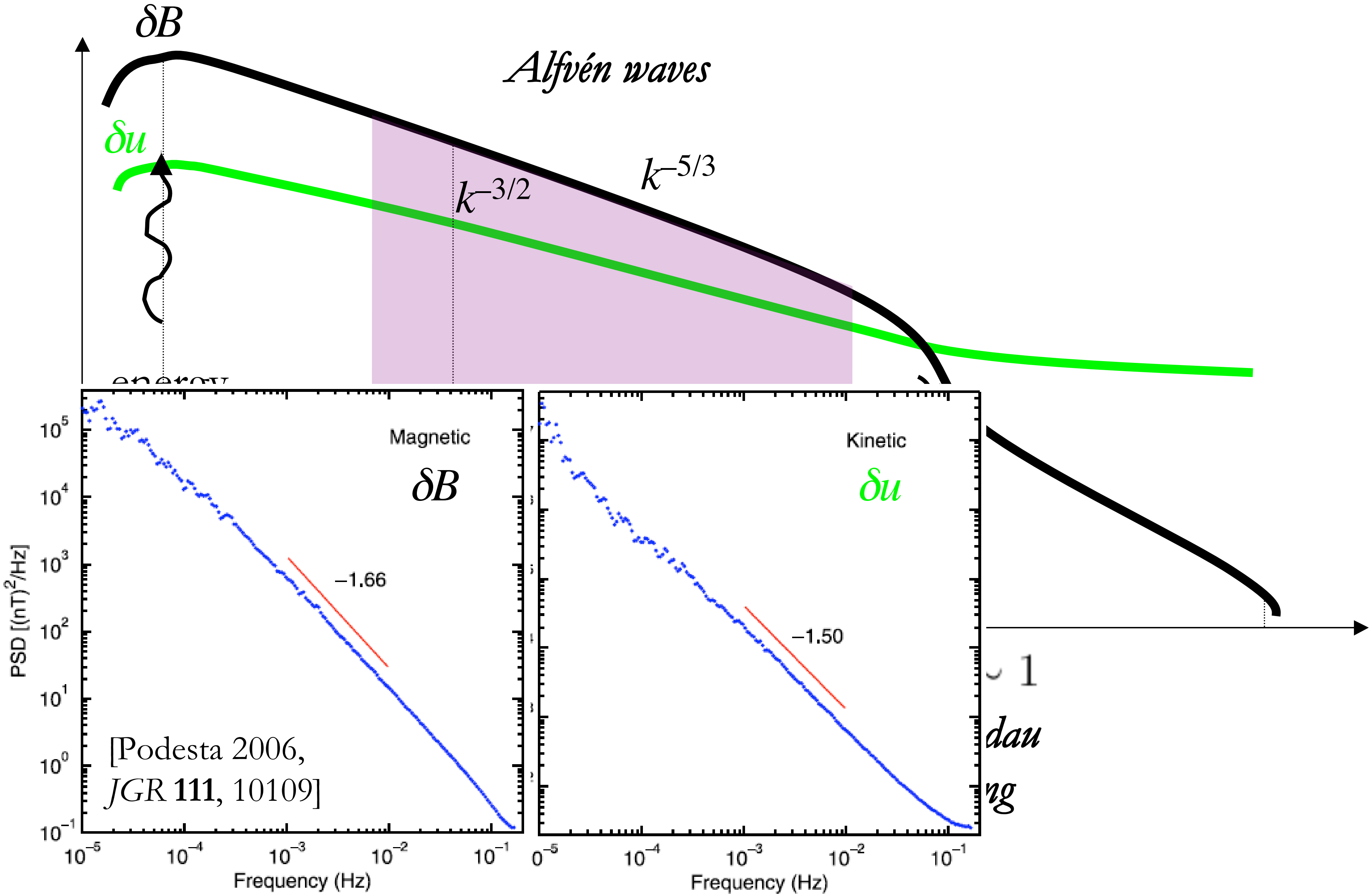

[Podesta 2006, *JGR* **111**, 10109]

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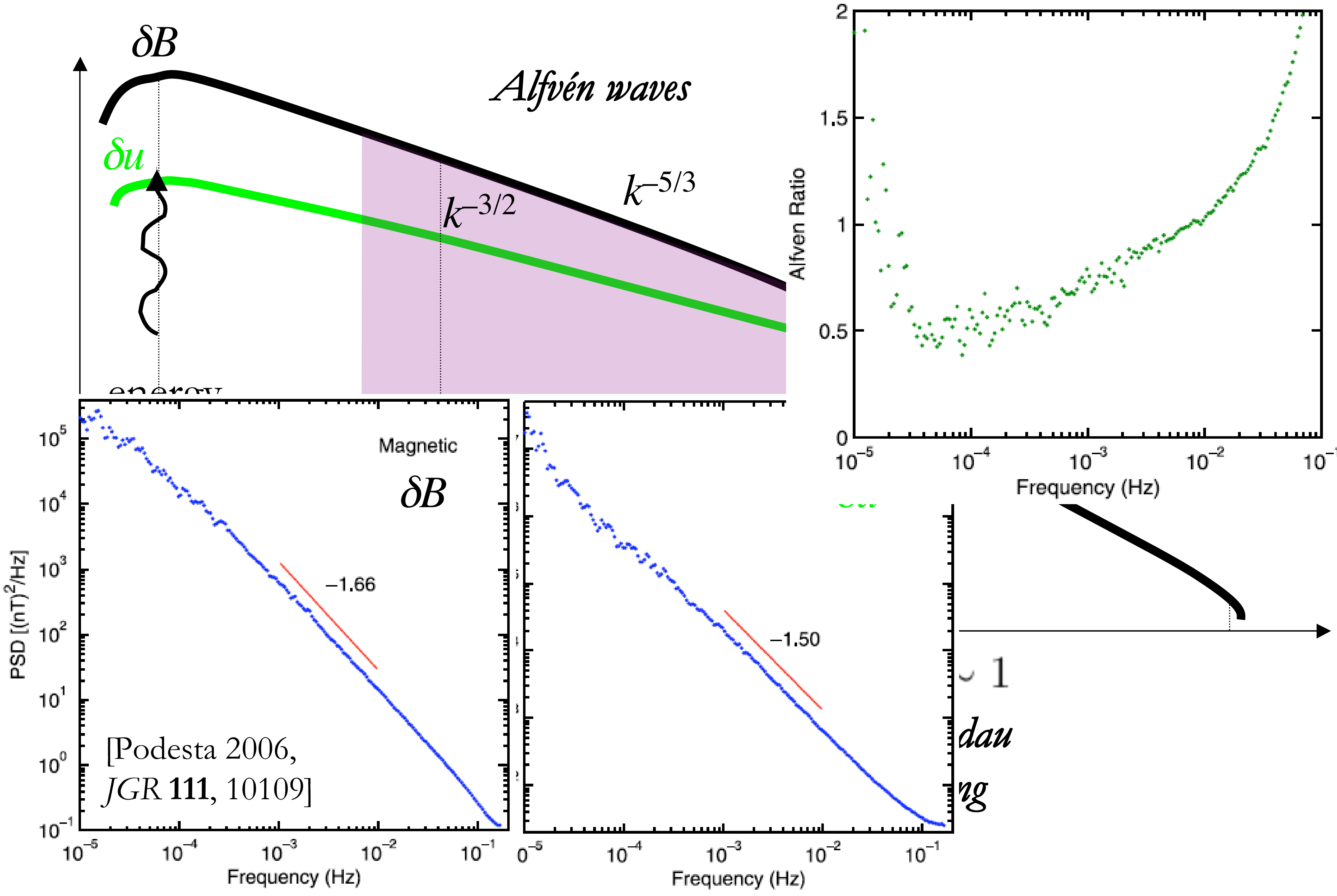

