# Ground State and Expansion Dynamics of 1D Fermi Gas


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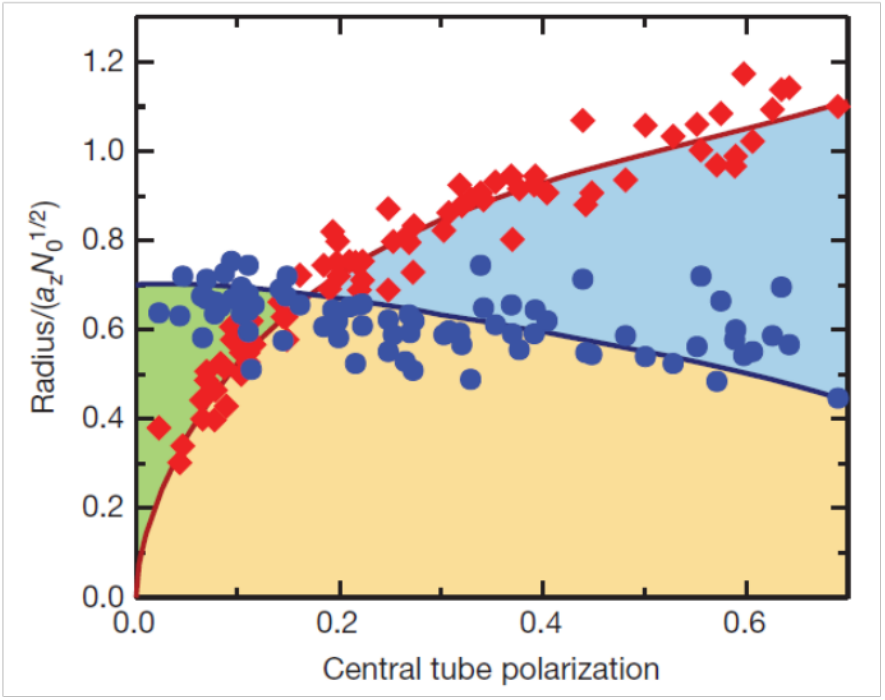
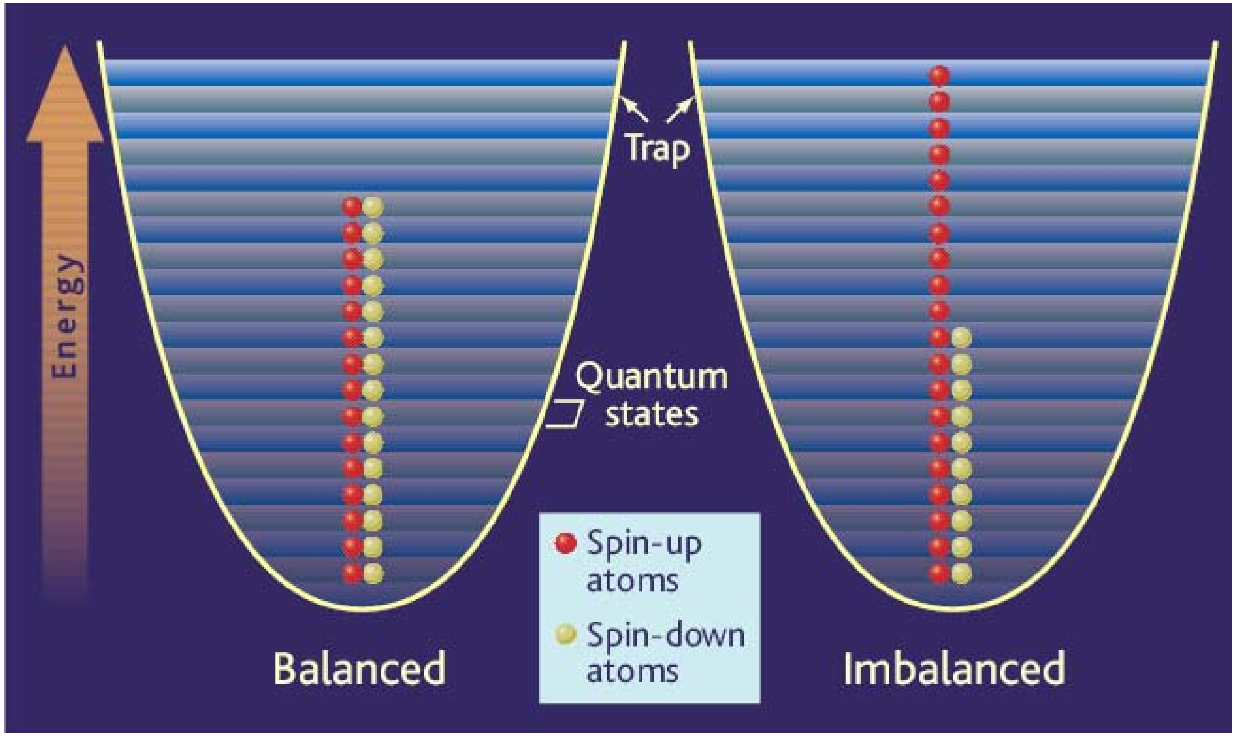
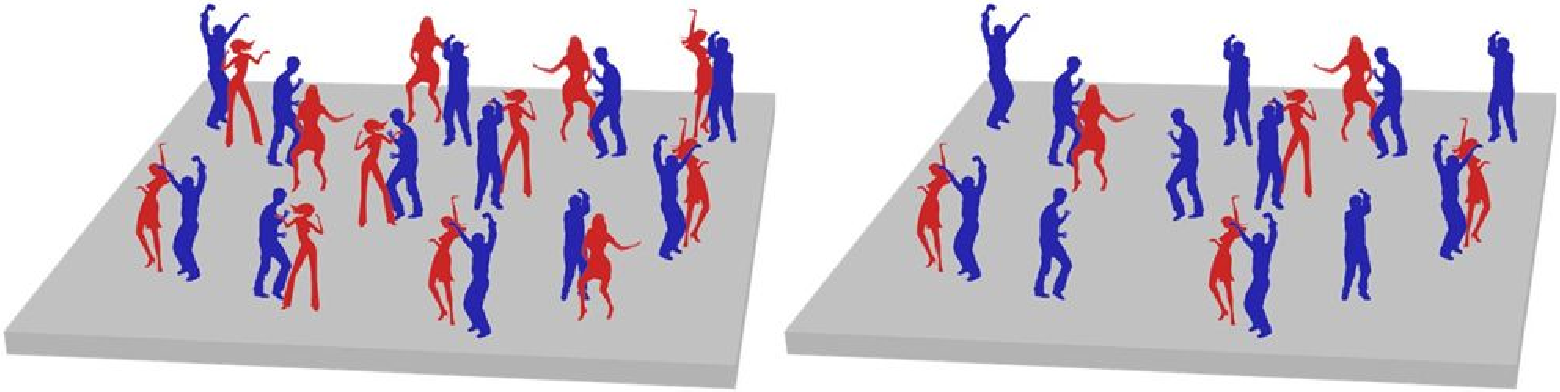



