SAKS SPACES AND VECTOR VALUED MEASURES

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Introduction: The purpose of this note is to give a sample of applications of Saks spaces to the theory of vector measures. For convenience we restrict attention to generalised Riesz representation theorems i.e. we consider representations of operators on spaces of continuous functions by integration with respect to a vector valued measure. A Saks space is a vector space with two structures, a norm and a locally convex topology, which are in some sense compatible. At first sight this may seem a rather strange object and its relevance to measure theory is not at all obvious. However, we hope that this paper demonstrates the thesis that they are a suitable tool for some aspects of the theory. Here we would like to mention one argument which may make this claim more plausible. One of the features of a σ-additive measure with values in a Banach space and defined on a σ-field is that it takes its values in a weakly compact set. This means exactly that it takes its values in a Saks space and indeed of a very special kind - one with compact unit ball. Now such Saks spaces are precisely those which are expressible as projective limits (in a suitable sense) of finite dimensional spaces. This allows us, for example, to reduce the proof of a Riesz representation theorem for operators with values in such Saks spaces to the finite dimensional (i.e. essentially the scalar valued) case by means of a simple formal manipulation with suitable functors. Surprisingly enough, although this result seems very special, it contains as Corollaries three important results and thus we obtain a simple and unified approach to them.

For the convenience of the reader we begin with a brief survey of the results and concepts on Saks spaces which we shall require. A detailed discussion can be found in COOPER [1]. In the second half of the paper we prove the Riesz representation theorem mentioned above and deduce three important representation theorems (cf. DIESTEL and UHL [2]).

This article is an extract from a systematic treatment of vector measures from the point of view of Saks spaces which is now in preparation.

1. <u>Definition</u>: A <u>Saks space</u> is a triple $(E,\| \|,\tau)$ where $(E,\| \|)$ is a normed space and τ is a locally convex topology on E so that $B_{\| \|}$, the unit ball of $(E,\| \|)$, is τ-closed and bounded.
We then write $\gamma[\| \|,\tau]$ or simply γ for the finest locally convex topology on E which coincides with τ on $B_{\| \|}$. We resume the most important elementary properties of γ in the following Proposition (cf. COOPER [1]):

2. <u>Proposition</u>: 1) $\tau \subseteq \gamma \subseteq \tau_{\| \|}$;
2) the γ-bounded subsets of E coincide with the norm-bounded sets;
3) a sequence (x_n) in E converges to zero with respect to γ if and only if it is norm bounded and τ-convergent to zero;
4) a subset of E is γ-compact if and only if it is norm-bounded and τ-compact;
5) (E,γ) is complete if and only if $B_{\| \|}$ is τ-complete;
6) the dual E'_γ of (E,γ) is the norm-closure of $(E,\tau)'$ in the dual of $(E,\| \|)$.

3. <u>Examples</u>: I. If E is a Banach space, then the following triples are Saks spaces:

$$(E,\| \|,\tau_{\| \|}),\quad (E,\| \|,\sigma(E,E')),\quad (E',\| \|,\sigma(E',E)).$$

II. If X is a completely regular space and $C^b(X)$ denotes the space of bounded, continuous complex-valued functions on X then $(C^b(X),\| \|,\tau_K)$ is a Saks space where τ_K is the topology of compact convergence.

III. If H is a Hilbert space and L(H) is the algebra of continuous linear operators on H we denote by τ_w and τ_s the weak resp. strong operator topology on L(H). Hence τ_w is defined by the seminorms $T \to |f(Tx)|$ $(x \in H, f \in H')$ and τ_s is defined by the seminorms $T \to \|Tx\|$ $(x \in H)$. Then

$$(L(H),\| \|,\tau_w) \quad\text{and}\quad (L(H),\| \|,\tau_s)$$

are Saks spaces.

IV. Let μ be a positive, finite, σ-additive measure on the space (Ω,Σ) and denote by $L^\infty(\mu)$ the corresponding L^∞-space. Then $(L^\infty(\mu),\| \|,\tau_1)$ is a Saks space where τ_1 is the topology induced by the L^1-norm and its dual is L^1.