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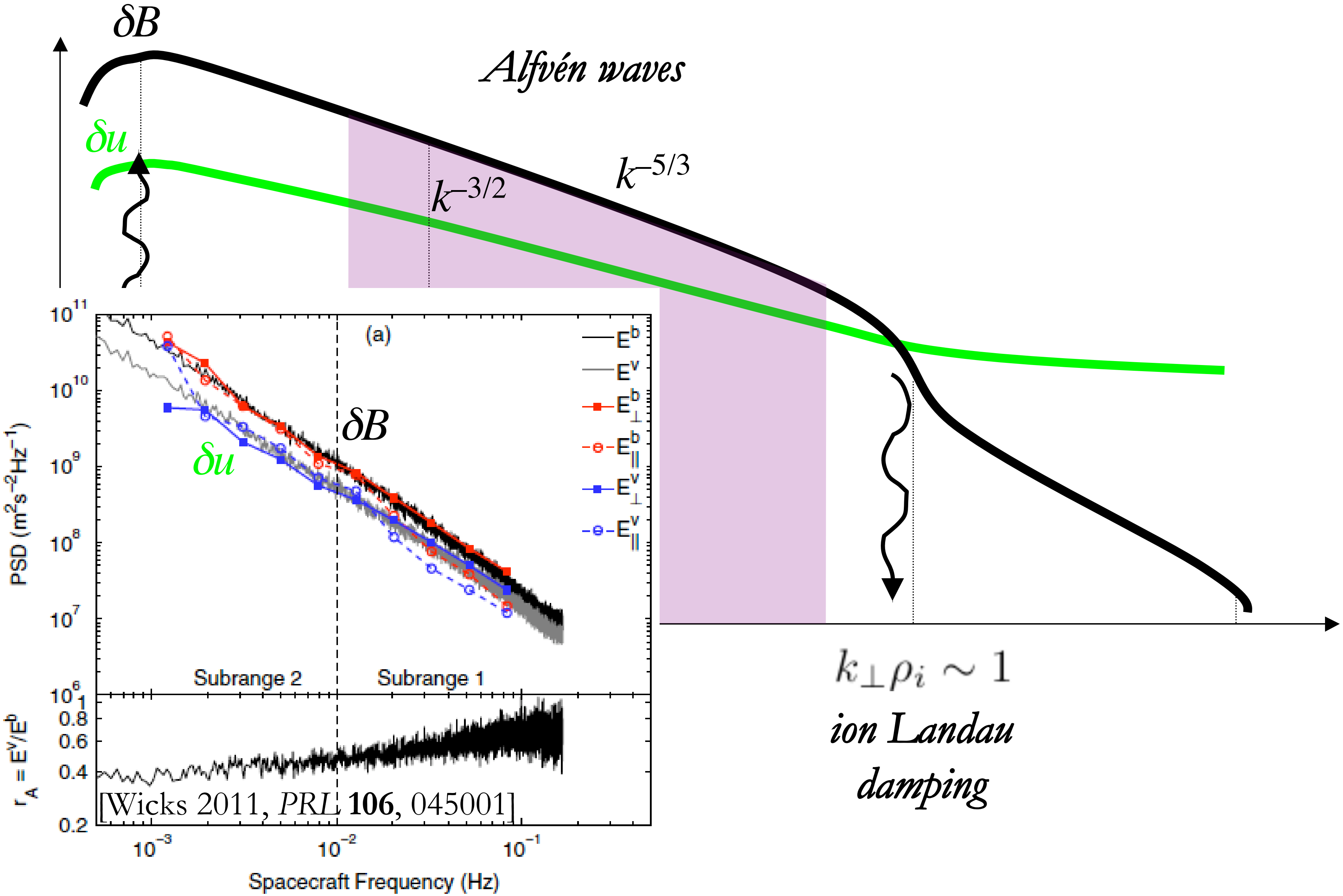
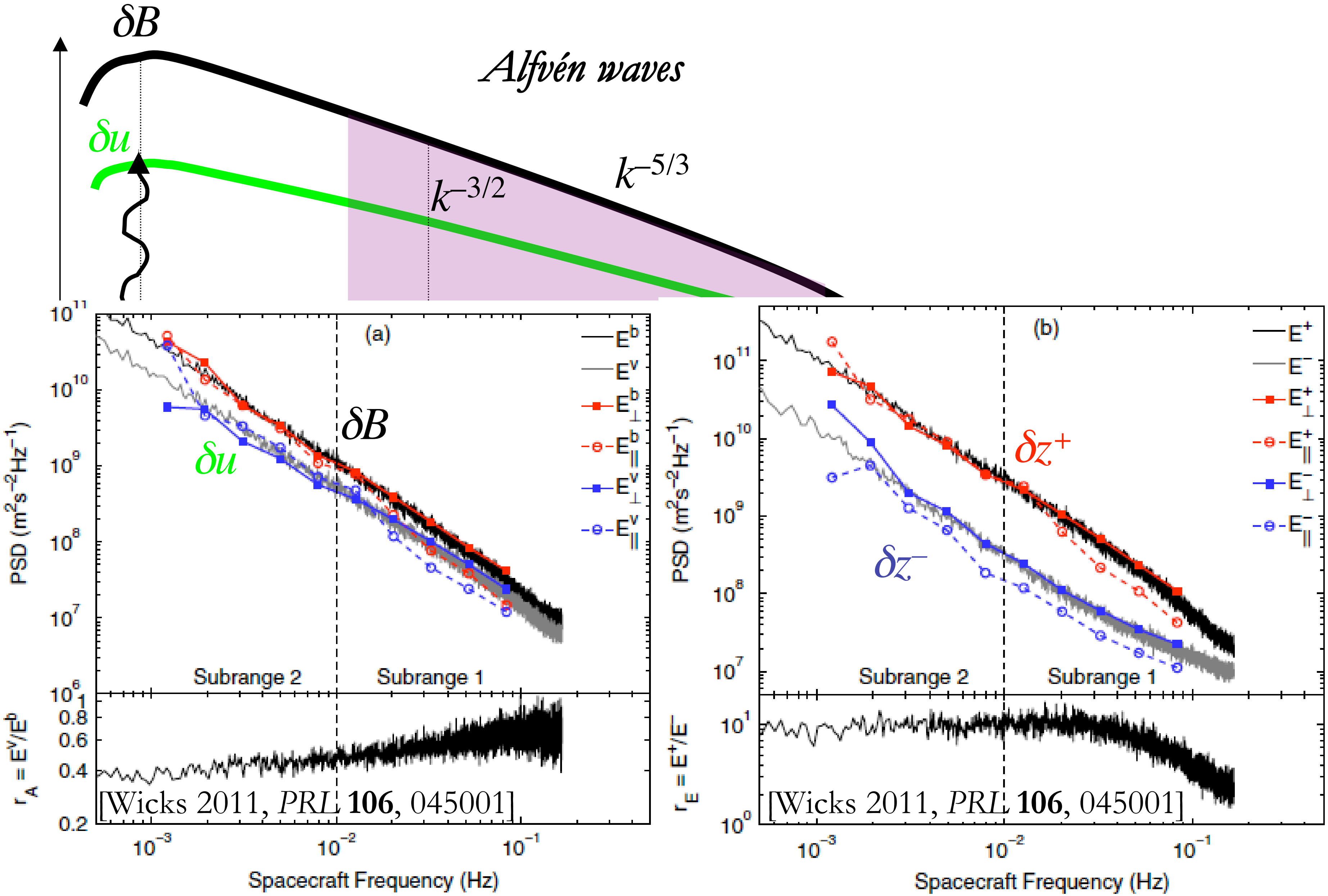
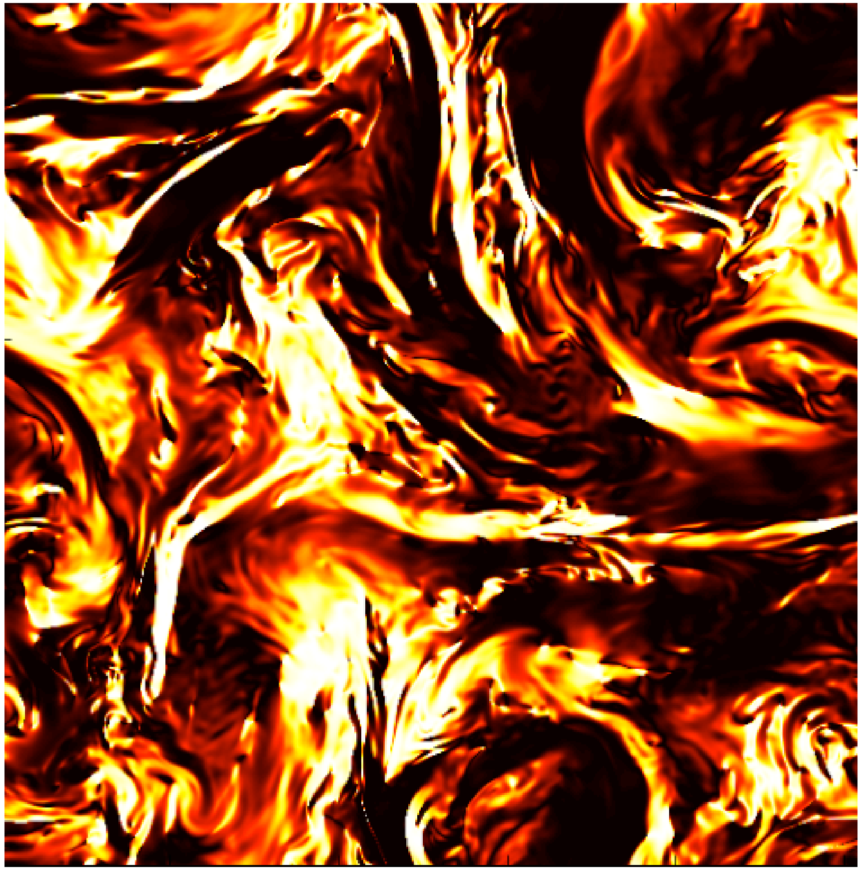


