Mathematical and Computational Modeling of Housing Market Dynamics

System engineering point of view

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Abstract—**Housing market dynamics and modeling** of housing market evolution **in Lugano (southern part of Switzerland) is the main goal of this work. An agent-based modeling has been used to model the individual behavior and decision-making process of people in the market. In fact this model is a representation of the decision making processes of humans in housing market. Every agent is identified through his position, time interval he exists in the system, his individual and social properties and his behavior. Different features make different mentalities in different agents to process uncertain information they gain from environment and the other agents to make two different types of events, i.e., supplies and demands. The model tries to forecast and analyze the dynamics of housing market in Lugano by modeling behavior of individuals who are living in this city and constructing the model on the interdependency of these behaviors and the information that flows in the system. In this paper the modeling approach we have used and the conceptual framework of the model will be presented.**

Keywords-component; agent-based modeling; multi-agent systems; decision making process; complex system engineering.

I. Introduction

Market is a macro-level phenomenon that its behavior depends on the behavior of human individuals. This type of macro-level phenomenon that is produced by the entities who exist on micro-level are simulated by individual-based simulation methods [1, 2]. When the entities in an individual-based simulation have cognitive processes capabilities, the individual-based simulation is called multi-agent approach [3,4].The problem we intend to deal with is housing market modeling which is a macro-level phenomenon that the significant actors in it are humans, so we decided to use a multi-agent based modeling approach to model this phenomenon. In this model, our focus is on individual behavior modeling but the goal is answering the questions that are related to the phenomena that occur in aggregated (macro) level. At any given time, each agent acts according to its current situation, the state of the world and the rules governing its behavior. By this model we are able to present a mathematical and engineering approach to model any financial economy as a complex system and analyzing the dynamics of the system in future.

Housing market in Lugano has been modeled by the agents and their spatial location. Two agent groups, i.e, supply makers and demand makers from Lugano and neighborhood town's population has been defined. In our model the agents don't behave just based on utility maximization strategy. They have some cognitive processes capabilities that enable them to make decisions considering their satisfaction, benefit, preferences, intentions, the features of the spatial point and all the uncertainty they confront during this decision making process. In this work the bounded rational behavior of individuals is modeled considering this matter that people process linguistic properties to make decision. By this type of models we are able to trace the intentions and preferences of the agents to discover those types of behaviors leading to disasters. By locating the agents in spatial points we model the interrelationship of the spatial dynamics and the decision making processes in the agents [5]. The spatial point unit is one hectare of Lugano. Every agent is characterized by its features and location.

In this paper we discuss the engineering of the housing market modeling as a complex system. Engineering of housing market systems is an important asset for business and economics of the nations. What is very challenging in this model is that we plan to model the behavior of a system through modeling the behavior of individual agents which are living in the system and make the emergent behavior of the system. The method we have used can be used for modeling the dynamics of every economic system. By this model we are able to observe how the behavior of an agent can affect the price of the house or the segregation phenomenon in Lugano during next 15 years. Model is detailed enough to find the roots of the emergent behavior we will have and it can be very useful to have a better risk management in future.

In this paper we explain the engineering methods we have used to explain and represent the model in three parts; i.e., mathematical modeling, structural modeling and behavioral modeling of the housing market. In mathematical modeling part we define the conceptual framework of the model and explain the important concepts we have used to define the model. Structure and behavior as an essential view of a system will be explained in the later parts.

II. Conceptual framework of the model

In this part of the paper, we explain the general framework we have used in this model. First we start to state the purposes of doing such a model in terms of the problems which will be solved or the questions that are answered through doing this model. Answering these two questions are the main purposes of doing such a model but it is clear that in addition of answering these questions we are able to address a few other questions such as the effect of house prices on segregation or interest rate on the prices of the houses. There is some background information to construct the model. This information will be injected to this model as the rules that govern the behavior of the agents. The sources of this information are either the economic information or the economic models outputs regarding the problem. For example we have used the results of residential choice models that have been used by economists to extract the rules that we need to specify the preferences of the agents to select their residence.

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These rules will be used as the rules of a fuzzy type-2 expert system to evaluate the satisfaction level of the household with their houses and the environment they live in. We will explain more the mentioned satisfaction level and the fuzzy expert system that has been used in this model.

Problem: We want to answer two types of questions through doing this model that are:

1- The prices of the houses and real estate products in Lugano during the next 15 years using the supply and demand concepts.