Workshop on “Confined Quantum systems”, Vienna, Austria, Feb. 2013

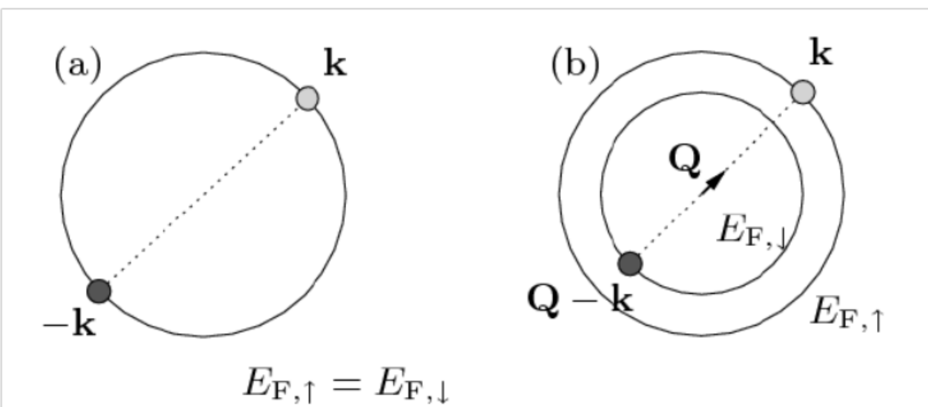
# Imbalanced Fermi mixtures

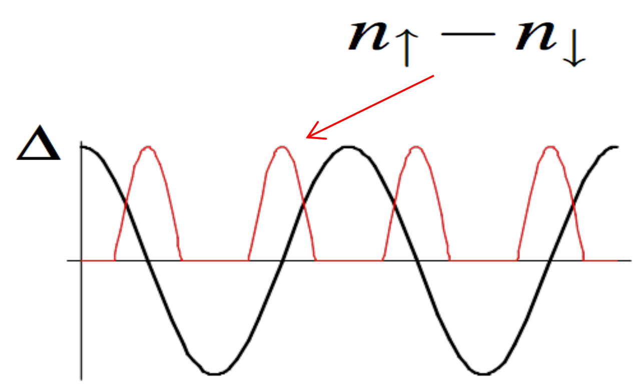


# Fulde-Ferrell-Larkin-Ovchinnikov instability

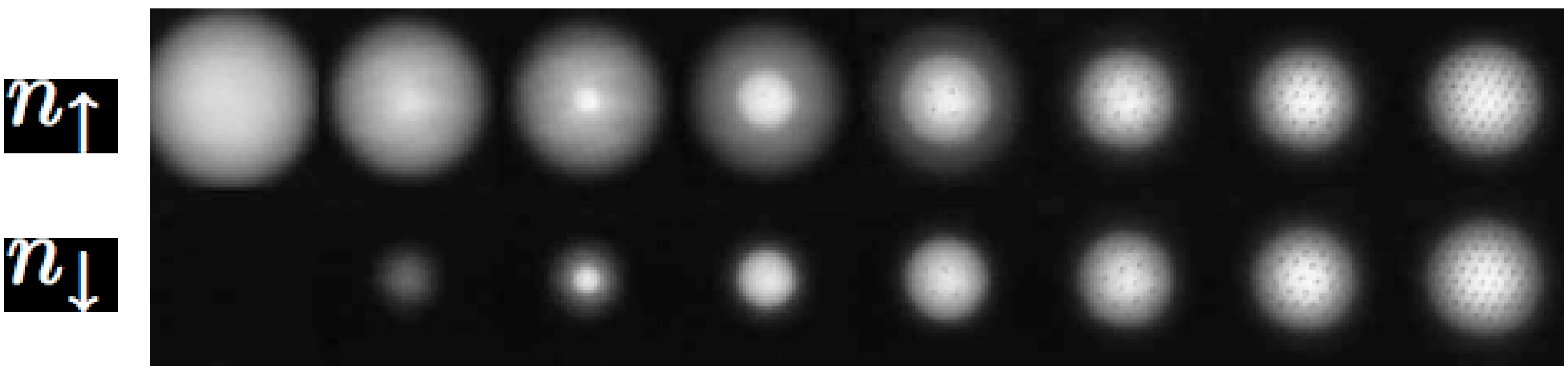
- BCS Cooper pairs have zero momentum
- Population imbalance leads to finite-momentum pairs
- FFLO instability results in textured states

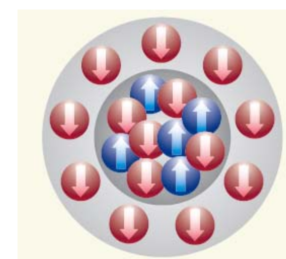


# Experiments on spin-imbalanced Fermi gas

- Rice (Hulet Group)
  - Science **311**, 503 (2006)
  - PRL **97**, 190407 (2006)
  - Nuclear Phys. A **790**, 88c (2007)
  - J. Low. Temp. Phys. **148**, 323 (2007)
  - Nature **467**, 567 (2010)
- ENS (Salomon Group)
  - PRL **103**, 170402 (2009)
- MIT (Ketterle Group)
  - Science **311**, 492 (2006)
  - Nature **442**, 54 (2006)
  - PRL **97**, 030401 (2006)
  - Science **316**, 867 (2007)
  - Nature **451**, 689 (2008)