4. Imbalanced Cascade



4. Imbalanced Cascade

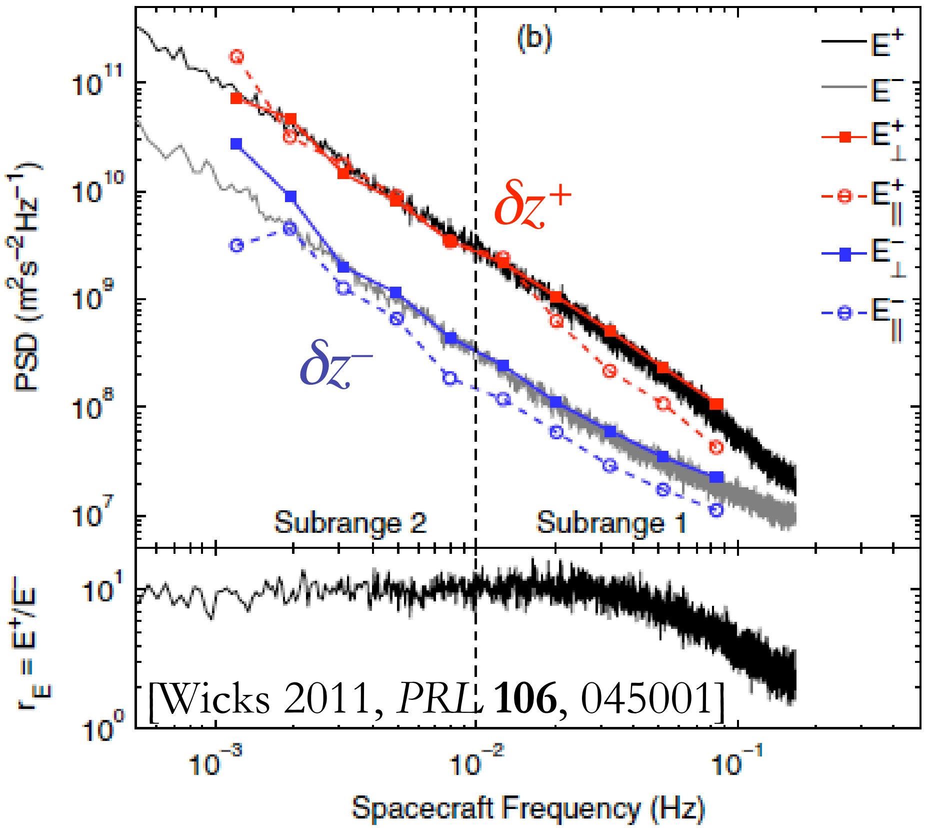
In fact, all MHD turbulence is locally imbalanced



$\mathbf{u} \cdot \mathbf{b}/|\mathbf{u}||\mathbf{b}|$

[From a balanced 512^3 RMHD simulation by A. Mallet (2010)]

[Perez & Boldyrev 2009, *PRL* **102**, 025003]



[Wicks 2011, *PRL* **106**, 045001]

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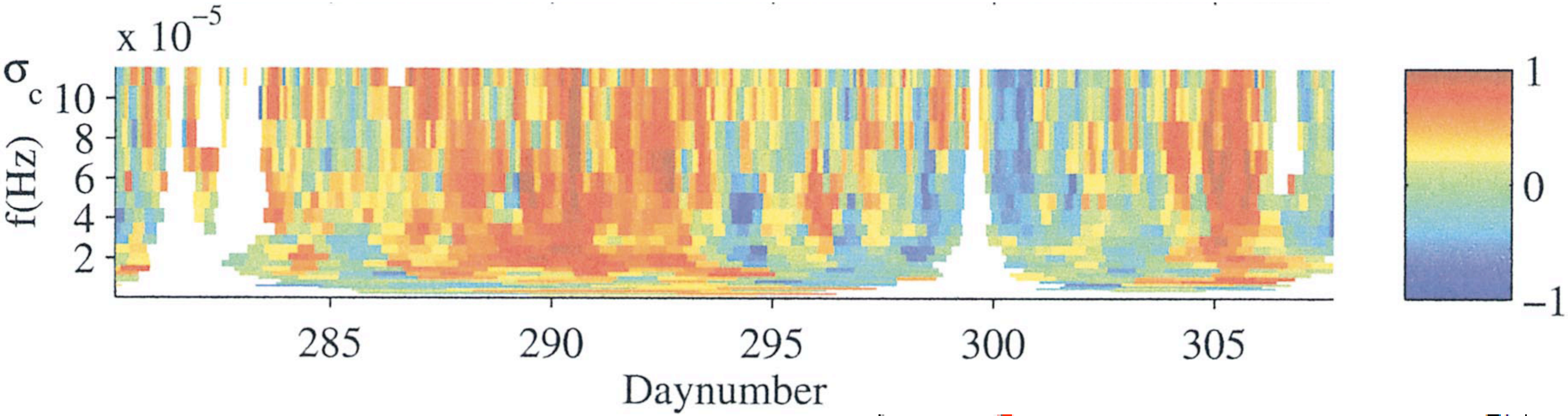
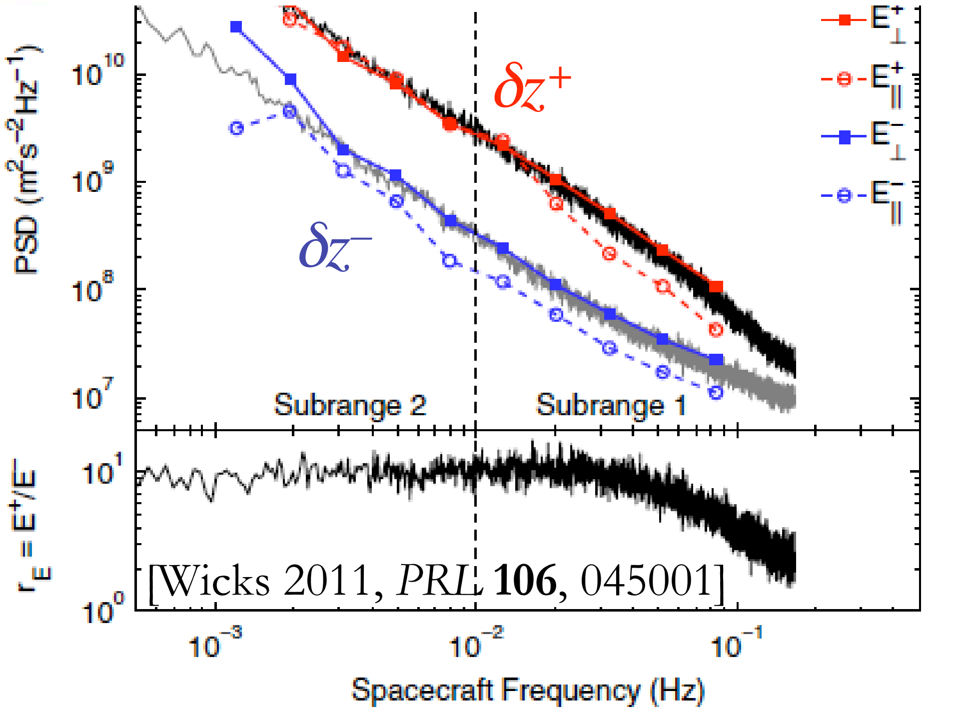


[Lucek & Balogh 1998, *ApJ* **507**,984]



[Wicks 2011, *PRL* **106**, 045001]