2- The impact of the current rate of immigration of foreigners to Lugano on the behavior of the households in southern part of Switzerland from segregation point of view.

The conceptual framework of the model is demonstrated in Fig.1 and Fig.2. To study the behavior of housing market in Lugano, we have focused on three significant factors, i.e., supply, demand and price. Demand is the rate at which people want to buy a new house. Demand is required to motivate the house suppliers to make houses to sell. When the demands of a property in a particular area increases and there is a shortage of supplies, then the power in the market shifts toward the seller and conversely when the demand decreases and there is a glut of properties available then the power of the market will shift toward the buyer. This means that they have this poverty to select a house among the available houses and negotiate a price that is lower than the published price [13].

There are two types of people in making the demands, i.e., people who are householders and the people who want to make their own households (Fig. 1). The people, who are not householders, can make demand in housing market by making their own household. Making own households causes by marriage or getting independent from the parents or by immigrating to Lugano (exogenous events). These types of reasons that cause demands are called exogenous because they are not produced inside the model and are the reasons that entre to the model from the outside.

The people who are households decides to change their current residence in order to buy a new house when they are not satisfied by the current house considering the features of the house, environment and their own socio-economic properties. The households are constantly evaluating their social and welfare position in society and considering the result of this evaluation they make new decisions. Satisfaction level of every agent in computed by a fuzzy type-2 expert system. For every agent the satisfaction level is computed by and the agents whose satisfaction level is under a stochastic threshold will be accumulated in Unsatisfied Agent. The unsatisfied agents will select a house in the available houses (Demand making) and produce the demand. In this model, unsatisfied households and the people who intend to make their own households make Demand. (Fig. 1)

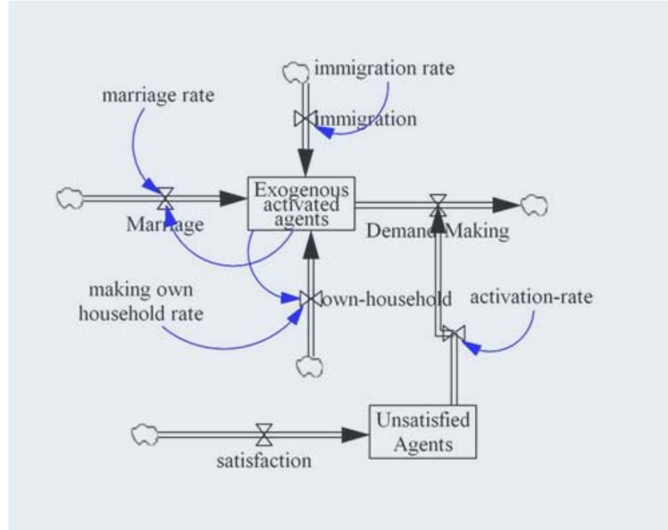


Fig 1.The conceptual framework of making Demand

The demand affects two things that are the outflow to the supply and the desired supply. In this model the rate of shipments from the supply is equal to the demand. The demand also specifies the supplier's desired stock. When the number of demand increases, the house suppliers start to build new houses and increasing its price.

. In this model the houses that are made by all the house suppliers in different hectares are accumulated in supply. The inflow supply represents the total production of the houses in Lugano. The outflow to sold houses is equal to the demand (the selected houses by the activated households). In this model the effects of excess or inadequate supplies has been modeled. The supply stock represents the total number of available houses in Lugano. The sold apartment flow is the weekly demand for houses and desired supply is the quantity of houses the suppliers would like to have in supply stock to cover several months of demands. This is because there are time lags between a change in demand and an increase in the supply of new properties becoming available, or other homeowners deciding to put their properties onto the market. Desired supply is the product of desired supply coverage ad demand. The supply ratio is the ratio of supply to desired supply and is a factor that determines price.

The behavior of housing market is specified by the desirable price for seller and the actual price (the price that is expected by the buyers). Price is modeled as a stock because prices cannot change instantaneously. People do not have immediate and exact information on the supply and demand of the commodity in question especially in housing market that there is time lag between making demand and providing supplies. Additionally, when the information becomes available, it takes time to make a decision about changing the price.

The model starts as an empty void with space and time coordinates. Time and Space are defined as below:

Time: $t \in T$ is a point in time, where $T \subseteq \mathbb{Z}$.

Space: $(x, y) \in X$ indicates a location in a vector space. X is a 2D vector space which determines the x and y coordination's of hectares in Lugano. Our model evolves in different hectares of Lugano, which are determined by 2D coordination's over a specific duration of time.