MW. Zwierlein, A. Schirotzek, C.H. Schunck, and W, Ketterle: Science 311, 492-496 (2006)

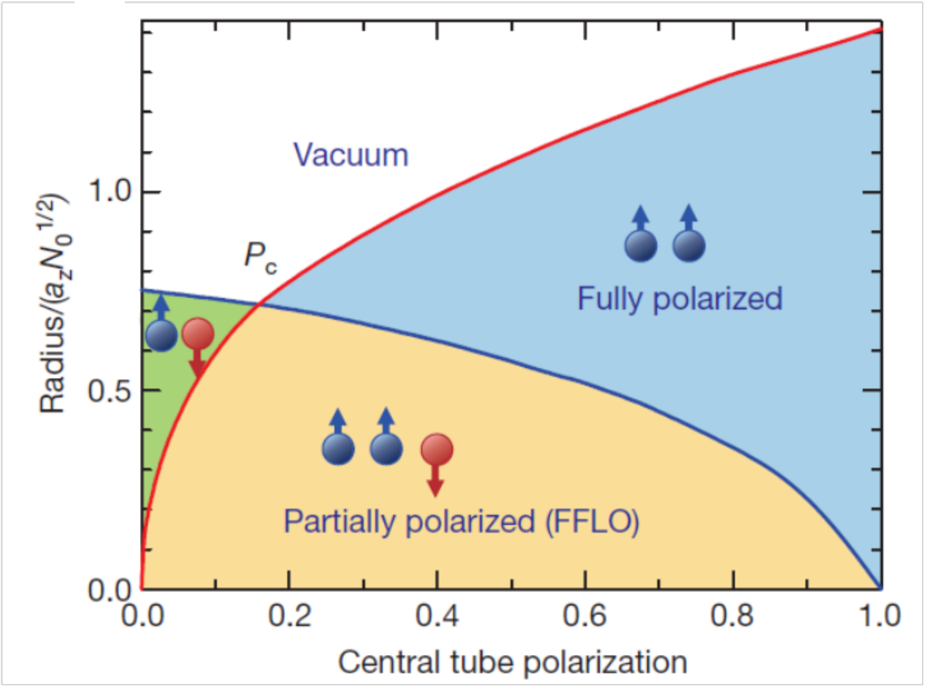
3D trapped gas: Superfluid core with polarized halo

# 1D trapped gas

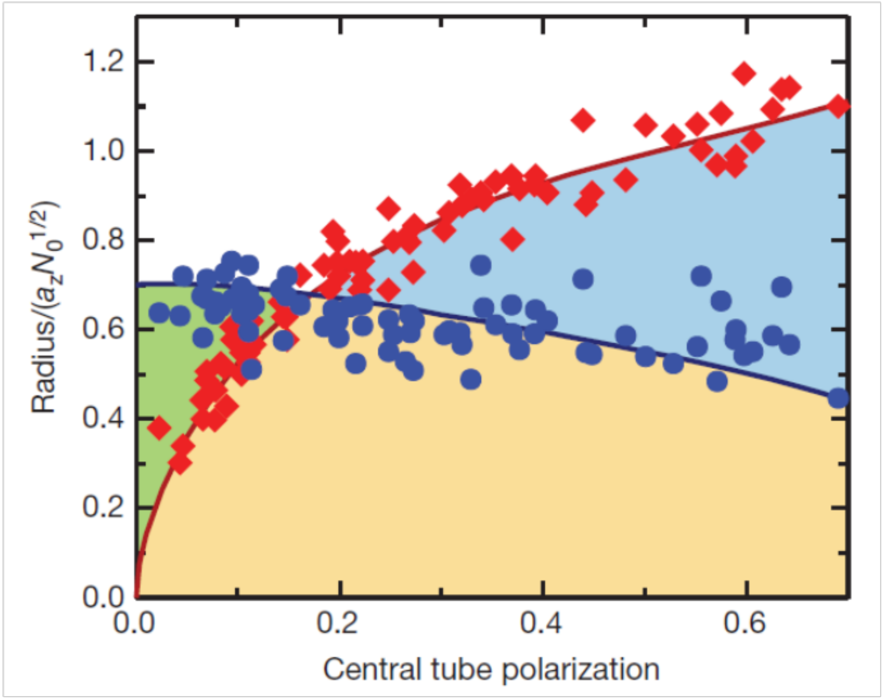
- **FFLO states are favored in 1D: nesting effect of the Fermi surfaces**

