GENERIC PROPERTIES OF FINITELY PRESENTED GROUPS AND HOWSON'S THEOREM.

G. N. Arzhantseva

Department of Mathematics and Mechanics, Moscow State University, Moscow 119899, Russia. E-mail: arjantse@nw.math.msu.su

0. Introduction. Let $X_m = \{x_1^{\pm 1}, \ldots, x_m^{\pm 1}\}$ be an alphabet with $m \geq 2$. For fixed n one can consider all group presentations

$$G = \langle x_1, \ldots, x_m \mid r_1 = 1, \ldots, r_n = 1 \rangle, \tag{1}$$

where $r_1, \ldots, r_n$ are cyclically reduced words in the alphabet X_m of length $\mid r_i \mid \leq t$. Let $N = N(m, n, t)$ be the number of all such presentations (1) nd $N_{\mathcal{P}}$ be the number of presentations among them such that the group G has a property $\mathcal{P}$. A property $\mathcal{P}$ of m - generated groups is said to be *generic* if for any n the ratio $N_{\mathcal{P}}/N$ tends to 1 as $t \to \infty$. We shall say that the genericity is exponential if this ratio tends to 1 faster than some function $1 - \exp(ct)$, where $c < 0$. Further, we will consider the exponential genericity only. The study of generic properties is useful because the combination of an arbitrary finite number of generic properties is again a generic property of group.

The following classes of groups are generic (see [1], [2]):

(1) torsion-free groups;

(2) groups satisfying the small cancellation condition $C'(\lambda)$ [3] for every fixed $\lambda > 0$;

(3) m - generated groups all of whose k - generated subgroups (for any $k < m$) are free;

(4) m - generated groups all of whose $\leq L$ - generated subgroups of infinite index are free (for some given L, possibly $L \gg m$) but all subgroups of finite index are not free.

To prove the genericity of class (4) we introduced in [2] a new condition on the defining relators of a finitely presented group such that the group satisfying it belongs to the considered class. The condition is formulated in terms of finite labeled graphs and is generic [2].

Recall that a subgroup P of G is *quasiconvex* in G if there exists $\varepsilon > 0$ such that any geodesic in the *Cayley graph* $C(G)$ of G with endpoints in P belongs to the ε - neighbourhood of P. Obviously, the quasiconvexity property depends on the group presentation chosen. In this paper we show that in the group G with presentation (1) satisfying the mentioned above condition every subgroup with the preassigned number of generators is quasiconvex for any presentation of G.

This allows us to prove the following version of Howson's Theorem for generic groups. Denote by $N_d = N_d(m, n, t)$ the number of all presentations (1) such that the intersection of any pair of $\leq d$ - generated subgroups is finitely generated one in the group G for some preassingned d.

Theorem 1. *There exists a constant* $a = a(m, n, d) > 0$ *such that the ratio* $N_d(m, n, t)/N(m, n, t)$ *is greater than* $1 - \exp(-at)$, *where* t *is positive integer.*

In other words, the intersection of any two subgroups with a given number of generators is finitely generated in a generic group.

In the sequel, we will use the presentations of finitely generated subgroups of a free group by finite connected X_m - labeled graphs. Such presentations were introduced by J.Stallings and were used for investigations of subgroups of a free group, free inverse monoids, and generic properties of finitely presented groups (see [4], [5] and [1] for details).

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