At $t = t_0$ model is an empty void with determined space points. Then we start to fill different space points (hectares) with the agents which are the apartments and the located households (Fig 3.).

We have proposed to use an agent-based model to connect the structure of the model to the behavior of individual agents. The events of the system are made by the behavior of the agents and the structure of the model is shaped by these events. In this model agents are defined as below:

Agent: an agent is an autonomous and identifiable object in space-time, which is able to make decision considering their own mentality, behavior and the information they get from environment and the other agents. The decisions of these agents cause happening of some events in the system leading to the creating new type of information and changing the state of the system. As we mentioned above, defining of the agents is done based on the questions we are interested to response. The questions are about the price and the behavior of the households so one of the agents that we define is households that we need to model their behavior. The price of housing is derived by supply and demand in a free market in this way that When demand increases and/or supply decreases, prices go up. The demand is made by the households considering the features of their houses and the supply is made by house suppliers. So to answer the mentioned questions we need to define three types of agents, i.e., Demand Makers, Supply Makers and the Houses. Demand and supply maker agents are called active agents and houses are called passive agents.

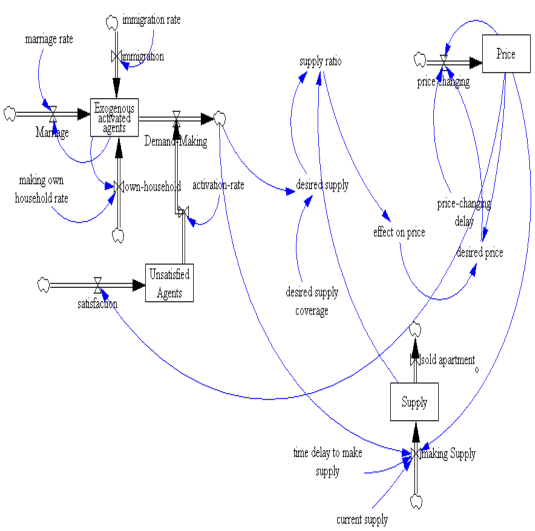


Fig 2. The conceptual framework of making Demand

Entity: Entity is an object that exists in space-time. In this model, the entities are the houses that exist in a space point (hectare) of the city in a time interval. The demand and supply is identified by the aggregated number of these entities.

Event: is an object that exists in the time dimension. It is created by the decisions of the agents in a specified time interval. When the event occurs, it may affect other agents, cause other events, the flow of the information and the future behavior of the other agents. I must emphasize that the events are the actions that are performed by the agents, and in this way they are related to space-time so they would be a factor of agent's behavior evaluating as well as the cause of other events.

State: our goal is modeling housing market and observing the evolution of the system in every time from the starting point and save the minimal amount of information from the initial state. In this case we stop the process at $t' > 0$ and observe save the achieved information at t' as the state of the system in different space points (hectares) of the model to observe and analyze the changes of the system.

At time $t_0 = 0$, the state of the system consist of the space that agents and entities exist in, the time that the model will be executed, the agents and the entities and the initial event.

A. *Mathematical Modeling of Housing Market*

In this work there are two different types of agents, i.e., demand makers ad supply makers that have been modeled behaviorally considering their features, location, residence, job, nationality and neighbors. These events are made through a behavioral decision making process by the mentioned agent. The conceptual model of this process has been shown in Figure 1,2. The events are created through the following steps:

1- Evaluation of satisfaction level for every agent through observing her environment, the place she lives in, her mentality and her social and individual properties. If an agent is not satisfied she tries to find a new place and enter in the market. This type of activation is called endogenous activation and happens in the mind of the people. This function is implemented by a fuzzy type-2 inference engine. This function is a multi-input, single output functions that is implemented on every agent that is take from the population.

2- Every activated agent tries to find a new house in a new position considering her abilities, needs and her reasons to change her previous house. Step 1 and 2 make the demand in the system. One of the significant factors in any market is demand. Demand has a positive effect on the market. In fact demand is made through a hierarchical fuzzy inference machine that the stages of this inference machine are modeling of unsatisfaction for every agent and modeling g of selection for every agent.

3- In every hectare the potential of building new house considering the number of demands for that hectare, the mortgage rate and the number of available supplies is computed to learn about the construction of new building.

4- Agents negotiate with each other to reach to an accepted price for the selected house in a specific space point.

