The Evaluation of Venture Capital As an Instalment Option:

Valuing Real Options Using *Real Options* †

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ABSTRACT

Many start-up companies rely upon venture capitalists to begin operations. Typically, after the initial injection of funds, additional funding is provided as the firm reaches certain performance targets. The payment of the first funding round is comparable to an initial option premium. Further payments are contingent claims: the right but not the obligation to continue financially supporting the project. If at any point, the venture capitalist ceases to pay, the project is assumed to end. Therefore, the venture capitalist can be thought of injecting funds that not only keep the project alive but also retain the right to pay the remaining payments in the future.

We interpret this type of corporate finance transaction as a multiple stage compound option, which is also known as an instalment option. In previous work by Davis, Schachermayer and Tompkins (2001, 2002), instalment options on traded assets were considered. In these papers, it is shown that static portfolios of simple European options can be formed that yield no arbitrage bounds on the value of instalment options.

In this research, we extend the analysis to venture capital contracts. For imitator (*clone*) projects, we derive bounds on how much a venture capitalist should initially invest in such start-up firms. Under suitable assumptions, such bounds rely solely upon no arbitrage arguments. Upper and lower bounds can be enforced by constructing portfolios of European options on firms in the same industry. To the best of our knowledge, the relationship between the financing of imitator start-up companies and instalment options has not been identified previously.

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1. INTRODUCTION

Venture Capital (VC) is an important source of funding for the development of innovative start-up companies. Yet, the finance literature provides little guidance as to how much the venture capitalist should pay to fund new projects. Cossin, Leleux & Saliasi (2002) state: "Valuing early-stage high-technology growth-oriented companies is a challenge to current valuation methodologies." The traditional approach to project analysis requires the forecasting of future cashflows and discounting these to present value at some risk adjusted rate. However, the structure of VC funding could be more appropriately analysed in terms of *Real Options*, as subsequent injections of funds are contingent on the project reaching performance targets.

Most of the literature on VC has focused on the contractual design of the investment contracts, with particular emphasis on skewing the distribution of the payoffs in favour of the VC investors. Research has examined the role of contracting design in such transactions and principal agent conflicts. Another line of research has concentrated on the empirical analysis of existing VC projects.[1] Recently some work has appeared identifying the possible application of real options methodology to the evaluation of VC transactions. So far, the pricing of VC real options is at a nascent stage and this research aims to address this issue.

The VC investor evaluates the project and, assuming favourable prospects, provides initial funding and further funding as time goes on, depending upon the firm reaching certain performance targets. If at any time (when such decisions for further funding are made), the firm has not met the targets, the VC investor can abandon the project and potentially receive some portion of the recovery value of the firm. For those projects (e.g. Internet firms) without fixed assets, there is no recovery value. We will consider this case here, without loss of generality.

We contend that this sort of VC funding is an instalment option. Such instalment options are multiple-period compound options. The initial introduction of project funding is comparable to the premium payment of an instalment option. Subsequent injections of funds into the project resemble further instalment premium payments. In previous research by Davis, Schachermayer & Tompkins [DST] (2001, 2002) tight bounds on the value of these options were deduced from the prices of European options (for traded assets) using the no arbitrage principle. Not only does this provide a tractable method for the pricing of instalment options, but also the bounds are robust to model assumptions such as the assumed price process (typically Geometric Brownian Motion with constant volatility). Furthermore, there is no need to estimate model parameters such as volatility or dividend yields if there is a liquid market for European options (with sufficiently long exercise times and comparable exercise prices). These results imply that such options can be super- and sub-replicated by standard European claims (which *really* exist and thus the sub-title to this paper). Therefore, the amount of initial funding provided by VC investors can be precisely determined and is preference-free.

In this research, we consider venture capital projects that are in a strong sense (made precise in Definition 1 below) imitators (or *clones*) of existing firms in some industry. Under the assumption that European options are traded for the existing firms (at appropriate strike prices and maturities), the Venture Capitalist can use this information to decide whether or not to fund a new project. This is achieved by comparison of the expected cash flows and future investment payments of the venture project to that of an instalment option on an existing firm in the same industry. We show that the VC investor can realise an arbitrage profit by funding the VC project and at the same time selling a portfolio of European options. This will occur if the required initial payment for the venture project is lower than the cost of the European option strategy. This provides a lower boundary on the amount that the VC investor would be willing to pay initially. Unlike the usual inclusion of investor risk preferences in the pricing of Real Options, this lower bound for the initial investment is preference-free.

The upper boundary is not determined by arbitrage, in a strict sense. However, if the VC investor is a profit-maximising rational agent, the upper boundary for initial project funding must be the upper boundary of the instalment option on the related firm. As this boundary is also enforced by a static portfolio of European options (assumed to be observable), this will be preference-free and model independent. In DST (2001), such bounds are typically within 5% of the theoretical value of the instalment option. Therefore, this research provides substantial guidance for the pricing of imitator VC projects.

This paper is organised as follows. In the first section, we will review the venture capital literature and concentrate on the evaluation of venture capital as a real option. This is followed by a discussion of instalment options pricing and the DST (2001) bounds with static portfolios of European options. Then, we show that the bounds for an instalment option on an existing firm are also the bounds for an imitator VC project when both the existing firm and the venture project share the same sources of external risk. Finally, conclusions and suggestions for further research appear.

