Kinetic models for chemotaxis

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Main Goal

- To study two different levels of description for chemotaxis and to study how these two levels are related.

General Picture

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Question:

Which is the set of equations that Φ obey?

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If

$$\Phi(t) = \lim_{\varepsilon\to 0} \Phi_\varepsilon(t)\ , \quad t < T$$

(in some sense) then $\mathcal{M}$ is the limit model of $\mathcal{M}_\varepsilon$.

Chemotaxis

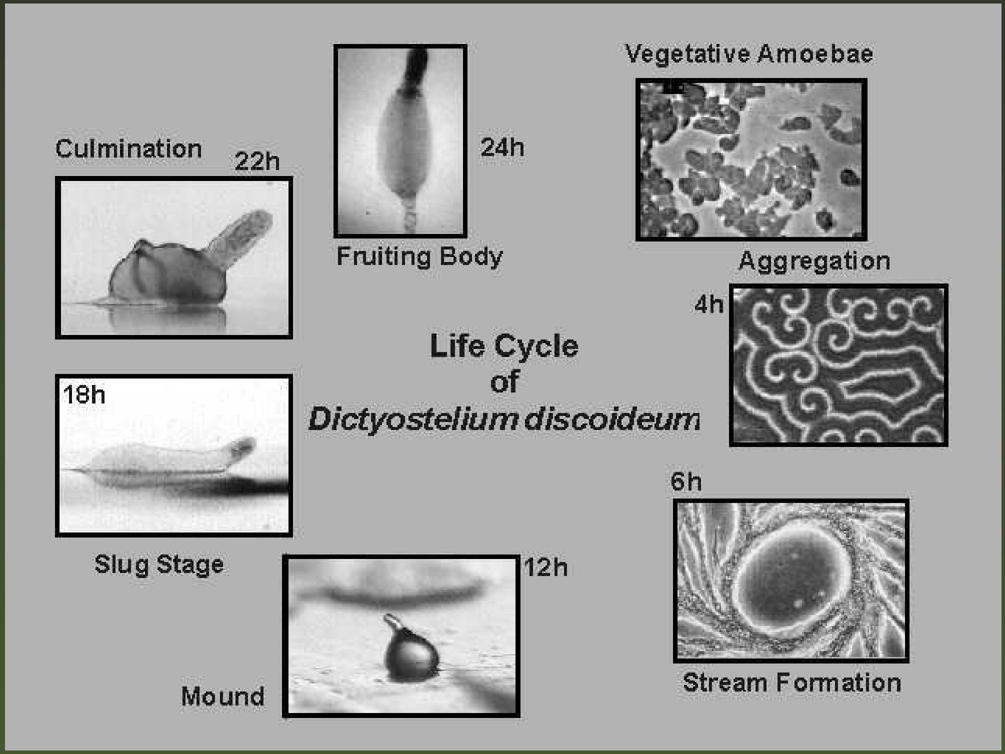

Figure 1: Life cycle of *Dictyostelium discoideum*. Picture made by Florian Seigert and Kees Wiejer (Zoologisches Institut München Ludwig-Maximilians-Universität München).

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- The cell goes in straight line for a certain characteristic time and then changes its direction from v' to v (in a space-time point (x, t) in the presence of the substance S and cell density ρ) according to a certain turning kernel $T[S, \rho](x, v, v', t)$.

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- The set of all possible velocities is given by a compact, spherically symmetric set V.

Velocity jump model

$$\partial_t f(x,v,t) + v \cdot \nabla f(x,v,t) =$$

$$\int_V (T[S,\rho](x,v,v',t) f(x,v',t) - T[S,\rho](x,v',v,t) f(x,v,t)) dv' \ .$$

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$f(x,v,t)$ = cell density in space-time point (x,t) with velocity v (*phase-space density*).

Velocity jump model

Notation

$$\begin{aligned} f &= f(x,v,t)\ , \\ f' &= f(x,v',t)\ , \\ T[S,\rho] &= T[S,\rho](x,v,v',t)\ , \\ T^*[S,\rho] &= T[S,\rho](x,v',v,t). \end{aligned}$$

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Equation

$$\partial_t f + v \cdot \nabla f = \int_V (T[S, \rho] f' - T^*[S, \rho] f) dv'\ .$$

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- We should consider also an equation for S:

$$\partial_t S = D_0 \Delta S + \varphi(S,\rho) \ .$$

Keller-Segel model

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Keller-Segel equations:

$$\begin{aligned}\partial_t \rho &= \nabla \cdot (D \nabla \rho - \chi(S)\beta(\rho)\rho \nabla S) \ , \\ \partial_t S &= D_0 \Delta S + \varphi(S, \rho) \ .\end{aligned}$$

$$\begin{aligned}\rho &= \text{cell density} \ , \\ S &= \text{density of chemo-attractant} \ , \\ \chi &= \text{chemotactic sensitivity} \ , \\ D, D_0 &= \text{diffusion coefficients} \ , \\ \varphi &= \text{interaction between } \rho \text{ and } S.\end{aligned}$$

Keller-Segel model

- Typical example of interaction

$$\varphi(S,\rho) = \alpha\rho - \beta S \ ,$$

with $\alpha > 0$, $\beta \geq 0$.