Phase diagram of trapped 1D Fermi gas:

Theory: Bethe Ansatz + LDA




Rice Experiment:




Liao *et al.*, Nature **467**, 567 (2010)

# Theoretical study: two complementary approaches

- **Mean-Field Bogoliubov-de Gennes study**

  -able to calculate mean-field superfluid gap

  -provide direct association between gap and density

- **Time-Evolving Block Decimation Method**

  -unbiased method retaining many-body correlations

  -will be able to handle stronger interaction strength

- **Combing two method with complementary advantages**

# Solving BdG equations

$$\begin{bmatrix} \mathcal{H}_\sigma^s - \mu_\sigma & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -\mathcal{H}_{\bar\sigma}^s + \mu_{\bar\sigma} \end{bmatrix} \begin{bmatrix} u_{j\sigma}(\mathbf{r}) \\ v_{j\sigma}(\mathbf{r}) \end{bmatrix} = E_{j\sigma} \begin{bmatrix} u_{j\sigma}(\mathbf{r}) \\ v_{j\sigma}(\mathbf{r}) \end{bmatrix}$$

$$H_\sigma^s = -\hbar^2\nabla^2/(2m) + V(r,z)$$

$$V(r,z) = \frac{1}{2} m\left(\omega_r^2 r^2 + \omega_z^2 z^2\right)$$

$$n_\sigma(\mathbf{r}) = \frac{1}{2}\sum_j \left[|u_{j\sigma}|^2 f(E_{j\sigma}) + |v_{j\bar\sigma}|^2 f(-E_{j\bar\sigma})\right]$$

$$\Delta(\mathbf{r}) = \frac{U}{2}\sum_j \left[v_{j\uparrow}^* u_{j\uparrow} f(E_{j\uparrow}t) - u_{j\downarrow} v_{j\downarrow}^* f(-E_{j\downarrow})\right]$$

Choose $T$ and $N_\sigma$

take initial guesses of $\mu_\sigma,\ n_\sigma(r),\ \Delta(r)$

→ diagonalize the matrix $\begin{bmatrix} H_\sigma^s - \mu_\sigma & \Delta \\ \Delta^* & -H_{\bar\sigma}^s + \mu_{\bar\sigma} \end{bmatrix}$

→ compute new $n_\sigma(r),\ \Delta(r)$

adjust $\mu_\sigma$

until: $N_\sigma = \int dr\ n_\sigma(r)$

until the input and output $\mu_\sigma,\ n_\sigma(r),\ \Delta(r)$ converge

Baksmaty *et al.*, PRA **83,** 023604 (2011)

NJP **13**, 055014 (2011)

# TEBD implementation

$$H = \int dx \sum_{\sigma} \psi_{\sigma}^{\dagger}(x)\left( -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_{ext} \right)\psi_{\sigma}(x) + U\psi_{\uparrow}^{\dagger}(x)\psi_{\downarrow}^{\dagger}(x)\psi_{\downarrow}(x)\psi_{\uparrow}(x)$$

*After spatial discretization:*

$$x \rightarrow i\Delta x, \quad \psi_{\sigma}(x) \rightarrow \psi_{\sigma i}, \quad \frac{d^2}{dx^2}\psi_{\sigma}(x) \rightarrow \frac{\psi_{\sigma i+1} - 2\psi_{\sigma i} + \psi_{\sigma i-1}}{(\Delta x)^2}$$

$$H = -t \sum_{<ij>,\sigma} \psi_{\sigma i}^{\dagger}\psi_{\sigma j} + U\sum_{i} n_{\uparrow i}n_{\downarrow i} + V_i \sum_{i}\left(n_{\uparrow i} + n_{\downarrow i}\right)$$