III. Model Implementation

The implemented model is a discrete-time model and the state of the system changes just at event occurrence time, which is a fixed time interval. As it was mentioned before model consists of two different types of agents, which are identifiable by their position and their properties. So in every hectare there are some buildings, households as demand makers and one potential house builder. So the state of the system in every hectare at location (x, y) at the current time t' is:

$$s(t', x, y) = (B_1^{t'}, \dots, B_n^{t'}, H_1^{t'}, \dots, H_k^{t'}, P^{t'})$$

Where $B_n^{t'}$ is the properties of the nth building and $H_k^{t'}$ is the characteristics of the kth householder as demand maker and $P^{t'}$ is the potential builder at time t' in the hectare which is located at position (x, y) .

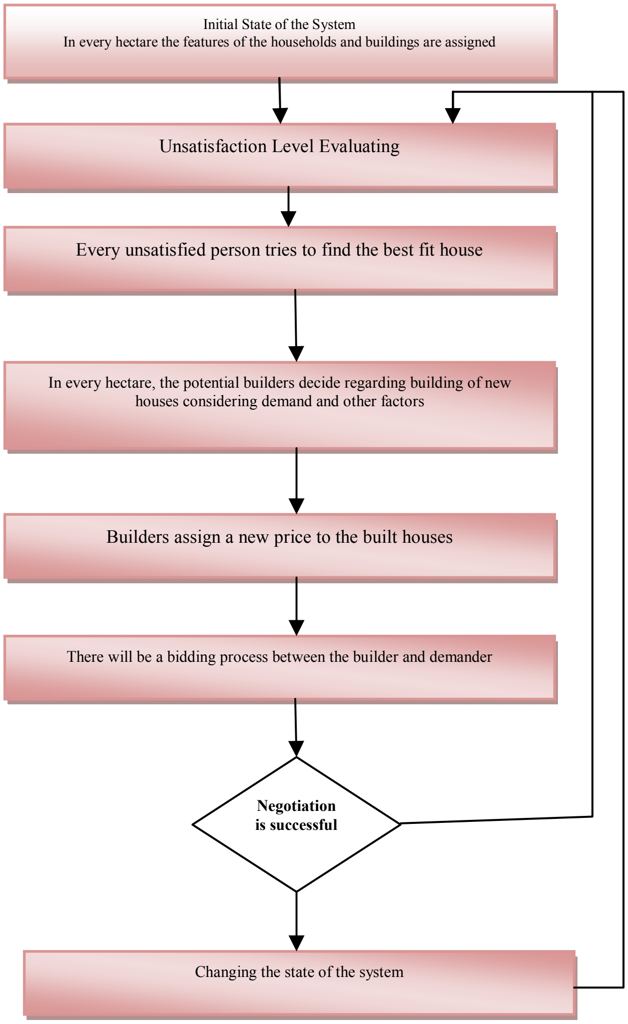


Fig. 3. Execution of the model

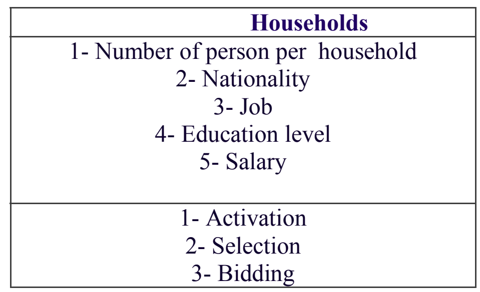
The state of the system at time t' is the state of different hectares in Lugano and is shown as below:

$$S(t') = (s(t', x_1, y_1), s(t', x_2, y_2), \dots, s(t', x_n, y_n))$$

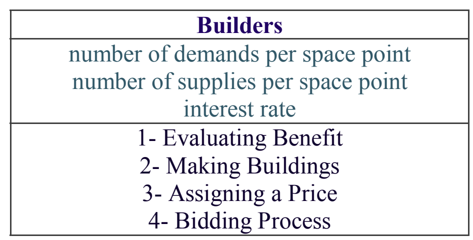
One turn execution of the model is shown in figure 4. In this work this time interval is one week and the state of the system changes considering the number of changes of the demands and supplies since initial time until the current time.