4. <u>Completions</u>: If $(E,\| \|,\tau)$ is a Saks space its completion is defined as follows: we let $\hat{B}$ denote the τ-completion of $B_{\| \|}$ i.e. the closure of $B_{\| \|}$ in the completion $\hat{E}_\tau$ of (E,τ). Then if $\hat{E}$ is the span of $\hat{B}$ and $\| \|^\wedge$

denotes the Minkowski functional of $\hat{B}$ $(\hat{E}, \| \|^{\wedge}, \tau)$ is the required completion. As an example, if E is a normed space then the completion of the Saks space $(E, \| \|, \sigma(E,E'))$ is the Saks space $(E'', \| \|, \sigma(E'',E'))$.

5. Saks space products and projective limits: If $\{(E_\alpha, \| \|_\alpha, \tau_\alpha)\}_{\alpha \varepsilon A}$ is a family of Saks spaces we form their product as follows: if E denotes the Cartesian product $\prod_{\alpha \varepsilon A} E_\alpha$ we put

$$E_o := \{x = (x_\alpha) \ \varepsilon \ E : \|x\| := \sup \|x_\alpha\|_\alpha < \infty\}$$

Then $(E_o, \| \|, \tau)$ is a Saks space where τ is the Cartesian product of the topologies $\{\tau_\alpha\}$. $(E_o, \| \|, \tau)$ is called the Saks space product of $\{E_\alpha\}$ and is denoted by $S \prod E_\alpha$.

Now let $\{\pi_{\beta\alpha} : E_\beta \to E_\alpha, \ \alpha, \beta \ \varepsilon \ A, \ \alpha \leq \beta\}$ be a projective spectrum of Saks spaces. As usual, we define the projective limit of this spectrum as the subspace of the product formed by the threads i.e. as

$$E_1 := \{(x_\alpha) \ \varepsilon \ S \prod E_\alpha : \pi_{\beta\alpha}(x_\beta) = x_\alpha \quad \text{for } \alpha \leq \beta\}$$

E_1 is denoted by $S \varprojlim \{E_\alpha\}$. As an example, if X is a locally compact space and $K(X)$ denotes the family of compact subsets of X then

$$\{\rho_{K_1,K} : C(K_1) \to C(K), \ K \subseteq K_1\}$$

forms a projective spectrum of Banach spaces (where $C(K)$ denotes the Banach space of continuous, complex-valued functions on K and $\rho_{K_1,K}$ is the restriction operator) and its Saks space projective limit is naturally identifiable with $(C^b(X), \| \|, \tau_K)$.

Now if $(E, \| \|, \tau)$ is a Saks space, we say that a family of seminorms S which generates τ is a suitable family if it satisfies the condition:

1) if $p, q \ \varepsilon \ S$ then $\max\{p,q\} \ \varepsilon \ S$;
2) $\| \| = \sup S$.

If $p \ \varepsilon \ S$, E_p denotes the Banach space generated by p (i.e. the completion of the normed space E/N_p where N_p is the kernel of p) and if $p \leq q$ then ω_{pq} denotes the natural mapping from E_q to E_p. Then $\{\omega_{qp} : E_q \to E_p\}$ forms a projective spectrum of Banach spaces.

6. Proposition: If $(E, \| \|, \tau)$ is a complete Saks space, then E is naturally identifiable with $S \varprojlim_{p \varepsilon S} E_p$.

7. <u>Proposition</u>: Let $(E, \| \|, \tau)$ be a Saks space. Then the following are equivalent:

1) $B_{\| \|}$ is τ-compact;

2) E is a Saks space projective limit of finite dimensional Banach spaces;

3) E has the form $(F', \| \|, \sigma(F',F))$ for some Banach space F.

Then $\gamma = \tau_c(F',F)$, the topology of uniform convergence on the compact sets of E, is the finest topology on E which agrees with τ on $B_{\| \|}$.

In fact, if 1) is fulfilled, then E is naturally identifiable with $S \varprojlim_{F \in \mathcal{F}(E'_\gamma)} \{F'\}$ where $\mathcal{F}(E'_\gamma)$ denotes the family of finite dimensional subspaces of E'_γ.