2. VENTURE CAPITAL – A LITERATURE REVIEW.

A number of papers have studied what venture capital firms do and theorise how they add value. Examples of these include Gorman & Sahlman (1989), Hellman and Puri (2000) and Lerner (1995). These papers examine what the VC investors tend to do after the initial investment in the firm. For example Kaplan & Strömberg (2001, 2002) and Gompers (1995) focus on the implications of contractual terms of VC arrangements. Their objective is to test various theories of the investor / principal agent conflict. Another line of research has been to use evidence from surveys of VC investment partnerships to describe the characteristics of these investments [see MacMillan, et al. (1985, 1987) and Fried and Hisrich (1994)].

Faced with valuation uncertainty, Sahlman (1990) suggests that the coping mechanism is to either design investment contracts which materially skew the distribution of the payoffs from the project to the VC investors or involve the active participation of the VC investor to assure that the project has the professional managerial expertise to succeed.

Sahlman (1990) identifies three key facets of the investment contract that skew payoffs in favour or the VC investor; (1) the staging of the commitment of capital, (2) the use of convertible securities instead of straight common shares and the associated senior claims on the assets of the firm in case of failure and (3) anti-dilution provisions to secure the VC investor's equity position in the new firm. Of these mechanisms, he concludes that staged capital infusions are the most potent control mechanism that a venture capitalist can employ. Cossin, Leleux & Saliasi (2002) examine the economic value of these legal features in a Real Option context.

The usual sequence of events in VC funding is that an entrepreneur either has previously developed a project (with prior revenues) or plans to start a new venture (without prior revenues). The entrepreneur approaches a VC partnership and seeks funding for this project. After submission of an appropriately detailed business plan and analysis by the VC investors, a funding proposal is made. Typically, a total amount of funding is approved for the project (committed funds) and payment is made in stages (or funding rounds). The first stage allows the project to begin and then at fixed points of time in the future, if certain performance targets are reached, the VC investors introduce additional funding. Ultimately, when the firm has reached sufficient size and has established a track record, the company is sold to the capital markets as an Initial Public Offering (IPO). The VC investors may not introduce additional funds to the project because certain performance targets have not been met. Also, as the project develops, competitive firms may enter the market place, copy the idea and the exclusivity value of the project is reduced. We will only consider the former case in this research.

For their investment, the VC investors obtain an equity share in the project and expect to profit when the IPO is launched. According to a survey of recent VC projects in the United States, Kaplan & Strömberg (2001) [Table 1], show that almost all VC investors receive convertible preferred stock in the firm when they pay in the funds. Optional redemption and put provisions are commonly used to strengthen the liquidation rights of the VC's investments. When discussing the expected profit from VC investment, Kaplan & Strömberg (2001) find that the median IPO stock price is 3.0 times greater than the cash infusion (the estimated value of the company) in the initial financing round (payment of the instalment options). Over a four-year horizon, this works out to a return of 31% per year (page 12).

VC projects also vary depending on how the level and timing of additional funding are initially defined. Kaplan & Strömberg (2001) state, "Even though redemption rights are the part of the VC contracts that most resemble debt, there are other ways that a VC investor can force a liquidation of badly performing firms. The most important mechanism is through staging of the investment [see Bolton & Scharfstein (1990), Neher (1999) provides a model of staging based on Hart & Moore (1998)]. We distinguish between two different forms of staging: ex ante (or within-round) and ex post (or between round) staging."

Kaplan & Strömberg (2001) further state, "In an ex-ante staged deal, part of the VC projects committed funding is contingent on financial or non-financial milestones (internal targets). This essentially gives the VC investor the right to liquidate the venture in the bad state of the world. Even though not all VC financings are explicitly staged ex ante, most of them are implicitly staged ex post, in the sense that even when all the funding in the initial round is released immediately, future financing will be needed to support the firm until the IPO." As Cossin, Leleux & Saliasi (2002) show, "ex-ante" funding or as they call it "Contingent Pre-Contracting" further funding is theoretically a better approach than the simple "right of refusal", which is an informal commitment of additional funds, as and when they are needed. Given this, we will restrict our analysis to those cases when future funding levels are explicitly set ex-ante.

"Of particular concern to VC investors is the liquidation cash flow rights that are assigned upon the failure of the venture.[2] By providing less funding in a given round, and hence shortening the time until the next financing round, the VC arrangement increases the ability to liquidate the venture if performance is unsatisfactory. Gompers (1995) analyses ex post staging using Venture Economics data. Time between financing rounds decreases with industry R&D intensity and market-to-book ration; it increases with industry tangible asset ratios". [Kaplan & Strömberg (2001) pages 28-29 and footnote 24].

VC projects also vary in terms of the nature of the proposed firm. According to Hellman & Puri (2000), these firms can be split into innovator or imitator firms. The difference is that innovator firms are launching a new product or service that has not been offered previously. For imitator firms, such a product or service has been introduced previously and the new firm contends that it can provide this more cost effectively or efficiently. In this instance, we have a frame of reference for comparison: existing firms in this industry. For our purposes, innovator and imitator firms are distinguished by how performance targets are set by Venture Capitalists (for additional introduction of funding). Innovator firms have targets set by internal performance, such as the development of patents or the successful completion of research projects. Imitator firms have targets set by external performance, which include sales targets, cashflows or attainment of predetermined market share levels. In the latter case, the success of these firms is assumed to be driven by "external" sources of risk that are general to the industry they belong to. This implies that both existing firms in the industry and the new imitator firm share the same sources of risk.