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with $\alpha > 0$, $\beta \geq 0$.

- Finite-time-blow-up:

$$\lim_{t \to T} \left(||\rho(\cdot,t)||_{L^\infty(\mathbb{R}^n)} + ||S(\cdot,t)||_{L^\infty(\mathbb{R}^n)} \right) = \infty \ .$$

Ex: $\rho(\cdot,T) \longrightarrow \delta_a$.

Re-scaling

Let us go back to the Othmer-Dunbar-Alt model:

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$$\begin{aligned} x' &= x/x_0 \ , \quad & t' &= t/t_0 \ , \quad & v' &= v/v_0 \ , \\ T' &= T/T_0 \ , & S' &= S/S_0 \ , & \rho' &= \rho/\rho_0 \ , \\ f' &= f/f_0 \ . \end{aligned}$$

Re-scaling

$$\frac{\partial f}{\partial t} + \frac{v_0}{x_0/t_0} v \cdot \nabla f = T_0 v_0^n t_0 \int_V (T f' - T^* f)\, dv' \,,$$

$$\frac{\partial S}{\partial t} = \frac{t_0}{x_0^2} D_0 \Delta S + \frac{\alpha_1 \rho_0 t_0}{S_0} \rho - \alpha_2 t_0 S \,,$$

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We impose the *diffusive scaling*: $t_0 \approx x_0^2$, normalizations and

$$
\varepsilon = \frac{x_0/t_0}{v_0}.
$$

Re-scaling

$$\frac{\partial f}{\partial t} + \frac{1}{\varepsilon} v \cdot \nabla f = \frac{1}{\varepsilon^2} \int_V (T_\varepsilon f' - T_\varepsilon^* f)\, dv' \,,$$

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The kernel T depends on ε...

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The solution depends on ε...

Formal computations

Consider the turning kernel

$$T_\varepsilon[S,\rho] = T_0[S,\rho] + \varepsilon T_1[S,\rho] + \cdots$$

such that

$$\begin{aligned} T_0[S,\rho] &= \lambda(S,\rho)(x,t)F(v)\ , \\ T_1[S,\rho] &= F(v)a(S,\rho)v\cdot\nabla S\ , \end{aligned}$$

where

$$F > 0\ , \quad \int_V F dv = 1\ , \quad \int_V vF dv = 0\ ,$$

$$\lambda \geq \lambda_{\min} > 0$$

Formal computations

Two possible examples are given by

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\begin{aligned}
T_\varepsilon[S,\rho] &= F(v)\lambda(S,\rho) + \varepsilon F(v) a(S,\rho) v \cdot \nabla S \ , \\
T_\varepsilon[S,\rho] &= \psi(S(x,t), S(x + \varepsilon\mu(\rho) v, t)) F(v) \ ,
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We consider the formal expansion of the solutions:

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f_\varepsilon &= f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \cdots \ , \\
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We put these expansions in the model, match terms with the same order of ε and solve the resulting system.

Formal computations

In both cases the *formal* drift-diffusion limit is given by the zeroth order equations (Othmer, Hillen)

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\begin{aligned}
\partial_t \rho_0 &= \nabla \cdot \left(D(S_0, \rho_0) \nabla \rho_0 - \chi(S_0, \rho_0) \rho_0 \nabla S_0 \right) \ , \\
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where

$$\begin{aligned} D(S_0, \rho_0) &:= \frac{1}{n\lambda(S_0, \rho_0)} \int_V v^2 F dv \mathbb{I} \ , \\ \chi(S_0, \rho_0) &:= \frac{a(S, \rho)}{n\lambda(S_0, \rho_0)} \left(\int_V v^2 F dv \right) \ . \end{aligned}$$

General Picture (again...)

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Rigorous results

Theorem (C., Markowich, Perthame, Schmeiser, Hwang, Kang, Stevens, Rodrigues): Consider turning kernels T_ε depending on S_ε, ∇S_ε and ρ_ε under mild assumptions. Then, the solution of the kinetic model $(f_\varepsilon, S_\varepsilon)$ is such that

$$\begin{aligned} \rho_\varepsilon &\to \rho_0 \ \text{in } L^2_{\text{loc}}(\mathbb{R}^n)\,, \\ S_\varepsilon &\to S_0 \ \text{in } L^q_{\text{loc}}(\mathbb{R}^n)\,, 1 \le q < \infty\,, \\ \nabla S_\varepsilon &\to \nabla S_0 \ \text{in } L^q_{\text{loc}}\,, 1 \le q < \infty. \end{aligned}$$

where (ρ_0, S_0) is the solution of the associated Keller-Segel model.