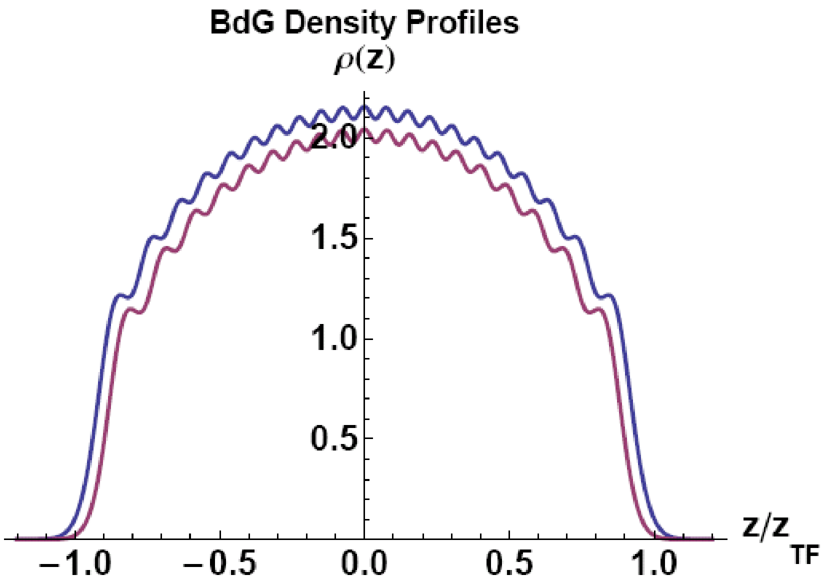
# Plan of investigation

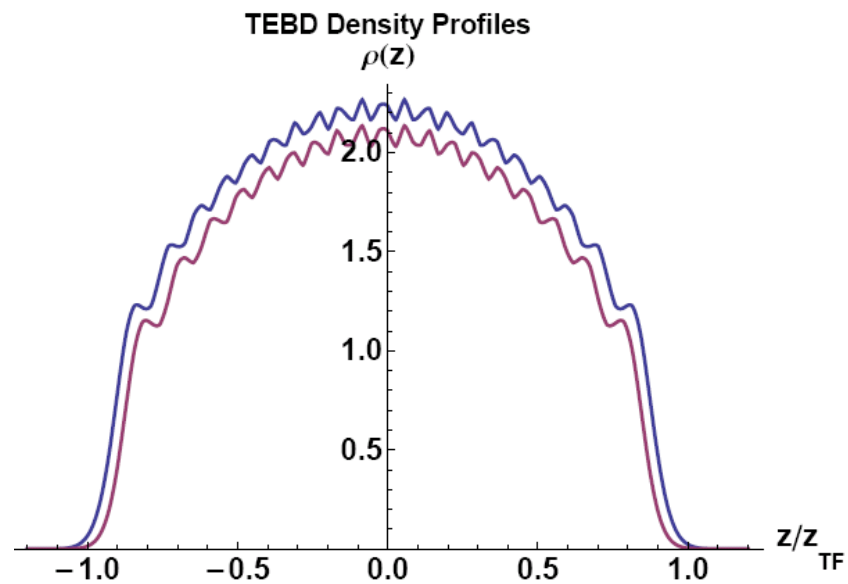
- Find the ground state within harmonic trap
  - BdG: Solving time-independent BdG Equation self-consistently
  - TEBD: Evolving many-body Schroedinger equation in imaginary time

- Turn off the trapping potential at $t=0$ and study the ensuing expansion dynamics
  - BdG: Solving time-dependent BdG Equation self-consistently
  - TEBD: Evolving many-body Schroedinger equation in real time

# Ground state: BdG vs. TEBD

$N_\uparrow = 21, N_\downarrow = 19, g_{1D} = -1.4$ ⟸ **Extremely weak interaction**