Further, the following are equivalent:

1) B is τ-compact and metrisable;

2) E is the Saks space projective limit of a sequence of finite dimensional Banach spaces;

3) E has the form $(F', \| \|, \sigma(F',F))$ for a separable Banach space F;

4) $B_{\| \|}$ is τ-compact and normable (i.e. there is a norm $\| \|_1$ on E so that $\tau = \tau_{\| \|_1}$ on $B_{\| \|}$).

8. <u>The Hom functor</u>: If $(E, \| \|, \tau)$, $(F, \| \|_1, \tau_1)$ are Saks spaces, then Hom (E,F) denotes the set of γ-continuous linear operators from E into F. Note that, as a vector space, this coincides with the space of norm-bounded linear operators from E into F if E is a Banach space. We regard Hom (E,F) as a Saks space with the supremum norm and τ_p, the topology of pointwise convergence, with respect to τ.

9. <u>Proposition</u>: 1) If $\{E_\alpha\}$ is an inductive system in BAN_1 and F is a complete Saks space then there is a natural isomorphism between the Saks spaces

$$\text{Hom}\,(B \varinjlim_{\alpha} E_\alpha, F) \quad \text{and} \quad S \varprojlim_{\alpha} \text{Hom}\,(E_\alpha, F).$$

In particular, if E is a Banach space, we have

$$\text{Hom}\,(E,F) = \text{Hom}\,(B \varinjlim_{G \in \mathcal{F}(E)} G, F) = S \varprojlim_{G \in \mathcal{F}(E)} \text{Hom}\,(G,F).$$

2) if $\{F_\alpha\}$ is a projective system of Saks spaces, E a Saks space, then there is a natural isomorphism between the Saks spaces

$$\text{Hom}\,(E, S \varprojlim_{\alpha} F_\alpha) \quad \text{and} \quad S \varprojlim_{\alpha} \text{Hom}\,(E, F_\alpha).$$

In particular, if F is a Saks space with $B_{\|\ \|}$ τ-compact, we have

$$\mathrm{Hom}\,(E,F) = \mathrm{Hom}\,(E,\ S\ \varprojlim_{G\in F(F'_\gamma)} G') = S\ \varprojlim_{G\in F(F'_\gamma)} \mathrm{Hom}\,(E,G').$$

10. Remarks on the Saks space $C^b(X)$:

The space $C^b(X)$ is one of the most important Saks spaces and we shall be interested in representations of operators on it. We note here that the dual of $(C^b(X),\gamma)$ is the space of bounded Radon measures on X, the duality being established by integration. This follows easily from 2. For the dual of $(C^b(X),\tau_K)$ is the space of Radon measures with compact support and the bounded Radon measures are just those which can be approximated by such measures. We remark in passing that there are natural Saks space structures on $C^b(X)$ so that the corresponding dual spaces consist of the bounded σ-additive Borel measures (resp. τ-additive Borel measures).
For the theory of $C^b(X)$ cf. COOPER [1], Ch.II.

11. Radon measures with values in a Saks space:

Let $(E,\|\ \|,\tau)$ be a Saks space, S a suitable family of τ-seminorms on E and X a completely regular space. A bounded Borel measure on X with values in E is a (finitely additive) norm bounded set function μ from Bo(X), the Borel field of X, into E. Such A measure is a Radon measure if it is inner regular with respect to τ i.e. satisfies the condition that

$$\lim_{\substack{K\in K(X)\\ K\subseteq A}} \omega_p \circ \mu(K) = \omega_p \circ \mu(A) \quad \text{in } E_p$$

for each Borel set A in X and each $p\in S$.

Note that we can then integrate functions in $C^b(X)$ with respect to such a measure. The integral $\int x\, d\mu$ is then in $\hat{E}_\gamma$, the completion of E.