Rigorous Results

By *mild assumptions* we mean:

$$\begin{aligned} \phi_\varepsilon^S[\rho, S] &\geq \gamma(1 - \varepsilon\Lambda(||S||_{W^{1,\infty}}))FF' \, , \\ \int_V \frac{\phi_\varepsilon^A[\rho, S]^2}{F\phi_\varepsilon^S[\rho, S]} dv' &\leq \varepsilon^2 \Lambda(||S||_{W^{1,\infty}}) \, , \end{aligned}$$

where

$$\begin{aligned} \phi_\varepsilon^S &:= \frac{T_\varepsilon[S, \rho]F' + T_\varepsilon^*[\rho, S]F}{2} \, , \\ \phi_\varepsilon^A &:= \frac{T_\varepsilon[S, \rho]F' - T_\varepsilon^*[\rho, S]F}{2} \, , \end{aligned}$$

and other more technical ones.

Global Existence

Theorem (C., Markowich, Perthame, Schmeiser, Hwang, Kang, Stevens): Consider turning kernels such that:

$$\begin{aligned} 0 \le T_\varepsilon[S_\varepsilon, \nabla S_\varepsilon] &\le c_1 + c_2 S(x+\varepsilon v, t) + c_3 S(x - \varepsilon v', t) \\ &\quad + c_4 |\nabla S(x+\varepsilon v, t)| + c_5 |\nabla S(x - \varepsilon v, t)| \,, \\ |\nabla T_\varepsilon[S_\varepsilon, \nabla S_\varepsilon]| &\le c_2 |\nabla S(x+\varepsilon v, t)| + c_3 |\nabla S(x - \varepsilon v', t)| \\ &\quad + c_4 |\nabla^2 S(x+\varepsilon v, t)| + c_5 |\nabla^2 S(x - \varepsilon v, t)| \,. \end{aligned}$$

The kinetic model has global existence of solutions.

Global Existence

Let us consider

$$T_\varepsilon[S](x,v,v',t) = \psi(S(x+\varepsilon v,t) - S(x,t))$$

with $\psi \geq \psi_{\min} > 0$, increasing and $\psi(y) \leq Ay + B$.

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$$T_\varepsilon[S](x,v,v',t) = \psi(S(x+\varepsilon v,t) - S(x,t))$$

with $\psi \geq \psi_{\min} > 0$, increasing and $\psi(y) \leq Ay + B$. Then the kinetic model has global existence of solution, converges (in the drift diffusion limit) to the *classical* Keller-Segel model (which presents blow up).

Global Existence

Theorem (C., Rodrigues): For certain classes of turning kernels $T_\varepsilon[S_\varepsilon, \nabla S_\varepsilon, \rho_\varepsilon]$, such that if $\rho_\varepsilon(x,t) \geq \bar{\rho}$, then

$$T_\varepsilon[S_\varepsilon, \nabla S_\varepsilon, \rho_\varepsilon](x,v,v',t) = T_0[S_\varepsilon, \nabla S_\varepsilon, \rho_\varepsilon](x,v,v',t) \ ,$$

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and for ε small enough, we conclude global existence of solution $(f_\varepsilon, S_\varepsilon)$. Furthermore,

$$||\rho_\varepsilon(\cdot,t)||_{L^\infty(\mathbb{R}^n)} \leq \max\{\bar{\rho}, ||\rho^{\mathrm{I}}||_{L^\infty(\mathbb{R}^n)}\} \ .$$

Global Existence

Example:

$$T_\varepsilon[S,\rho] = \lambda(S,\rho)F + \varepsilon a(S,\rho)Fv\cdot\nabla S\ ,$$
$$T_\varepsilon[S,\rho] = \psi(S(x+\varepsilon\mu(\rho)v,t) - S(x,t))F$$

with

$$a(S,\rho) = 0\ ,\ \ \mu(\rho) = 0\ ,\ \ \rho \geq \bar{\rho} > 0\ .$$

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Corollary: For the limit Keller-Segel model, we can conclude global existence of solutions. Furthermore, the cell density is bounded. (Hillen, Painter)

$$
\partial_t\rho = \nabla\cdot(\nabla\rho - \beta(\rho)\rho\nabla S)\ ,\quad \beta(\rho) = 0\ , \rho \geq \bar{\rho} > 0\ .
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Beyond Keller-Segel

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Q: Is blow up good or bad?
A: *Bad:* Blow up does not exists in nature, we should look for models without blow up.
A: *Good:* Keller-Segel model cannot be valid after certain time, so a realistic model should indicate that it breaks down.