In this research, we will restrict ourselves to case of imitator firms with ex ante funding. Our rationale for this choice is twofold. Firstly, Hellman & Puri (2000) contend that: " For imitators the provision of funds may be the most important aspect of venture capital., whereas for innovators, the product market dimension can be more important." (page 963). As we are interested in the value of the real option (the precise amount of initial funds) and not the impacts of expertise by the VC investor, we will restrict our analysis to these. Secondly, under our rather restrictive assumptions that both imitator firms and existing firms share *exactly* the same sources of risk, we can derive precise no arbitrage bounds.

2.1. VENTURE CAPITAL AS A REAL OPTION

Such ex ante funding (like our instalment options), is according to Kaplan & Strömberg (2002) "[such] staging, on the other hand, does not seem to be related to internal risk, but instead to the amount of risk *external* to the firm. This suggests that the driving force for ex ante staging is not asymmetric information, but rather the *option to abandon the project*, which will be more valuable in volatile environments." (page 24). Berger, Ofek and Swary (1996) have considered such an abandonment option in such a context.

As our research will only examine the initial amount that should be paid for VC projects, we contend that we are justified in restricting our analysis to imitator firms. Hellman and Puri (2000) show that in real VC practice, funding is of prime importance to imitator firms. Furthermore, in their sample of Silicon Valley start-up firms, "We find that the presence of venture capital increases the amount of funds raised by imitators, by not by innovators." (page 979).

Kulatiliaka and Perotti (1998) point out that for both innovator and imitator firms, the value of the option must take into account competing firms. In financial options markets, actions taken by one investor will not affect other investors. For example, the holder of an option on a financial asset has the exclusive right to exercise that option, and exercise by one agent does not effect the exercise decision by other firms. The agent has monopoly over the exercise opportunity. From Bruun & Bason (2001) "Not always so in real options analysis. When a firm is undertaking, in example, an R&D investment, it is in effect purchasing a option on possible commercialisation or further development. But a competitive firm can make similar investments and thus exercise by one firm will affect the market value of the option for other firms - possibly drive it to zero". In this research, we will not assume such a feedback effect. We assume that the decision by the VC investor to exercise does not affect competitive projects that may be introduced during the tenure of the project.

Bruun & Bason (2001) have applied real options to VC investments. For a broad based review of the literature on real options see Dixit & Pindyck (1994), Trigeorgis (1996) and Lander and Pinches (1998). Growth options and VC are seen as the same thing.

The obvious starting point for the pricing of the VC real option is the Black & Scholes (1973) model. Benaroch & Kauffman (1999) and Panayi & Trigeorgis (1998) have used the BS (1973) model in this context[3]. However, as is pointed out by Bruun &

Bason (2001), “There is a growing body of evidence, however, that the assumptions underlying the standard Black Scholes option pricing model are either too simplistic, or downright false when it comes to pricing options on many real assets.” (Page 3). They go on to suggest that modifications of the Black & Scholes (1973) model as proposed by Merton (1973, 1976) and by Cox and Ross (1976) may be more appropriate.

The Merton (1976) jump diffusion model is of particular interest, due to the discrete nature of VC project evaluation and the fact that such projects experience jump-like behaviour (as technical breakthroughs are achieved in the project). Pennings and Lint (1997) have argued that a more appropriate approach to modelling VC & R&D projects would be a jump process, and this would lead to an intuitive economic interpretation of the volatility. Changes in price are likely to be caused by technical discoveries and the arrival of information affecting the particular project (e.g. competitor entry), and these occurrences often happen at discrete intervals.

They formulate such a model with the value of the firm driven by stochastic jumps, plus a deterministic drift term. They successfully apply this to the analysis of R&D projects at Philips [see a discussion in Lint & Pennings (1998)]. A similar model has been presented by Willner (1995) where the impacts of jumps are decreasing with time (the continued existence of the firm).

A particular challenge for real option modelling is that the underlying asset must be defined. For the articles previously referenced, the underlying asset is assumed to be the overall value of the firm. This is assumed to follow Geometric Brownian Motion (GBM). However, Angelis (2000) extends the Black & Scholes (1973) model by substitution of predictions of revenues and costs rather than the value of the project. She also assumes that such revenues and costs conform to Arithmetic Brownian Motion (ARM) rather than assuming that the process follows GBM. Thus, this approach harkens back to Bachelier's (1900) original option pricing model.

A number of authors have proposed models with multiple state variables. For financial options, the striking price is fixed at the beginning of the contract. In VC & R&D projects, this is not always the case. A fixed strike price in this instance suggests that the VC investors know with certainty the cost of commercialising the project. In many instances this is *only known* after the project has been accepted and had the chance to run. Thus, the exercise price adds an additional source of uncertainty. One approach would be to use the Margrabe (1978) model with the right to exchange one risky asset for another. In the context of venture capital (R&D) projects, Kumar (1996, 1999) did this. In a similar vein, Fischer (1978) examined exercise price uncertainty for financial options and Pindyck (1993) extended this to real options. Schwartz and Moon (2000a) apply this approach to the evaluation of R&D projects as do Berk, Naik & Green (1998) and Schwartz & Zozoya-Gorostiza (2000) for high tech investments.