We use the notation $M_R(X;E)$ for the (vector) space of E-valued Radon measures on X.
We regard $M_R(X;E)$ as a Saks space with the following structures. The norm is the semivariation norm and the subsidiary locally convex topology is that defined by the seminorms

$$\mu \mapsto p(\int x\, d\mu) \qquad (p\in S,\ x\in C^b(X))$$

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12. Proposition: If $(E,\| \|,\tau)$ is a complete Saks space, then there are natural identifications:

$$M_R(X;E) = S \varprojlim_{p \in S} M_R(X;E_p).$$

Proof: If μ is in the left hand side, then the elements $(\omega_p \circ \mu)$ form a thread which defines an element of the right hand side. On the other hand, a thread (ω_p) on the right hand side pieces together to form a bounded E-valued measure which is clearly Radon.

We consider Riesz representation theorems for continuous linear operators from the Saks space $C^b(X)$ (cf. 3.II) into a Saks space. We prove one main theorem for operators with values in a Saks space with compact unit ball. Using the machinery developed above, this can be proved in a few lines. We then show how the classical representation theorems for bounded, weakly compact and compact operators from $C(K)$ into a Banach space follow immediately from this.

First we recall that if F is a Banach space, X a completely regular space, then if $\mu : Bo(X) \to F$ is a Radon measure

$$T_\mu : x \mapsto \int x \, d\mu$$

is a β-continuous linear operator from $C^b(X)$ into F. $\mu \mapsto T_\mu$ is an isometry from $M_R(X;F)$ into $Hom(C^b(X),F)$. In general it is not onto as the example of the identity operator on $C[0,1]$ shows. However, if F is finite dimensional it clearly is (once again, we can easily reduce to the case where F is one-dimensional and then the result can be found, for example, in COOPER [2], II.3.3).

Hence we have, for finite dimensional Banach spaces F,

$$M_R(X;F) \cong Hom(C^b(X),F).$$

Once again, this is a natural isomorphism of functors.

The above isomorphism is a Saks space isomorphism if we give the left hand side the topology of pointwise convergence on $C^b(X)$ as auxiliary topology.

13. Proposition: Let $(E,\| \|,\tau)$ be a Saks space with $B_{\| \|}$ τ-compact, X a completely regular space. Then if $T : C^b(X) \to E$ is a β-γ-continuous linear operator there exists a Rdon measure $\mu : Bo(X) \to E$ representing

T i.e. T is the operator

$$T_\mu : x \mapsto \int x \, d\mu$$

Conversely, every Radon measure μ defines a β-γ-continuous linear operator T_μ in the above manner.
Hence integration establishes a Saks space isomorphism

$$M_R(X;E) = \mathrm{Hom}(C^b(X),E).$$

Proof: We put $G := E'_\gamma$ and calculate:

$$\begin{aligned}\mathrm{Hom}(C^b(X),E) &= \mathrm{Hom}(C^b(X), \; S \varprojlim_{F \in \mathcal{F}(G)} F') \\ &= S \varprojlim_{F} \mathrm{Hom}(C^b(X),F') \\ &= S \varprojlim_{F} M_R(X,F') \\ &= M_R(X, \; S \varprojlim_{F} F') \\ &= M_R(X,E).\end{aligned}$$

14. Remark: A less formal demonstration of the above result goes as follows: since T maps bounded sets in $C^b(X)$ into relatively compact subsets of (E,γ), then

$$T'' : C^b(X)'' \to E''$$

actually takes its values in E.

Noting that if A is a Borel set in X then its characteristic function χ_A defines, by integration, an element of $C^b(X)'' = (M_R(X), \| \, \|)'$ we may define

$$\mu(A) := T''(\chi_A)$$

which is an E-valued measure.

By the continuity of T'' (with respect to the norm in $C^b(X)''$) we can deduce that

$$T''(x) = \int x \, d\mu$$

for every bounded, Borel-measurable function on S and so in particular, for $x \in C^b(X)$. The converse fact is easy.

Using the above result, we can now easily obtain a result for general operators T with values in a locally convex space.

15. Proposition: Let E be a locally convex space, X a completely regular space. Then any continuous, linear operator $T : C^b(X) \to E$ may be represented by integration with respect to a Radon measure μ from Bo(X) into $(E'', \sigma(E'',E'))$. In fact, μ takes its values in the $\sigma(E'',E')$-closure of $T(B(C^b(X)))$.