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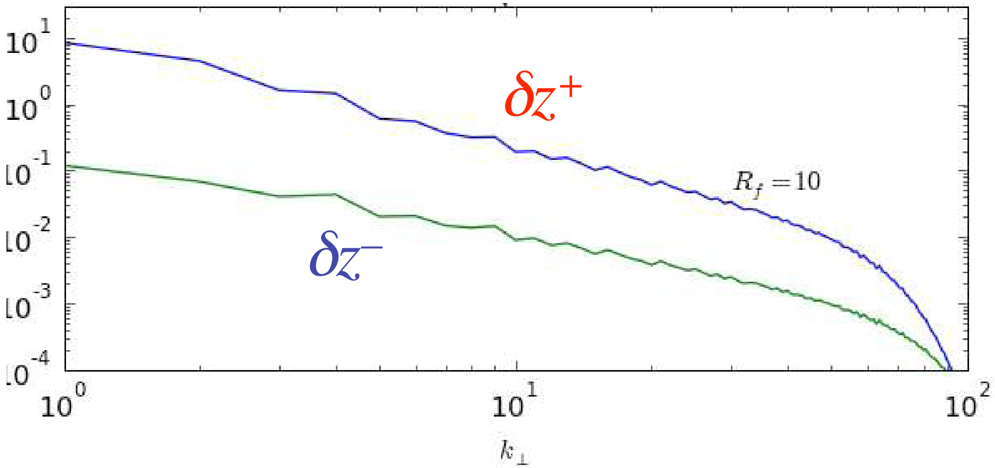

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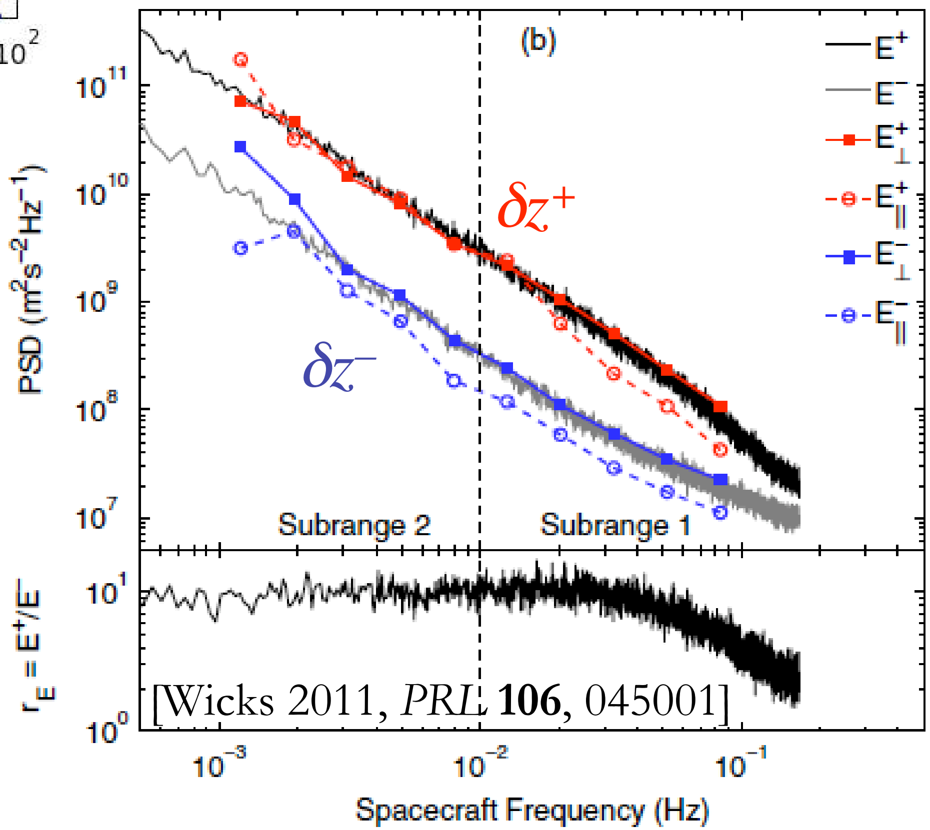


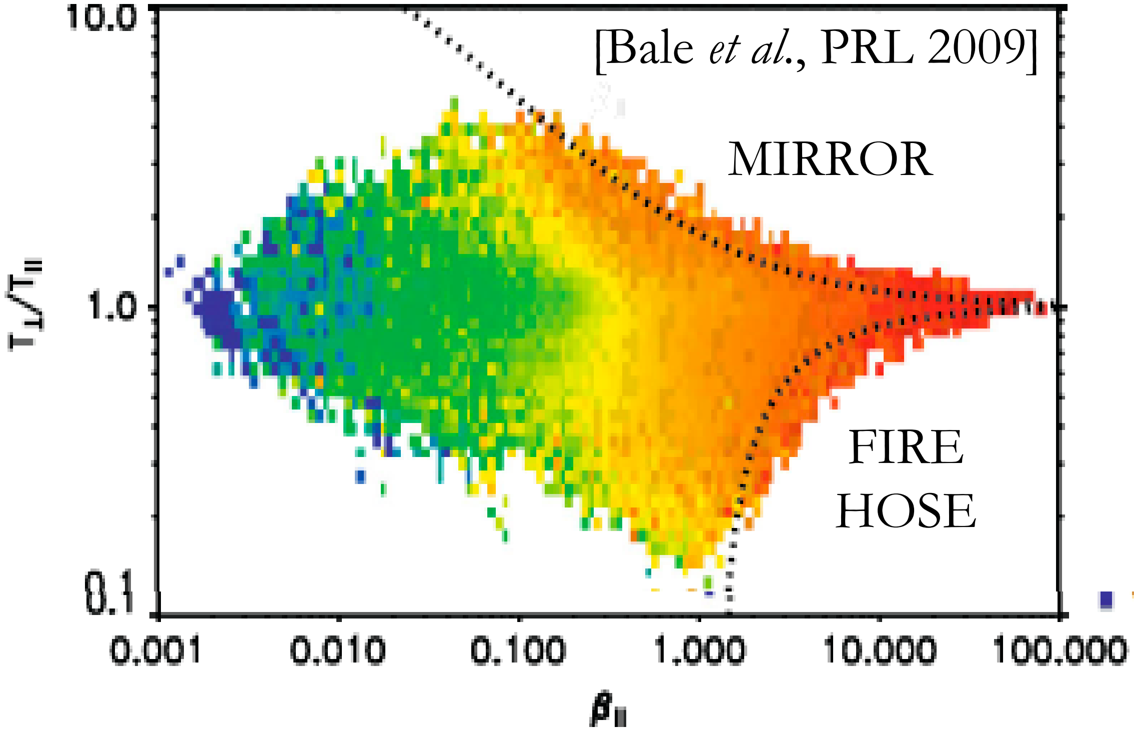
[Wicks 2011, *PRL* **106**, 045001]

Scaling theories that are simple extensions of those for the balanced case do not describe correctly either numerics or measurements (which also disagree with each other)

5. Microphysical Energy Injection

This means that there is energy injection just above the ion Larmor scale

NB: The firehose ($k_{||} \sim k_{\perp}$) or the nonlinear state of mirror modes ($\delta B/B \sim 1$) are not described by GK!