However, in the special case of VC real options, the multiple rounds of financing introduce path dependency, which are not dealt with by these one-period European option pricing models. According to Bruun & Bason (2001) "Another feature of real projects is that they often consist of a succession of discretionary investment opportunities. Thus part of the payoff from investing in a real option consists of further options. This is commonly known as compound options, and the issue was first dealt with by Geske (1977, 1979). When a VC or R&D project consists of several rounds of financing, they should be thought of as compound options. When the length of each financing round can be extended, Longstaff's (1990) model for pricing options with extendible maturities can be applied."

When the compound option model is applied to this problem [for example, Perlitz, et al. (1999) and Li (2000)] only a two period compound option is considered. While it seems obvious that some compound option-pricing model would be appropriate, the typically high number of possible rounds of funding (greater than two), makes analytic option pricing intractable. Antikarov (2001) states "The complexity of multiple uncertainty factors is difficult to capture. Some strategies represent an intricate set of compound options, making detailed modelling intractable." Cossin, Leleux & Saliasi (2002) also side-step this problem, "this is beyond the scope of this paper where the combination of multiple complex optional features presents in itself a challenging problem."(p. 1).

While such approaches provide some guidance as to these structures, little has appeared on providing specific prices for projects and how to estimate the value of what is a complicated multiple payment compound option. Such multiple period compound options (instalment options) can be priced using numerical methods. Cortazar (2002) discusses these procedures for Real Options, generally and Jagle (1999) uses a binomial approach for pricing investment projects, specifically DST (2001, 2002) use such a procedure for determination of theoretical prices of instalment options and later compare these to the prices bounded by European options on the same underlying asset. Cossin, Leleux & Saliasi (2002) prove that the pricing of a VC contract is similar to that of a complex package of financial options. They fail to identify exactly what this package is and that this package is an instalment option. This will be done in this research.

Given an assumed model, the lack of observable parameters is an important challenge for practical implementation. This is particularly true for the volatility parameter, for which few (if any) observed values of the underlying asset exist. As Majd and Pindyck lament "it may be difficult or impossible to estimate [the project's volatility and dividend yield] accurately" (1987, p. 25). One such approach to volatility estimation may be to estimate the standard deviation of the project returns. Unfortunately, a historical time series of project market values is seldom available, especially when valuing the option to invest in a new venture project or in R&D. In any case, Cossin, Leleux & Saliasi (2002) show that for VC projects, volatility will not be constant but will be inversely related to the life of the project. They state; "Empirical results (Gompers (1995)) and research studies (Berk, Green & Naik (1998)) have shown that the systematic risk as well as the volatility levels are highest early in its life and decrease as the project approaches completion. As per Myers and Howe (1997) cited in Berk, Green and Naik (1998), the cost of capital should thus decrease through the life of the project, due to higher "leverage" of the project early in its life." (page 7).

One alternative proposed by McDonald and Siegel (1986), Majd and Pindyck (1987) and Dixit and Pindyck (1994) is to model the project volatility as the average percentage standard deviation of overall stock market equity returns. Pickles and Smith

(1993) suggest using the standard deviation of the underlying state variable if the project is commodity related (crude oil for modelling underground oil reserves). Similarly, Davis (1998) showed that the dividends and volatilities for real option projects should be based upon the volatility of the underlying state variable (in his case mining companies and the price of Gold , Copper or Crude Oil), adjusted for the elasticity of the project to this state variable. He also considers the impacts of changing cost features in production and either fixed or variable production capacity.

For many VC projects, the underlying asset is not a project with fixed commodity assets, but based upon innovations. In Hellman and Puri (2000), the focus of the VC activities was on High Tech firms in Silicon Valley. In Kaplan and Strömberg (2001, 2002) the vast majority of VC projects were in IT/Software, Telecom and Biotechnology firms. Thus, the analysis of Davis (1998) probably does not apply here.

There is little doubt that both the approach to pricing the options and the estimation of the input parameters are problematic. This is where the instalment option approach can again provide some insights. If a VC project aims to imitate an existing venture, the rational VC investor should compare it to the alternative investments currently available in the same field. As was indicated above, the choice of appropriate input parameters may come from a time series analysis of firms in the same industry as the VC project. However, such estimation (by definition) is based upon historical analysis and subject to sampling errors (and possible non-stationarity). If options on the existing firms are traded, this provides information about the parameters required to value the VC real option. If in turn, a static portfolio of traded options can be purchased (or sold) which bound the value of the VC real option, then the need for both choice of a model and parameter estimation is eliminated. This is the aim of this research.

Thus, our analysis will provide bounds on what should initially be paid for the VC project given a known sequence of further cash injections into the venture. Since an imitator firm is driven by exactly the same sources of external risk, it can be defined as a "*clone*" of the existing firm: driven by the same stochastic process up to a scaling factor.

In this case, the valuation of an imitative VC project (or R&D) can be framed relative to other firms in that industry.

3. INSTALMENT OPTIONS – AN INTRODUCTION

An instalment option is a European option, where the premium is paid at predetermined dates throughout the life of the option. If, at any of the dates the buyer fails to pay the additional premium, the option ceases with no further obligations from either the buyer or the seller. If the buyer pays the entire schedule of premia, he holds a standard European call that will potentially provide a payoff at some known future date.