Beyond Keller-Segel

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A: Yes. This was done in the kinetic level. Other possibility is given by Velazquez:

$$\partial_t \rho = \nabla \cdot \left(\nabla \rho - \frac{\rho}{1 + \mu\rho} \nabla S \right)$$

For any $\mu > 0$, there is global existence of solutions. For $\mu = 0$, solutions blow up in finite time.

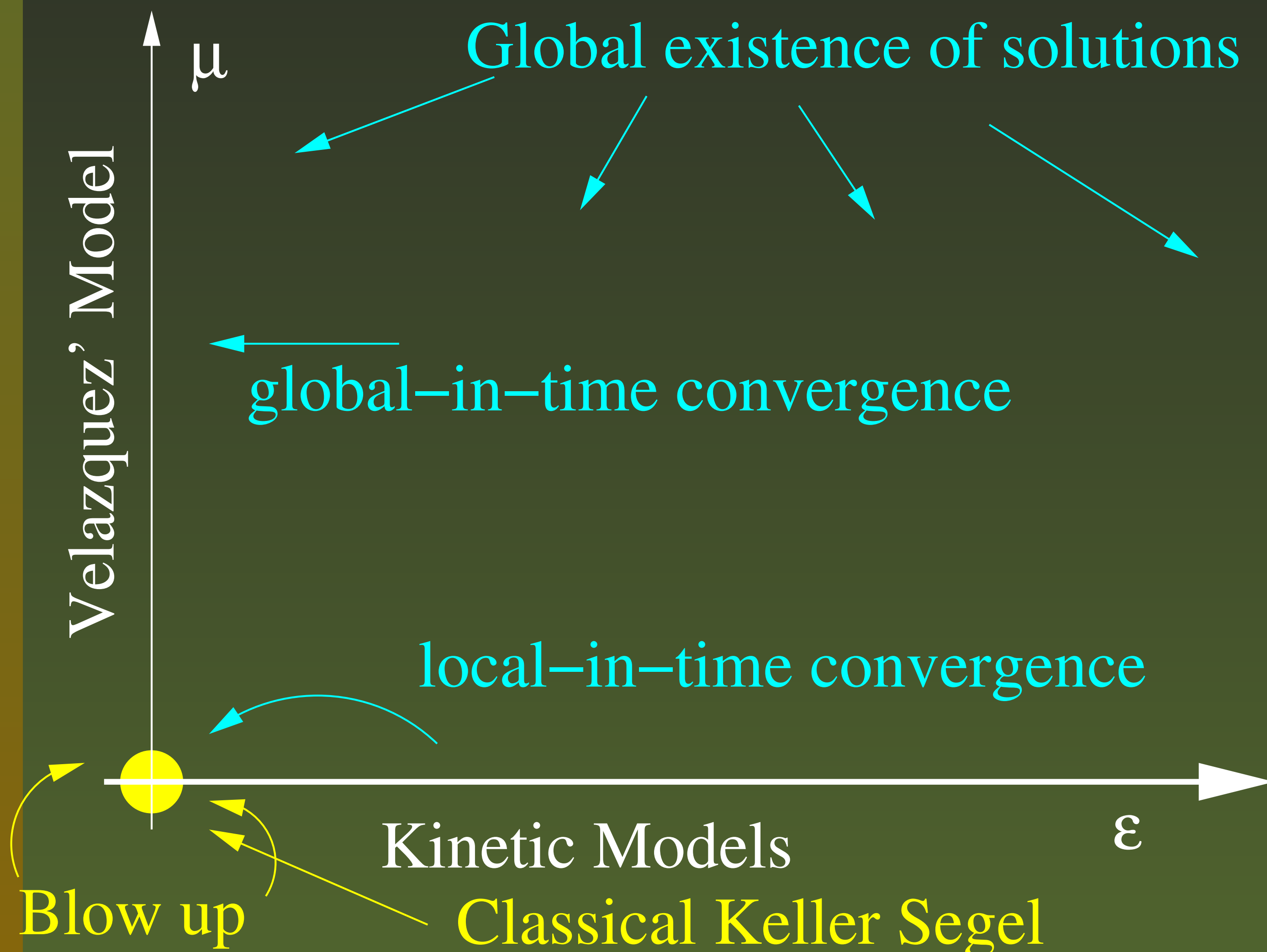
Beyond Keller-Segel

With

$$T_{\varepsilon,\mu}[S,\rho] = \psi\left(S\left(x + \frac{\varepsilon}{1+\mu\rho}v\right) - S\left(x,t\right)\right)$$

the solution exists globally and the drift-diffusion limit (*globally in time*) is the Velazquez' model (C., Kang).

Beyond Keller-Segel



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THE END