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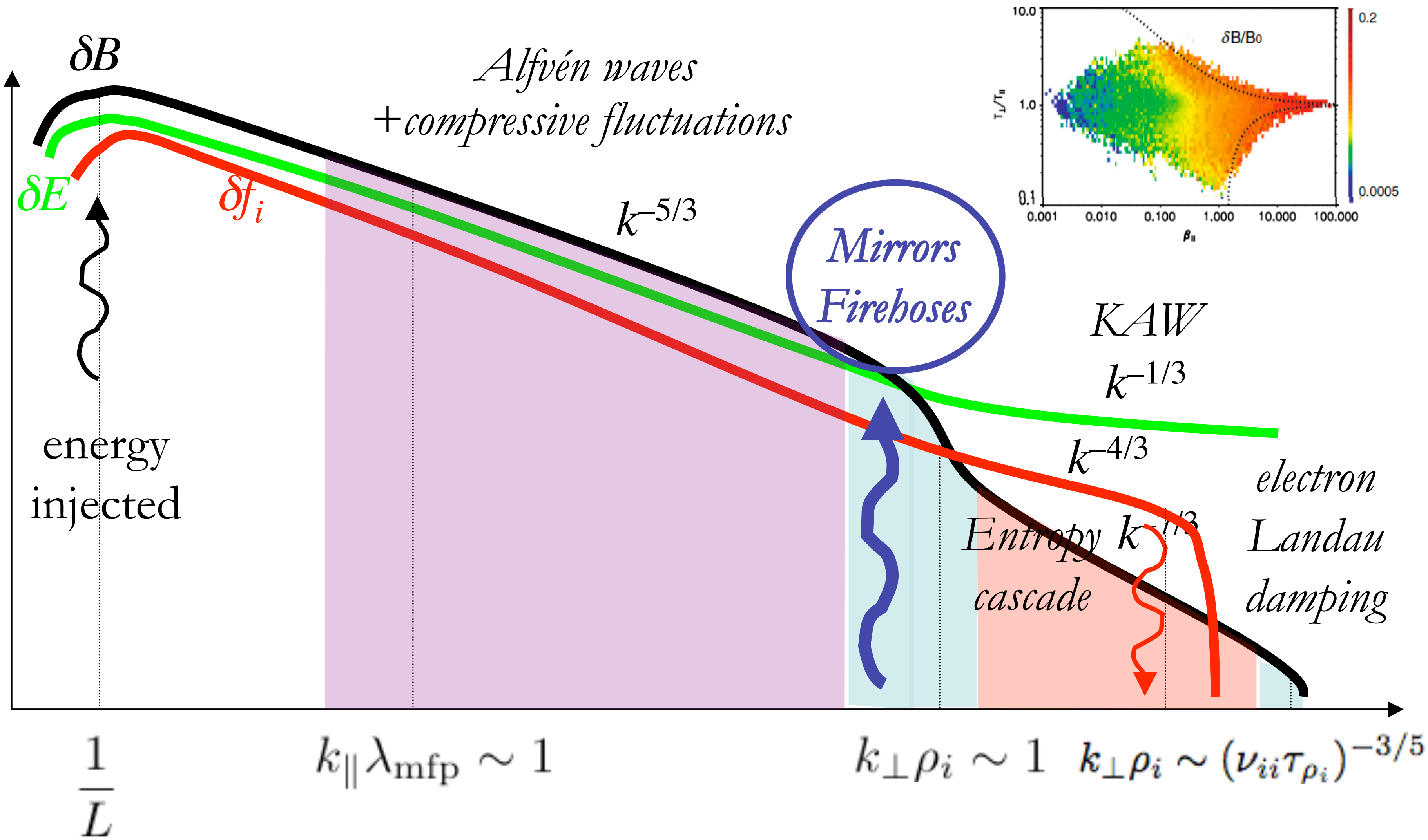
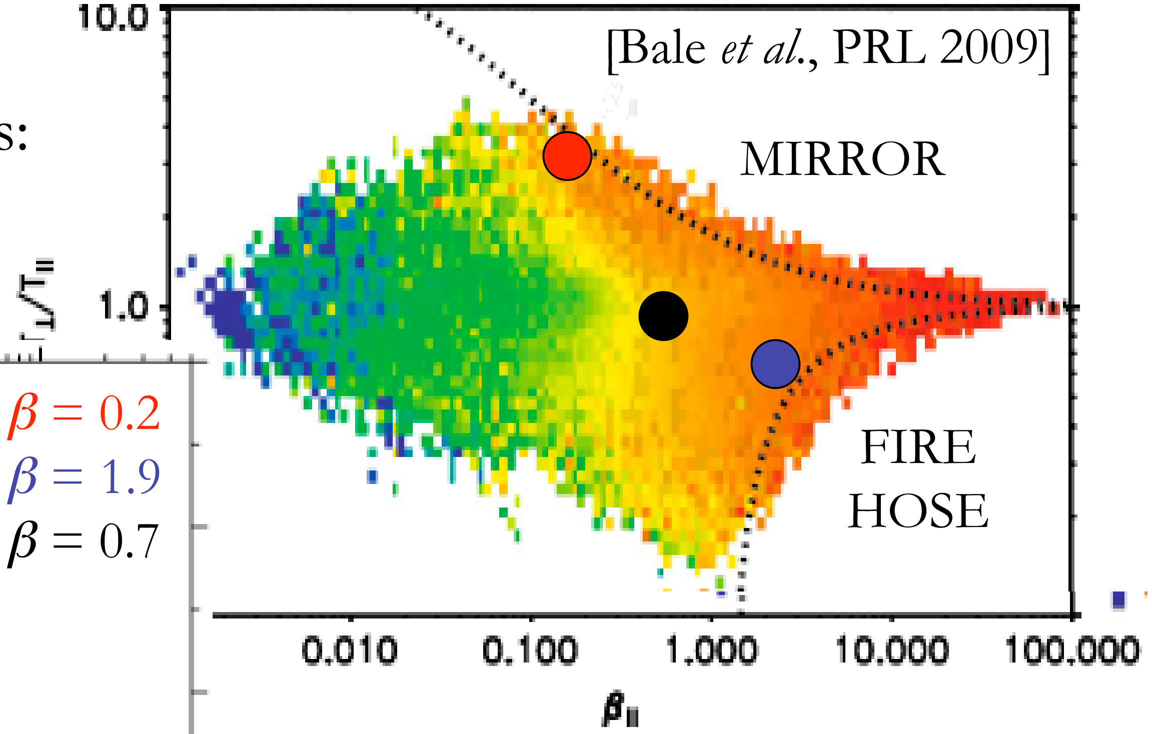
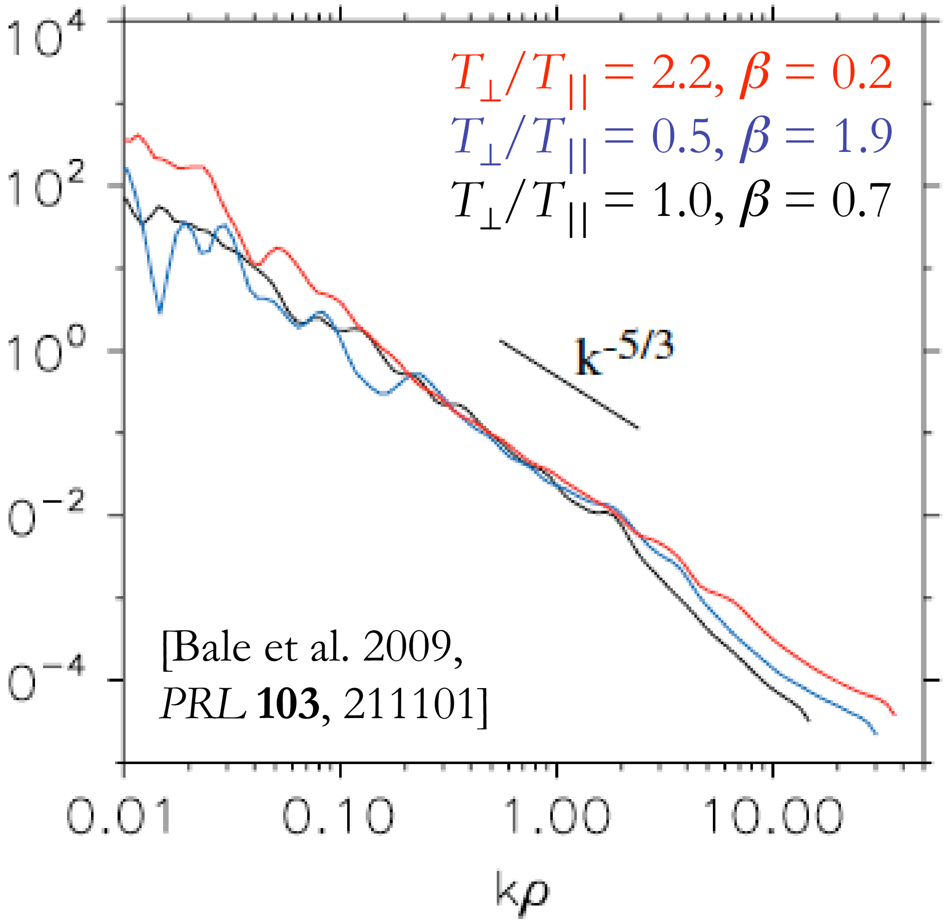


5. Microphysical Energy Injection

The injection from microinstabilities not obvious: swamped by cascade?