These products are compound options with more than one additional payment. The simplest type of an instalment option (compound option) consists of two premium payments - two instalments - where the first payment needs to be made to launch the option, and the next payment is optional. Geske (1977, 1979) and Selby & Hodges (1987) previously considered these products with applications to Real options by Perlitz, et al. (1999) and Li (2000).

For an (n>2) instalment option, the payment schedule is established at the beginning of the contract and payments are due periodically - monthly, quarterly, or according to any predetermined schedule - as long as the option exists. Cessation of the payments automatically implies termination of the option on the date of the first missed payment. Previously, DST (2001) considered instalment options with a series of n premium payments, $(p_i)_{i=0}^{n-1}$, starting with p_0 paid at the initiation of the instalment option t_0, with additional payments p_i at times t_1 ,…., t_{n-1}. If all payments, $p_0,....,p_{n-1}$, have been paid by the purchaser of the instalment option at each point in time, $t_0,.....,t_{n-1}$, the instalment option is a European (call) option terminating at date t_n with strike price K. For the sake of convenience, we will define $t_n=T$.

DST (2001) proved that for a known schedule of future instalment payments, $p_1,.....,p_{n-1}$ at predefined points in time, $t_1,.....,t_{n-1}$, a known expiration date, T, and a fixed strike price K, bounds for the initial premium payment, p_0, paid at t_0, can be determined solely by arbitrage relationships with European options.[4]

3.1. No-arbitrage bounds for instalment Options

We start by giving a precise definition of our model of a financial market. Let $(S_t)_{t_0 \leq t \leq T}$ denote the price process of a risky asset, based on and adapted to the filtered stochastic basis $(\Omega, (\mathcal{F}_t)_{t_0 \leq t \leq T}, P)$, and $r \geq 0$. We will ignore interest rate volatility, assuming for notational convenience that the riskless rate is a constant, r, in continuously compounded terms. We assume that this process is free of arbitrage in the following sense: the set $\mathcal{M}^e(S)$ of probability measures, Q, under which the discounted process $(e^{-rt}S_t)_{t_0 \leq t \leq T}$ is a martingale, is non-empty.

An instalment option on one share of S is given by a natural number, $n \geq 1$, real numbers $t_0 < t_1 < .... < t_{n-1} < t_n = T$, positive numbers $p_0, p_1, ..., p_{n-1}$ and a strike price $K > 0$. This instalment option entitles the holder to receive the random amount $\left[S_T - K\right]_+$ at time T, if all instalments, $p_0, p_1, ..., p_{n-1}$, have been paid at all time points, $t_0, t_1, ..., t_{n-1}$. We define $\hat{p}_1$ as the upcounted (to time T) value of the instalments $(p_i)_{i=1}^{n-1}$:

$$\hat{p} = \sum_{i=1}^{n-1} e^{r(T-t_i)} p_i \tag{1}$$

For $i = 1, ..., n-1$, we denote by K_i the discounted (to time t_i) value of the instalments to be paid at times, $t_i, ..., t_{n-1}$:

$$K_i = \sum_{j=i}^{n-1} p_j e^{-r(t_j - t_i)} \tag{2}$$

Assume that for any $s \in [t_0, T]$ there is a liquid market for European calls with maturity, T, and all strike prices $K > 0$. From the principle of no arbitrage[5], we know that there is some $Q \in \mathcal{M}^e(S)$ such that the actual market prices at t_0 are given by:

$$C(t_0, T, K) = E_Q[e^{-r(T-t_0)} \left(S_T - K\right)^+] \tag{3}$$

where Q is a martingale measure for the process S. Note that we do not restrict the choice of our model or the special choice of the equivalent martingale measure, Q (which is not unique in the incomplete case). We only assume that there is some equivalent martingale measure, Q, such that equation (3) gives the observed market prices, for each $K \in \mathbb{R}_+$ and for s = t_0. (i.e. today). Essentially, we observe today's prices $C(t_0, T, K)$ and that is *all* we know and *need* to know about the process S and the measure Q. We will also assume (mostly for simplicity of exposition) that both the existing firm and the venture capital project do not pay dividends over the period of the analysis. This makes sense for the venture capital project as all earnings should be retained and reinvested to allow the maximum rate of growth and the highest probability of success.

<u>THEOREM 1:</u>

Under the above assumptions, the inequalities:

$$C(t_0,T,K+\hat{p}) \geq p_0 \geq [C(t_0,T,K) - e^{-r(T-t_0)}\hat{p}]_+ \qquad (4)$$

can be derived from the principle of no arbitrage in the following sense: if either inequality is violated, an arbitrage is possible for an investor who can trade (long and short) in the instalment option as well as on the market for European Options. This is an application of Equation (2).

<u>PROOF:</u>

Assuming only that there is a liquid market for the European options, DST (2001) show that there is an arbitrage opportunity if the payment p_0 at t_0 does not satisfy the two inequalities of (4).[6] In fact, DST (2001) assumed that the instalments, $p_1, \ldots\ldots, p_{n-1}$, are exactly equal, but using equations (1) and (2) the argument carries over in a straightforward way to the present setting.