IV. STRUCTURAL MODELING OF HOUSING MARKET

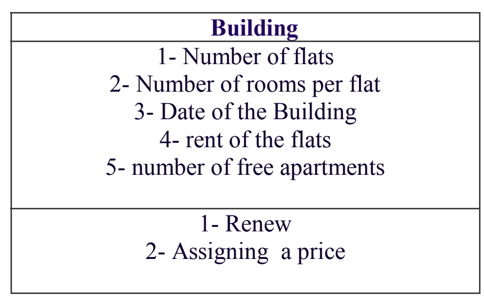
The model evolves by the decisions which are made by the agents. The decisions of the agents make the events of the system which are number of available supplies and demands. As it was mentioned before, the model is specified completely through a space-time vector. At any time, every space point of the model (1 hectare of Lugano) is composed of a class of Households and Buildings and the system is composed of a class of Builders. We show builders, buildings and households as an object to capture the information pertaining to them and to specify the associations between the object classes. The class diagram which has been used in this model is shown in figure 5. Our block diagram has three components that are class name, features and activities. Every class has a name as an identifier and some features as the properties of the class that are shared with the other objects of the class or the information that is used by the objects of the class to be processed by the activities . In this model two types of information is used, i.e., the attributes which are calculated like "number of demands" in Builders class and the attributes that are extracted directly from the data set we have like "nationality" in households class. The activity part of every class represents the behavior of the instances of the class considering the features of every instance and the features of the other classes.



A. Class Diagram of Households class per every space point



B. Class Diagram of Builders for the system



C. Class Diagram of Buildings per every space point

Figure 4. The Class Diagrams for different agents of the model

In this model we have instances diagrams which are similar to class diagrams and describe the objects of the classes. In every hectare there are a number of instance diagrams for households and buildings. Depending on the features of every hectare an instance diagram for the builder can be produced.

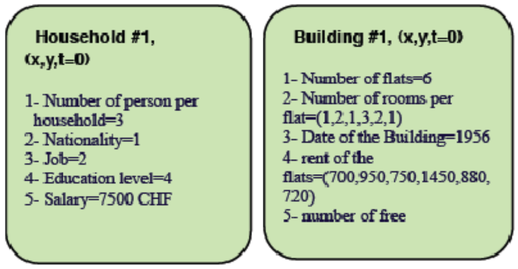


Figure 5. Instance diagram for one household and one building in the hectare (x,y) at the initial time.

For having the structural model of Housing market we need to aggregate all the instance diagrams in Lugano and start to run the simulation for all the instance diagrams.

In this model agents are able to make interconnection and have a cooperative decision making process. Households evaluate their satisfaction level and if they are not satisfied start to change their place and make the demands of the market. If the number f demands are high and the number of supplies is low, Builders starts to make houses and sell them. The buyers try to have negotiation with the sellers to reach to an ideal price. So in all part of the model and in every turn of execution some agents make interconnection with some other agents and cause the flow of information in the system.

V. BEHAVIORAL MODELING OF HOUSING MARKET

In this part we explain the methodology we have used to show the behavior of the agents in this model. Behavior means what the agents do. The behavior of the agents is implemented through the functions, the input and the outputs of functions and the ordering of the functions. One of the most important challenges in this model is processing of linguistic and uncertain features and properties as the inputs and outputs of the functions. The processing of the linguistic inputs is done by fuzzy type-2 expert systems. The inputs and the outputs are fuzzy type-2 membership grades.

The inputs in this model are the attributes that are captured for every object in every space point at any time interval. After categorizing the attributes we need to specify that what type of linguistic variables will specify the status of these attributes and how these linguistic variables are represented and processed in their mind of different agents. Different people have different deductions about a same quality. For example when we say the rent of the apartment is low has different meaning for a person with 2000 CHF/Month income in compare to a person with 10000 CHF/Month. Low rent is represented differently in the mind of these two agents from two different countries and cultures and when we are modeling dissatisfaction we must consider these type of differences. This is the uncertainty of the values and in modeled by fuzzy type-2 membership functions. To introduce the fuzzy type-2 membership function we need to review the fuzzy type-2 set very briefly. A type-2 fuzzy set, denoted A, is characterized by a type-2 membership function $\mu_A(x,u)$, where $x \in X$and $u \in J_x \subseteq [0,1]$, i.e.,

$$A = \{((x,u),\mu_A(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\}$$

In which $0 \le \mu_A(x,u) \le 1$. A can also be expressed as:

$$A = \int_{x\in X}\int_{u\in J_x} \mu_A(x,u)/(x,u) \qquad J_x \subseteq [0,1]$$

As it is clear by the definition, the type-2 fuzzy set is the extension of the type-1 fuzzy sets to a higher degree. Type-2 fuzzy sets have grades of membership that are defined by type-1 membership functions, which are called secondary membership degree. When all secondary grades as $\mu_A(x,u) = 1$ then it is an interval fuzzy type-2 system. The uncertainty that exists in the primary memberships of interval type-2 fuzzy system consists of a bounded region that is called the footprint of uncertainty (FOU). It is the union of all primary memberships, i.e.,