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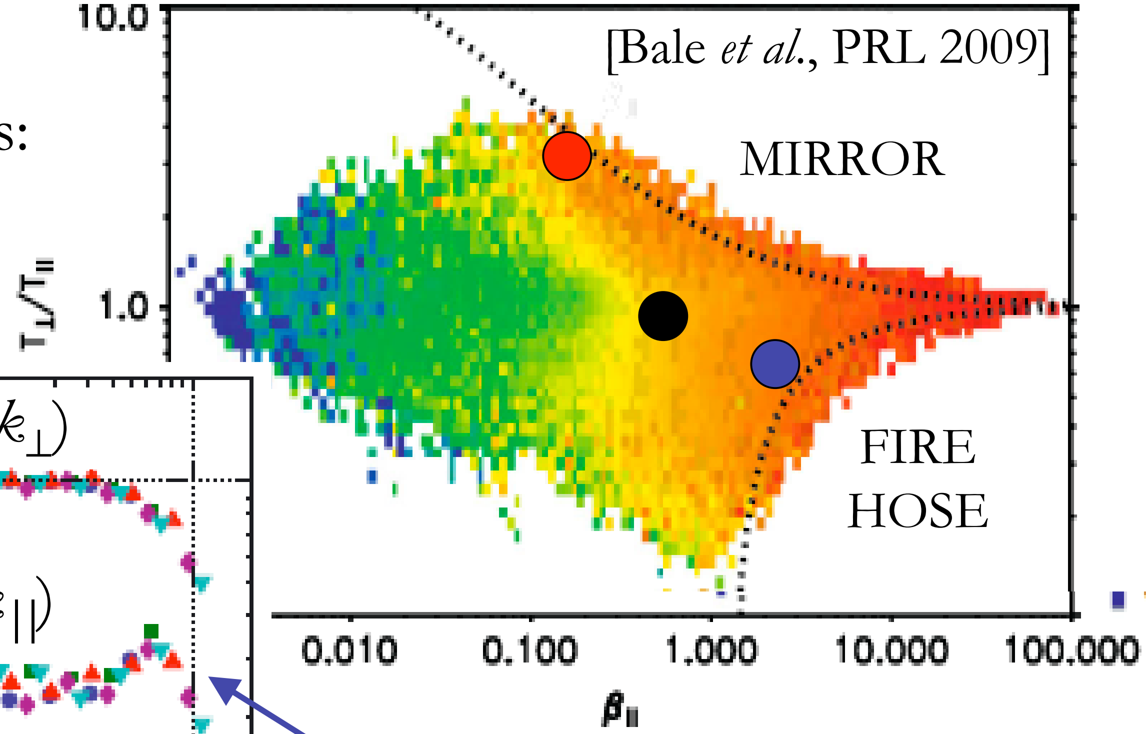
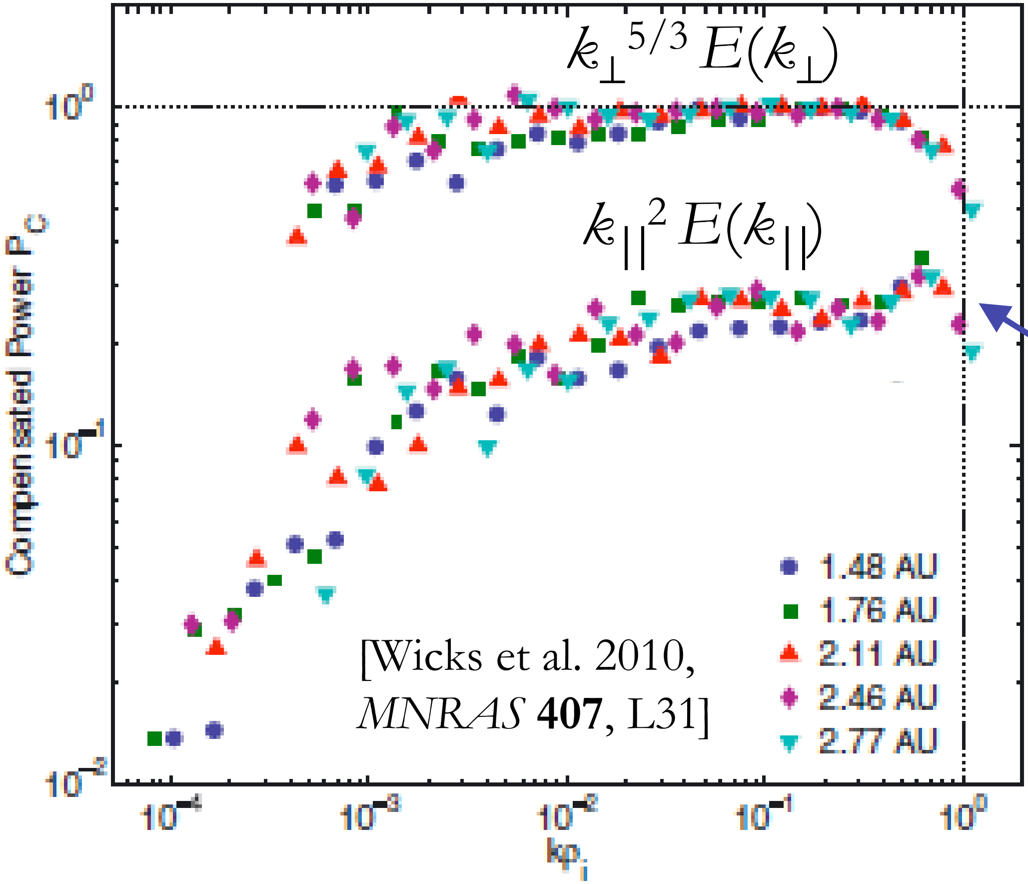
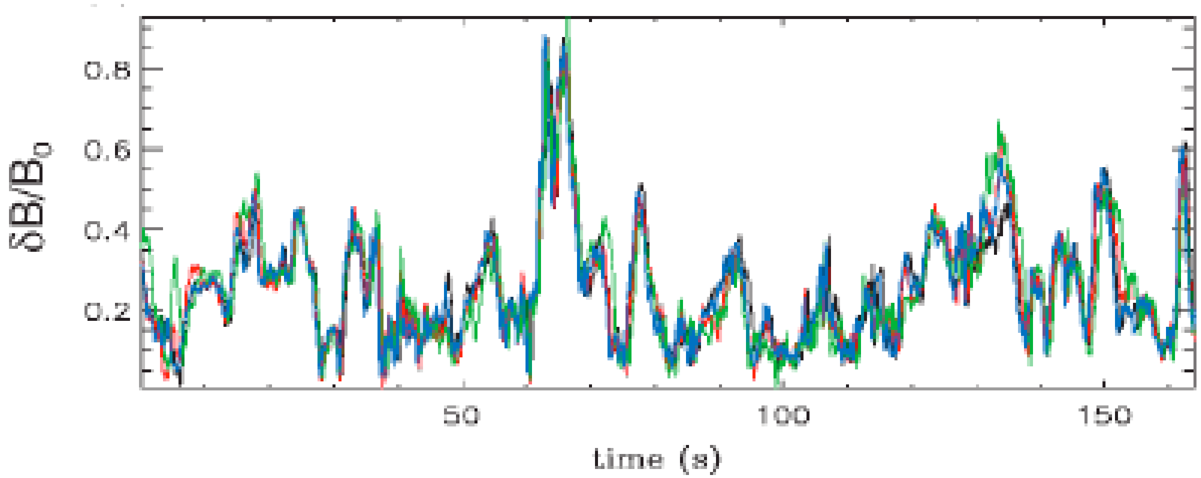
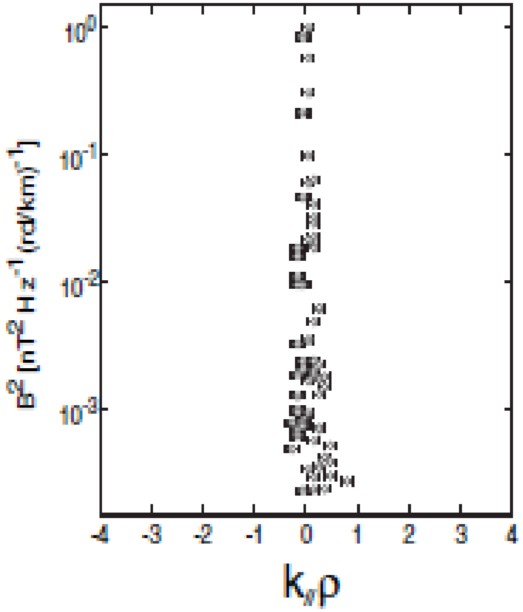


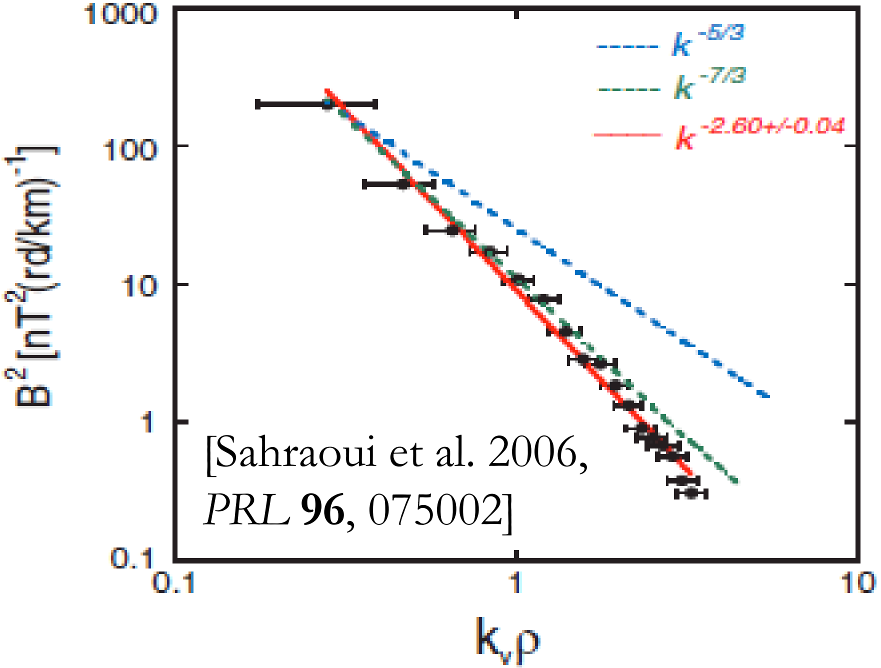


Firehose?
(less anisotropic than the cascade)

“Mirror Cascade”?