<u>REMARK:</u>

The message of the above theorem is that by considering super-replicating portfolios, we can determine easily computable bounds on the price (from the prices of European options). Since violation of these bounds will yield an arbitrage, any pricing model must produce a price within these bounds. If the bounds are sufficiently tight, pricing can be achieved without the choice of either a model or require the estimation of parameter inputs. The only assumption is that European options with the appropriate striking prices and terms to maturity [as required by equation (4)] are observed.

4. NO-ARBITRAGE BOUNDS FOR *CLONE* VENTURE CAPITAL PROJECTS

We formalise the concept of a venture capital project in the following way: there is a fixed funding scheme, $f_0, f_1, \ldots, f_{n-1}$ of payments to be made at predetermined times $t_0 < t_1 < \cdots < t_{n-1}$ to keep a project Y alive. Similarly as in the case of instalment options, we assume that if at some time, t_i, the venture capitalist decides not to pay the funding tranche, f_i, the project dies, without leaving the venture capitalist with any rights or obligations. As was mentioned previously in the literature review, Kaplan & Strömberg (2001) indicate that this is a similar mechanism as occurs in venture capital financing.

There is also a time horizon, $t_n = T$ (satisfying $t_{n-1} < t_n$) which might be thought of as the time of IPO. At this point, the venture capitalist receives an amount of money, which is a random variable, provided that he/she has paid all the funding tranches, $f_0, f_1, \ldots, f_{n-1}$ and pays the (deterministic) cost of launching the IPO, which is denoted by f_n.

How should we model this random variable describing the proceeds of the IPO for the venture capitalist? To do so, we make the following "mind experiment": suppose that the venture capitalist would give the unconditional commitment to pay the funding plan $f_0, f_1, \ldots, f_n$ at predetermined times $t_0 < t_1 < \cdots < t_{n-1} < t_n = T$. We recognise that this eliminates all the flexibility of staging the payments (and removes the essential element in VC financing), but for this mind experiment, we *assume it*. The venture capitalist offers the project Y (equipped with his/her full payment commitment) to a third party. We also assume that the venture capitalist is default-free, i.e. that he/she will fulfil this commitment with probability one. What is the price the venture capitalist can achieve for the project if it is offered at time $t \in [t_0, T]$? We model this (hypothetical) price by a stochastic process $(\tilde{S}_t)_{t_0 \le t \le T}$, based on and adapted to the stochastic base $(\Omega, (\mathcal{F}_t)_{t_0 \le t \le T}, P)$ introduced above.

Our basic economic assumption is that the venture capital project is an imitator of an existing company, X, whose price process is modelled by the process $(S_t)_{t_0 \le t \le T}$ defined in the previous section. The term "imitator" in the literature often is interpreted as "being driven by the same random shocks [external sources of risk]". What does this mean precisely? One way to formalise this idea mathematically is to assume that the two processes, S and $\tilde{S}$ are identical, up to a scaling factor $c>0$. This is a strong way of interpreting the term: "imitator company". In fact, we do not need the full strength of this assumption: for our purposes it suffices that this assumption holds true for the terminal date, T, the time when the project should be launched as an IPO. This is restated in the subsequent definition.

DEFINITION 1:

*Fix the time horizon T>0. A venture capital project Y is called a "**clone**" of company X if the random variable* S_T *modelling the price of a share on company X at time T, and the random variable* $\tilde{S}_T$ *modelling the value of the venture capital project Y at time T (equipped with the unconditional obligation of the Venture Capitalist to pay all the payments of the funding plan) satisfy the equality* $\tilde{S}_T = cS_T$, *almost surely, where c>0 is a scaling factor denoting the number of shares.*

This definition allows us to link the capital project, Y to options on the company, *X*.

PROPOSITION 1:

Assume that the venture capital project Y is a *clone* of the company X and denote by c the associated scaling factor. Fix the time schedule $t_0 < t_1 < \cdots < t_{n-1} < t_n = T$ and positive numbers $f_1 = p_1, \ldots, f_{n-1} = p_{n-1}$, as well as $f_n = cK$.

Let f_0 and p_0 be two numbers and consider the following two investment possibilities:

(a) the venture capital project, *Y*, defined by $t_0 < t_1 < \cdots < t_{n-1} < t_n = T$ and $f_0, f_1, \ldots, f_n$, and

(b) the instalment option on c shares of the company, X, defined by $t_0 < t_1 < \cdots < t_{n-1} < t_n = T$ and $p_0, p_1, ..., p_{n-1}$ and cK.

We then have the following relations between f_0 and p_0.

(i) If $f_0 < p_0$, the venture capitalist being able to invest into Y according to the payments scheme $(f_0, ..., f_n)$ can make an arbitrage by taking a position in the venture capital project and selling the above defined instalment option.

(ii) If $f_0 > p_0$, the venture capitalist can not act rationally by purchasing the venture capital project as he/she would be better off by purchasing the appropriately scaled instalment option.

PROOF:

It is clear that Definition 1 was formulated to make the above assertions hold true. In the first case (i), where the venture capitalist invests in the project and sells the portfolio of European options that super-replicates the instalment option, the difference $p_0 - f_0 > 0$ is earned without risk. This assumes that the venture capitalist funds the payments for the project Y, $f_1, ..., f_{n-1}$ by the receipt of the instalment option premia, $p_1, ..., p_{n-1}$ on c shares of company X. If the buyer of the instalment option fails to pay the premium p_i at any point, t_i, then the arbitrageur also ceases to pay the funding f_i and they walk away. The venture capitalist abandons the venture project at the same moment, he/she fails to receive the instalment premium. Finally, if all payments are made, the final payoff of the two investments can be expressed as $c(S_T - K)_+ = (cS_T - cK)_+ = (\tilde{S}_T - f_n)_+$, which cancels out by definition.