If T maps the unit ball of $C^b(X)$ into a relatively weakly compact subset of E, then μ takes its values in E (actually in $T(B(C^b(X)))$ and is a Radon measure with respect to the original topology in E.

Proof: For the first assertion, let B be the $\sigma(E'',E')$-closure of $T(B(C^b(X)))$ in E" and let F be the Saks space spanned by B in E" with $\| \|_B$ as norm and $\sigma(E'',E')$ as auxiliary topology. Then this is a Saks space with compact unit ball and the result follows immediately from 14.

In the second case, take $B := \overline{T(B(C^b(X)))}$, the closure now being taken in E, and define the Saks space F to be $(E_B, \| \|_B, \sigma(E,E'))$. Then T is represented by an F-valued Radon measure μ (i.e. Radon with respect to $\sigma(E,E')$). Now the Orlicz-Pettis theorem for Radon measures implies that μ is a Radon measure with respect to the finest topology on F compatible with the duality between (F, F'_γ) and in particular with the respect to the topology of E.

16. Remark: We note that if $\mu \in M_R(X;E)$ is such that its range is contained in a weakly compact, absolutely convex set, then the associated integration operator $T : C^b(X) \to E$ sends the $C^b(X)$-ball into this same set and T is weakly continuous and so continuous if $(C^b(X), \beta)$ is a Mackey space (e.g. if S is locally compact, paracompact).

17. Proposition: Let (E,τ) be a quasi-complete locally convex space, $T : C^b(X) \to E$ a β-continuous linear operator. If T does not map the unit ball of $C^b(X)$ into a relatively weakly compact subset of E there is a sequence (x_n) of functions in $C^b(X)$ with mutually disjoint supports so that if j is the mapping $(\lambda_n) \to \Sigma \lambda_n . x_n$ from c_o into $C^b(X)$ and A denotes the β-closed span of $\{x_n\}$ in $C^b(X)$ then in the following diagram

$$\begin{array}{ccccc} & & C^b(X) & \xrightarrow{T} & E \\ & & \cup & & \cup \\ c_o & \xrightarrow{j} & A & \xrightarrow{T|A} & T(A) \end{array}$$

j and $T|_A$ are isomorphisms. More informally, T fixes a subspace of $(C^b(X),\beta)$ which is isomorphic to c_o.

Consequently, if E fails to contain a copy of c_o then every continuous linear operator $T : C^b(X) \to E$ takes the unit ball into a relatively weakly compact subset of E.

Proof: If T fails to satisfy the given condition, then by a standard result, $T' : E' \to M_R(X)$ takes some equicontinuous set H in E' to a subset of $M_R(X)$ which is bounded but not relatively $\sigma(M(X),M(X)')$-compact. Then by GROTHENDIECK's characterisation of weakly compact sets in $M_R(X)$ (cf. BUCHWALTER and BUCCHIONI [3], p.76) there exists a sequence (f_n) in H and a sequence (U_n) of disjoint open sets in S and an $\varepsilon > 0$ so that $|T'(f_n)(U_n)| > \varepsilon$ $(n \in \mathbb{N})$. i.e. $|f_n \circ \mu(U_n)| > \varepsilon$ (where μ represents T). By ROSENTHAL's Lemma (cf. DIESTEL-UHL [2], I.4.1) we may suppose that

$$|f_n \circ \mu| \, (\bigcup_{m \neq n} U_m) < \varepsilon/2 \qquad (n \in \mathbb{N}).$$