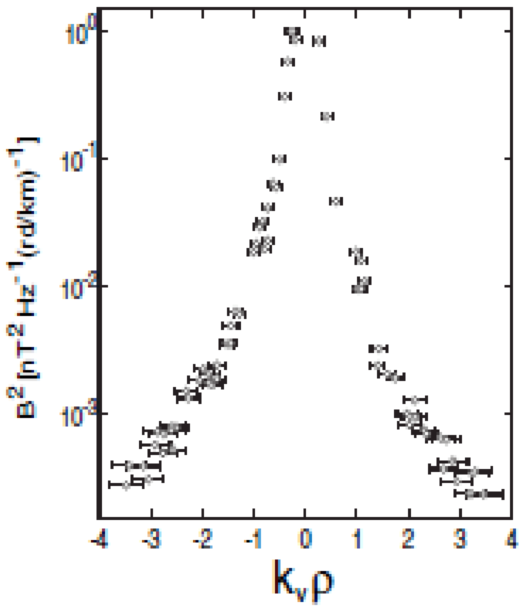




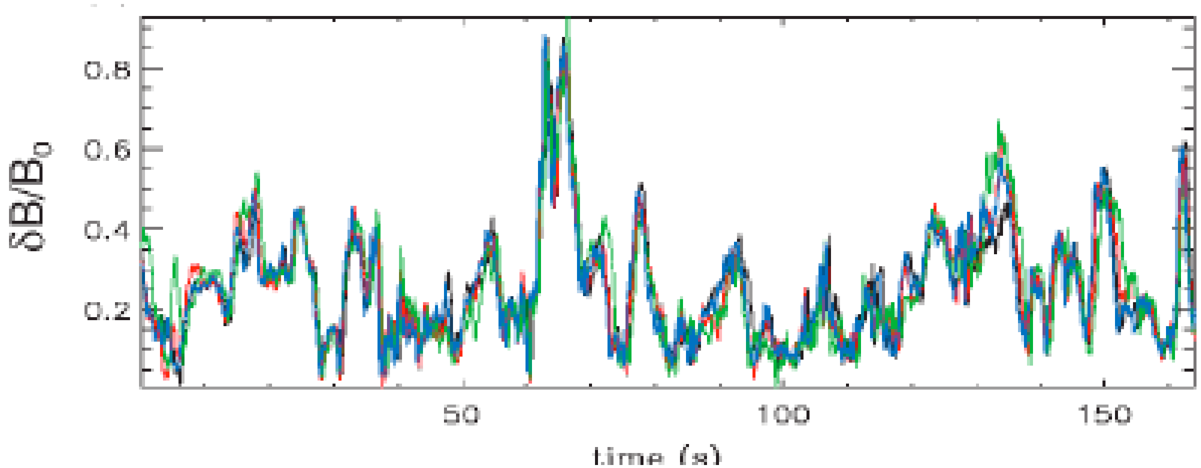


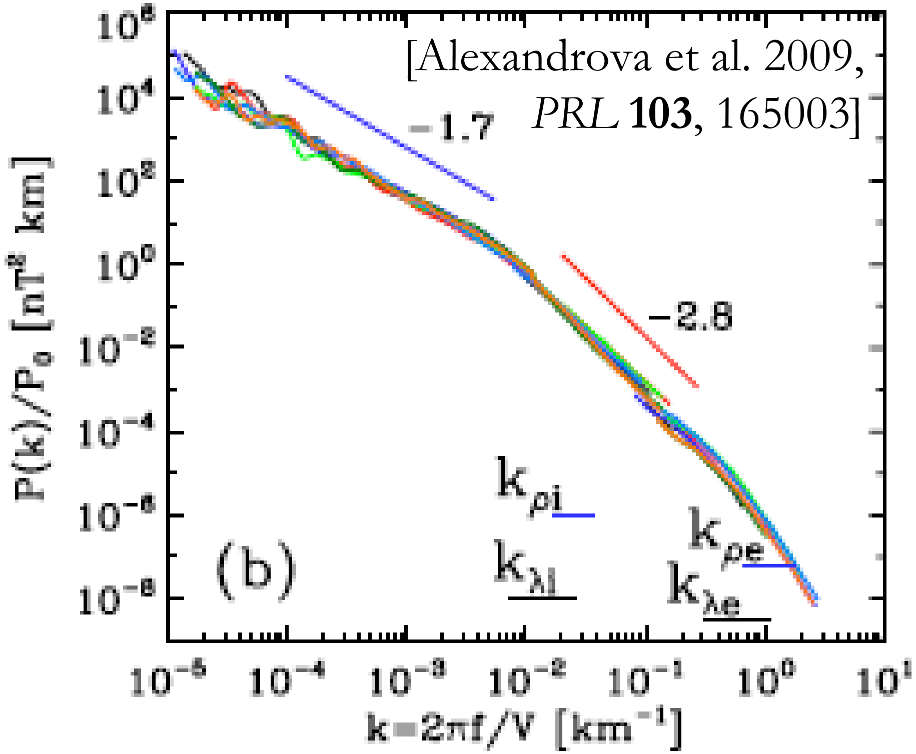
[Sahraoui et al. 2006, *PRL* **96**, 075002]

- $\delta B/B \sim 1$
- $k_{||} << k_\perp$
- Spectrum closer to –8/3 than –7.3



6. Universal Not-Quite-KAW Cascade?





Another interesting problem to sort out

- $\delta B/B \sim 1$
- $k_{||} << k_{\perp}$
- Spectrum closer to –8/3 than –7/3
- But it tends to be –2.8 anyways (remarkable **universality**, btw!)

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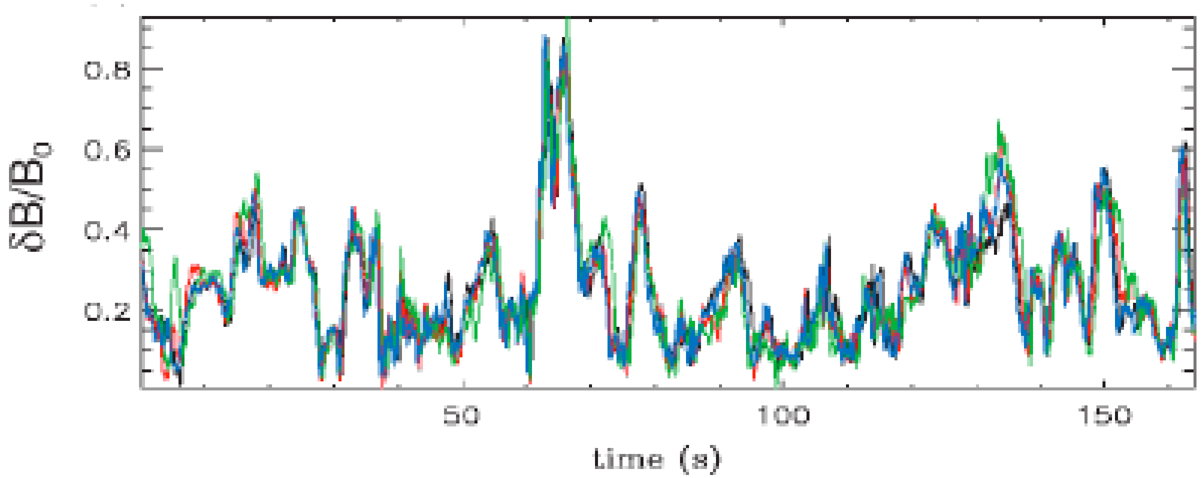
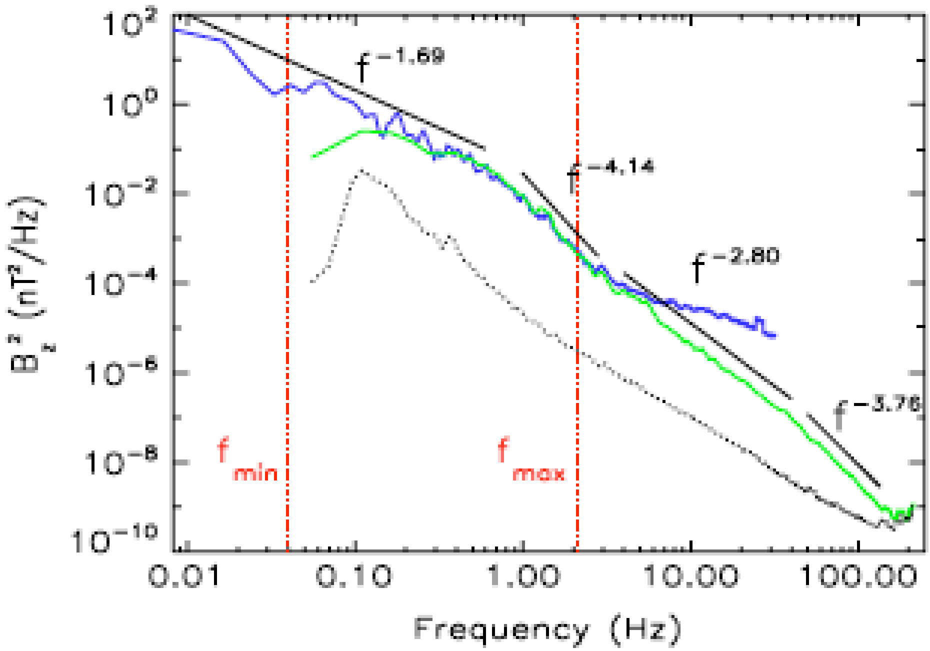