The proof of (ii) is – from a mathematical point of view - analogous. From an economic point of view it seems important to point out that, for $f_0 > p_0$, the venture capitalist cannot make arbitrage in a strict sense (as he/she cannot "go short" on the venture capital project). The economic rationale against the possibility of $f_0 > p_0$ is simply that in this case, the venture capital project would obviously be an irrational investment.

Now, we can combine Proposition 1 with the bounds on instalment options obtained in Theorem 1 above:

THEOREM 2a

Under the above assumptions, an upper bound for a *rational* Venture Capitalist on a *Clone* Venture Capital project (*Y*) is the no arbitrage upper bound given in equation (4) of the corresponding instalment Option on company *X* as described in Proposition 1.

$$f_0 \le cC[t_0, T, K + (\hat{p} / c)] \tag{6}$$

where
$$\hat{p} = \sum_{i=1}^{n-1} e^{r(T-t_i)} p_i .$$

PROOF

From Proposition 1, if the initial cost of the project f_0 is higher than the cost of the super-replicating portfolio for the instalment option, $cC(t_0, T, K + \hat{p})$, a rational investor will not choose the project but rather the instalment option. Therefore, the super-replicating portfolio for the instalment option is also –a fortiori- super-replication of the venture capital project. ¦

THEOREM 2b

Under the above assumptions, a lower no arbitrage bound for a *Clone* Venture Capital project (*Y*) is the no arbitrage lower bound given in equation (4) of *c* units of the corresponding instalment Option on company *X* as described in Proposition 1.

$$f_0 \ge c[C(t_0, T, K) - e^{-r(T-t_0)} \hat{p} / c]_+ \tag{7}$$

PROOF:

From Proposition 1, if the initial cost of the project f_0 is lower than the cost of the sub-replicating portfolio which enforces the lower bounds for the instalment option, $c[C(t_0, T, K) - e^{r(T-t_0)} \hat{p}]_+$, a profit maximising investor will invest in the project and would then sell the sub-replicating portfolio whose price equals the lower boundary of the instalment option. Given that both projects will either cease to exist at exactly the same point in time t_i or pay $c(S_T - K)_+ = (cS_T - cK)_+ = (\tilde{S}_T - f_n)_+$ at time *T*, this strategy would yield the investor any difference p_0 - f_0 as an arbitrage. ¦

As pointed out in the proof of Proposition 1, the above results hold true due to our carefully chosen Definition 1 of a “clone” project. From an economic perspective, this assumption is of course, very strong and hardly met in practice (although, there are some practical cases, which come closer to the above assumptions as the step-up expansion option considered below).

Nevertheless, the above results can be used as ***guidance*** for determining economically meaningful funding schemes for venture capital projects also in more general circumstances. Indeed, consider the following variant of Definition 1:

DEFINITION 2:

Fix the time horizon T>0. A venture capital project Y is said to have a ***similar risk profile*** *as company Z if the random variable* $\overline{S}_T$ *modelling the value of company Z at time T, and the random variable* $\tilde{S}_T$ *modelling the value of the venture capital project Y at time T (equipped with the unconditional obligation of the Venture Capitalist to pay all the payments of the funding plan) coincide in distribution up to a scaling factor c>0:*

$$Law(\tilde{S}_T) = Law\ (c\overline{S}_T)\ .$$

The idea underlying this definition is that Y and Z are not necessarily "driven by the same sources of randomness"; in fact, the random variables, $\tilde{S}_T$ and $c\overline{S}_T$ may even be independent. What we do assume is that Y and Z have the same risk profile in the sense that they offer the same probabilities (under the "physical" probability measure P) for gains and losses up to the horizon date T. For practical purposes, it should be possible in many cases to find for a venture capital project, Y, some appropriate company, Z, for which there is a liquid market for European options and to which Y has *a similar risk profile*. We still note that the case, when we impose in addition some assumptions on the correlation between $\tilde{S}_T$ and $c\overline{S}_T$ is left for future research.

What can we deduce from the above arguments for a venture capital project Y that has *a similar risk profile to* (but not necessarily a *clone* of) a company, Z ? The unfortunate answer is that the no arbitrage arguments made previously break down completely, as we can no longer find trivial sub- or super-replicating portfolios. However, we may still apply the above formulae to calculate bounds for the funding of the venture capital project, Y, which may serve as a guide to the initial investment. The economic rationale for these bounds now reads as follows: suppose *there existed* such a company, call it *X*, with a liquid market for European options, such that its price process defined by $(S_t)_{t_0 \le t \le T}$, is identical in distribution to the price process $(\overline{S}_t)_{t_0 \le t \le T}$ of *Z*, and such that $cS_T = \tilde{S}_T$ almost surely.

If the prices for the European options on the companies *X* and *Z* were all equal, then the bounds for the venture capital project *Y* obtained from the above formulae would be implied by no arbitrage arguments on European options on company, *X* as argued above.

