Entropy method for hypocoercive and non-symmetric Fokker-Planck equations with linear drift

Anton Arnold (Vienna University of Technology)

Abstract. In the last 15 years the entropy method has become a powerful tool for analyzing the large-time behavior of the Cauchy problem for linear and non-linear Fokker-Planck type equations (advection-diffusion equations, kinetic Fokker-Planck equation of plasma physics, e.g.). In particular, this entropy method can be used to analyze the rate of convergence to the equilibrium (in relative entropy and hence in L^1). The essence of the method is to first derive a differential inequality between the first and second time derivative of the relative entropy, and then between the entropy dissipation and the entropy.

For degenerate parabolic equations, the entropy dissipation may vanish for states other than the equilibrium. Hence, the standard entropy method does not carry over. For hypocoercive Fokker-Planck equations (with drift terms that are linear in the spatial variable) we introduce an auxiliary functional (of entropy dissipation type) to prove exponential decay of the solution towards the steady state in relative entropy. We show that the obtained rate is indeed sharp (both for the logarithmic and quadratic entropy). Finally, we extend the method to the kinetic Fokker-Planck equation (with non-quadratic potential) and non-degenerate, non-symmetric Fokker-Planck equations. For the latter examples the "hypocoercive entropy method" yields the sharp global decay rate (as an envelope for the relative entropy function), while the standard entropy method only yields the sharp local decay rate.

References:

* A. Arnold, J. Erb: Sharp entropy decay for hypocoercive and non-symmetric Fokker-Planck equations with linear drift; arXiv 2014.
* F. Achleitner, A. Arnold and D. Stürzer: Large-time behavior in non-symmetric Fokker-Planck equations; Riv. Mat. Univ. Parma 6 (2015) 1–68.

Mathematical Physics Seminar
07.06.2016, 13:15
1090 Wien, Oskar-Morgenstern-Platz 1, Besprechungszimmer 2nd Floor