Now choose a sequence (x_n) in $C^b(X)$ so that $|x_n| \leq \chi U_n$ and

$$|f_n \circ T(x_n)| = |\int_S x_n d(f_n \circ \mu)| > \varepsilon$$

which is possible since $f_n \circ \mu$ is a Radon measure. Then j, as defined in the statement of the theorem, is clearly a well-defined, continuous injection. We claim that

$$\|T \circ j((\lambda_n))\|_H \geq \varepsilon/2 \, \|(\lambda_n)\|_{c_o}$$

for each $(\lambda_n) \in c_o$, where $\| \|_H$ denotes the seminorm of convergence on the equicontinuous set H. Indeed for any $(\lambda_n) \in c_o$ and any $k \in \mathbb{N}$,

$$\|T \circ j((\lambda_n))\|_H \geq |\langle T \circ j((\lambda_n)), f_k\rangle| = |\int_S \sum_{n \in \mathbb{N}} \lambda_n x_n d(f_k \circ \mu)|$$

$$\geq |\int_S \lambda_k x_k d(f_k \circ \mu)| - \|(\lambda_n)\|_{c_o} \, |f_k \circ \mu|(\bigcup_{l \neq k} U_l)$$

$$\geq |\lambda_k|\varepsilon - \|(\lambda_n)\|_{c_o} . \varepsilon/2$$

Taking the supremum over k on the right hand side we get the required estimate.

This shows that $(T \circ j)^{-1}$ is well-defined and continuous on $T \circ j(c_o)$, from which it follows that j, as an operator from c_o to $j(c_o)$ and T, as an operator from $j(c_o)$ to $T \circ j(c_o)$ are isomorphisms (by the following trivial Lemma).

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18. <u>Lemma</u>: Let X,Y,Z be topological spaces, $f : X \to Y$ and $g : Y \to Z$ continuous, surjective mappings such that $g \circ f$ is an isomorphism. Then f and g are also isomorphisms.

<u>Proof</u>: The injectivity of $g \circ f$ implies that of g and f so that f^{-1} and g^{-1} are well-defined. But as $g^{-1} = f \circ (g \circ f)^{-1}$ and $f^{-1} = (g \circ f)^{-1} \circ g$ it is clear that f and g are continuous.

With these results it is now easy to prove the following Proposition:

19. <u>Proposition</u>: Let $(E, \| \|, \tau)$ be a complete Saks space, $T : C^b(X) \to E$ a β-γ-continuous linear operator with representing measure $\mu \in M_R(X;E'')$. Then the following are equivalent:

1) T does not fix a copy of c_o;
2) T maps the unit ball of $C^b(X)$ into a relatively weakly compact set;
3) T maps weakly summable sequences to summable sequences;
4) T maps weakly Cauchy sequences into convergent sequences;
5) T maps sequences which tend weakly to zero convergent sequences;
6) if (x_n) is a bounded sequence of functions in $C^b(X)$ with mutually dusjoint supports, then $Tx_n \to 0$ in E;
7) T maps weakly compact sets in $C^b(X)$ into compact sets;
8 μ takes its values in E;
9) μ is a Radon measure with values in E;
10) μ is a strongly additive measure with values in E.

<u>Proof</u>: 1) $\Longleftrightarrow$ 2) is Proposition 17

2) $\Longrightarrow$ 9) is Proposition 15

9) $\Longrightarrow$ 2) - 6) are all simple applications of the Lebesgue-dominated convergence theorem. The reverse implications all follow from the fact that if μ is not Radon then the (x_n) constructed in the proof of Proposition 17 supply counter-examples.

9) $\Longrightarrow$ 8) is clear and 8) $\Longrightarrow$ 10) follows from the weak σ-additivity of μ and ORLICZ-PETTIS. 10) implies 2) is embedded in the proof of Proposition 17.

9) $\Longrightarrow$ 7) Note that $(C^b(X),\beta)$ is not necessarily complete so that we cannot use EBERLEIN-SMULIAN. Firstly, we may assume that E is a Banach space (using the characterisation of compactness in Saks spaces) (cf. COOPER [1], I.1.12). Then $\mu : Bo(X) \to E$ has tight semi-variation

norm so that for $\varepsilon > 0$ we may find $K \in K(X)$ so that if T_K denotes the operator associated to $\mu|_K$ then $\|T - T_K\| < \varepsilon$. Now if B is weakly compact in $C^b(X)$, then $T_K(B)$ is compact in $(E, \| \ \|)$ (using factorisation through $C(K)$ and EBERLEIN-SMULIAN) and we may conclude that $T(B)$ is compact in $(E, \| \ \|)$.

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