As it is economically quite reasonable to assume that, for two companies *X* and *Z* with equal (in distribution under the "physical" measure P) price processes, we also have equal prices for the corresponding options. We conclude that the above results can be used to give indications for economically feasible funding schemes $(f_i)_{i=0}^n$ for the project *Y*. We stress, however, that - unless *Y* is an exact clone of *Z* – the hedging aspects of the above arguments break down (if we use Definition 2, alone) and we cannot indicate any non-trivial hedging portfolios.

4.1. Applications to other types of real options

The present analysis would apply more directly to an existing firm, which wishes to expand current operations in the next year. Such an expansion has been considered in the Real Options literature as a Step-up or Expansion option [see Kulatiliaka and Perotti (1998)]. Consider an existing firm with a given successful product line. An existing firm produces 10,000 units of a product and sells all these units. The firm assumes that the demand for these products would be totally inelastic up to a number N, so that up to 20,000 units would all be sold at the same price (if N $\geq$ 20,000). However, the overall demand, N, for the product depends on some stochastic process (which will only be gradually revealed over the next year) and the sole effect on the firm in the existing or expansion phases is scalar in nature. If the firm expands from 10,000 units to 20,000 units and sells all these units, the value of the firm would double. Thus, as in the previous example with the venture project and an existing firm, the value of the expansion is exactly equal to the previous value of the firm. If, however, the firm can choose to fund the expansion over-time and if adverse circumstances occur, can abandon the project with no further obligations, then this can be seen as an instalment option (with possibly a rebate from any salvage value of the expanded project). Furthermore, as we observe the price process of the firm prior to the expansion and if we assume that European options trade on the firm, the value of the expansion must be the value of the instalment option on the pre-expansion firm. The firm can also determine the bounds on the instalment option via the portfolio of European options. In this example, our analysis holds directly if we assume that the salvage value of the expansion is zero, when the decision to abandon the project occurs. While we assume no further payment is required at the decision to abandon the expansion (such as cleaning up or shut down costs), such payments can easily be included in the analysis.

5. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEACH

In this research, we have evaluated venture capital projects as instalment options. We have argued that the usual approach to staggering the introduction of funding of such projects over time can be compared to an instalment option as previously examined by DST (2001, 2002).

As has been pointed out in the literature, such a Venture Capital (VC) project can be evaluated as a multi-stage compound option, the pricing of which is – even in the framework of the Black-Scholes (1973) model - intractable analytically but possible numerically. In addition and as is the case for many Real Options applications, the determination of model parameters is difficult. We show that under the assumption that a new VC project is an imitator firm (a "clone" sharing exactly the same external risk as an existing firm in the same industry), no arbitrage bounds on the value of the venture capital project may be derived from no arbitrage bounds on an instalment option on the existing firm. These bounds, in turn, are enforced by a static portfolio of European options on the related firm. Therefore, the problems of model selection and parameter estimation are eliminated as the bounds proposed by DST (2001, 2002) are robust with respect to model mis-specification and stochastic volatility.

In our setting, we provide clear guidance both to Venture Capitalists and entrepreneurs, if the funding requirement for a VC project is above a comparable instalment option (and thus a standard assumed observable European Option) on an existing firm, the Venture Capitalist should rationally choose the instalment option. Likewise, if the funds required for the VC project are below the lower bound of the instalment option on the existing firm, the entrepreneur should sell this (portfolio of European Options) and use the proceeds to invest in the project.

In this research, the assumptions of the "*clone*" relationship between the venture capital project and an existing firm is rather restrictive. This was done to allow for rigorous mathematical proofs to be derived. In reality, such assumptions will most certainly be violated. Nevertheless, it is likely that while the almost sure identity $\tilde{S}_T = cS_T$ (assumed in the definition of a "clone") may be violated, the variables may be related in distribution as was pointed out in Definition 2 above. In this instance, this analysis will still provide guidance to venture capitalists as to the bounds for the initial funding that should be paid.

Future research could attempt to sharpen these results for such cases where the random variables are no longer perfectly correlated. Recent work by Davis (2000) on the implications of basis risk on the hedging of non-perfectly correlated assets may shed light on this problem.

Another line of research not considered here is how VC investors should structure the sequence of future cash infusions. Previously, DST (2001) have considered the impacts of the payment sequence for instalment options on existing firms on the probability of exercising the option. As with this paper, it may prove fruitful to link these results with the empirical record of how venture capital structures sequence the cash injections.

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[1] For an excellent review of the three main classes of research on venture capital, see Cossin, Leleux & Salisasi (2002)

[2] While most such VC investors receive convertible preferred securities, Repullo & Suarez (1999) shows that the optimal contact for the VC investor is a multiple period warrant-like claim.

[3] Detailed summaries of these papers are provided in Schwartz and Zozoya-Gorostiza (2000) pg. 2.

[4] While the concept of instalment options can be generalised to allow the underlying option to be non-standard (for example, American or an exotic option), we will restrict our analysis here to standard European options as the underlying. This is done as most of these products assume an underlying European option and most extensions of compound option methodology to other areas of finance make similar assumptions.

[5] See Harrison & Kreps (1979), Harrison & Pliska (1981) and Delbaen & Schachermayer (1994) for a review of the principle of no arbitrage.

[6] In DST (2001, 2002), the lower bound includes a Bermudan Put option. For the sake of convenience (and without loss of generality) this is omitted here. The result is that the lower bound is not as tight as in DST (2001, 2002). However, this difference is slight.