$$FOU(A) = \bigcup_{x\in X} J_x$$

The lower and upper membership function that is associated with the lower and upper bound of $FOU(A)$ and is defined as below:

$$\overline{\mu}_A(x) \equiv \overline{FOU(A)} \qquad \forall x \in X$$

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Therefore we conclude that in every interval type-2 fuzzy system,

$$J_x = \left[\underline{\mu}_A(x), \overline{\mu}_A(x)\right], \qquad \forall x \in X$$

After listing the attributes and specifying the category of them it is the time to convert the crisp values of them into some membership degree in a way that it is able to express the subjectivity of the people in evaluating the properties and the uncertainty they must handle when they are analyzing linguistic variables.. Fuzzyfying in fact means intersecting the input value to the fuzzy set associated with every linguistic term. In this work we have used Gaussian type-2 membership function with uncertain mean that takes on values in $[m_1, m_2]$, i.e,

$$\mu_A(x) = \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right] \qquad m \in [m_1, m_2]$$

In this way different attributes and the linguistic terms that specify their status is represented in the mind of every agent considering the differences that exist among different agents. The knowledge of the agents (consumers) are represented as some rules as below;

$$R^l = if\ x_1 is\ F_1^l and \ldots and\ \ x_p\ \ is\ F_p^l, then\ y\ is\ G^l$$

$$l = 1, .., M$$

All the variables (inputs and outputs) are modeled by fuzzy type-2 membership function to model and minimize the effects of uncertainty. Having p inputs (attributes) $x_1 \in X_1, \ldots, x_p \in X_p$ and one output (dissatisfaction level) $y \in Y$ we are able to make M rules which in fact demonstrate the knowledge of the households about the residence they live in and are uncertain. Uncertain rules mean the rules who's the antecedent and consequent parts are uncertain or we can say more precisely membership functions. By using fuzzy type-1 we are not able to handle such uncertainty on the rules (behavior) and so we proposed to use fuzzy type2 systems to model every agent's particular kind of intelligent behavior.

By observing the attributes of a thouse, each rule R^l determines a type-2 fuzzy set $B^l = A_x \circ R^l$ such that:

$$\mu_{B^l}(y) = \mu_{A_x \circ R^l} = \coprod_{x \in X} \left[\mu_{A_x}(x) \cap \mu_{R^l}(x, y)\right] \quad y \in Y$$
$$l = 1, 2, \ldots, M$$

$\mu_{B^l}(y)$ which is called induced consequent membership function and in fact is the firing strength of this rule. By this equation, the input-output relation between the type-2 fuzzy set that activates one rule in the mind of the agent and the output (dissatisfaction) is represented. Our model is an interval type-2 fuzzy sets and meet under product t-norm, so the result of the input and antecedent operations, which are contained in the firing set , is an interval type-1 set,

$$F^l(x') = \left[\underline{f}^l(x'), \bar{f}^l(x')\right] \equiv [\underline{f}^l, \bar{f}^l]$$

Where

$$\underline{f}^l(x') = \underline{\mu}_{F_1^l}(x_1') * \ldots * \underline{\mu}_{F_p^l}(x_p')$$

And

$$\bar{f}^l(x') = \bar{\mu}_{F_1^l}(x_1') * \ldots * \bar{\mu}_{F_p^l}(x_p')$$

Where * is the product operation.

Three control operations were used in this model and are sequence, selection and iteration. Sequence determines the sequence of the execution of the functions, selection selects one function to be executed among some other functions and iteration iterates the model over time to make the dynamics of the market. These operations have been shown in Figure 2.

CONCLUSION

Creating a detailed model of economic systems needs a huge amount of work on the modeling of the behavior of individual agents. The engineering part of these types of model is very important and needs to be done very carefully. The type of the structure and framework of the model effect the behavior type of the agent and it can affect the accuracy of the system so it is clear that the validation, analysis and implementation of the model are strongly connected to the engineering part of the model. We have represented a general approach to represent the model mathematically, structurally and behaviorally. Modeling of these types of systems can rapidly lead to a complicated simulation where it is difficult to recall parts of the model, so having an initial representation of the model seems crucial. In this work I have presented some part of our engineer work on the housing market modeling as a complex system modeling and will represent all the work more precisely in the came ready of the paper.

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