Another interesting problem to sort out

[Saharaoui et al. 2010, *PRL* **105**,131101]



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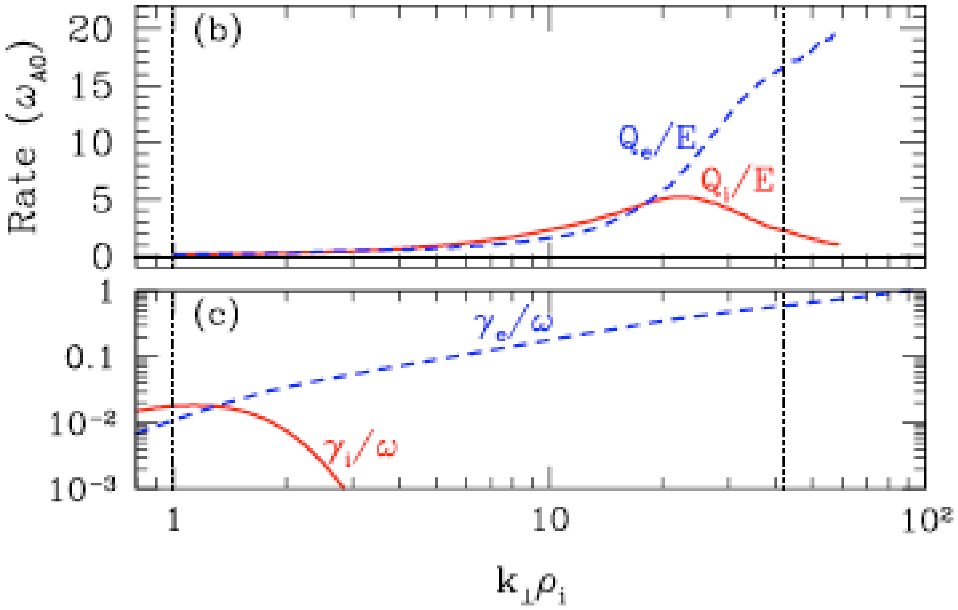
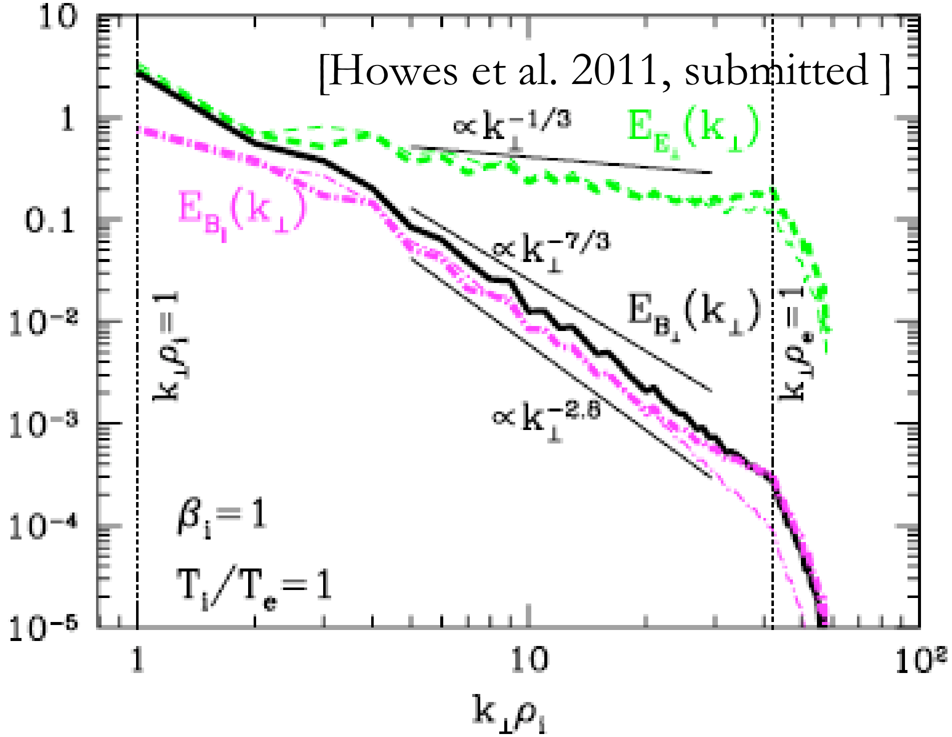
- Spectrum closer to –8/3 than –7/3
- But it tends to be –2.8 anyways (remarkable **universality**, btw!)

6. Universal Not-Quite-KAW Cascade?

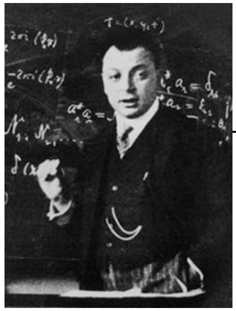
It might be just the usual critically balanced KAW cascade (–7/3) steepened a bit by electron Landau damping (but no theory for that…)



GK simulations:



- $k_{||} << k_\perp$
- Spectrum closer to –8/3 than –7/3
- But it tends to be –2.8 anyways (remarkable **universality**, btw!)



Part II. The Known Unknowns

1. Ion vs. Electron Heating

What sets T_i/T_e?

2. Compressive fluctuations in the inertial range

Why are they not damped?

3. Velocity (3/2) and magnetic (5/3) spectra in the inertial range

4. Nature of imbalanced Alfvénic cascade

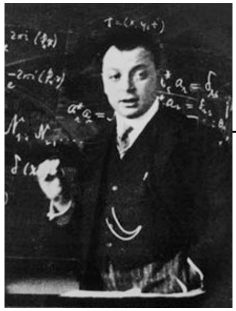
5. Microphysical energy injection

How do the mirror/firehose fluctuations and the KAW cascade coexist in the sub-Larmor range?

6. Universal scaling in the sub-Larmor range?

[7. Perpendicular vs. parallel heating]

Schekochihin *et al.*, ApJS **182**, 310 (2009)
Schekochihin *et al.*, MNRAS **405**, 291 (2010)
Rosin *et al.*, MNRAS, in press; arXiv:1002.4017



Part I. The Knowns

1. **Kinetic turbulence is a generalised (free) energy cascade in phase space towards collisional scales.**
   The free energy cascade splits into various channels:
   AW + *compressive* above ion gyroscale ("inertial range")
   KAW + *entropy cascade* belowion gyroscale ("dissipation range")
2. **Turbulence is anisotropic at all scales**
   Scaling theories based on the **critical balance** conjecture give results that seem broadly to be consistent with SW evidence and GK simulations
3. **Plasma is marginal to microinstabilities** (firehose, mirror etc. driven so by spontaneous generation of pressure anisotropies)

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