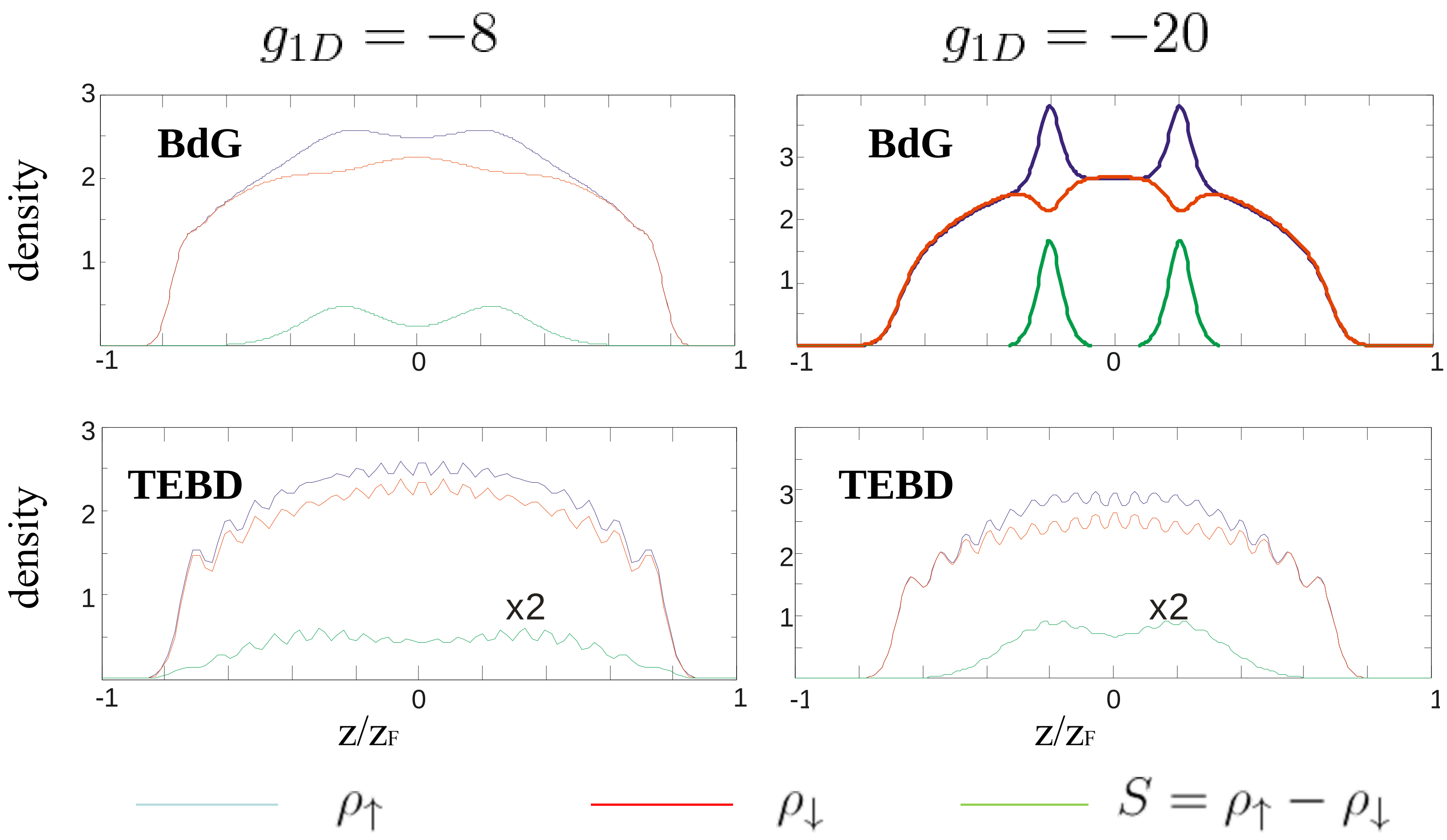



$\rho_\uparrow$ $\rho_\downarrow$

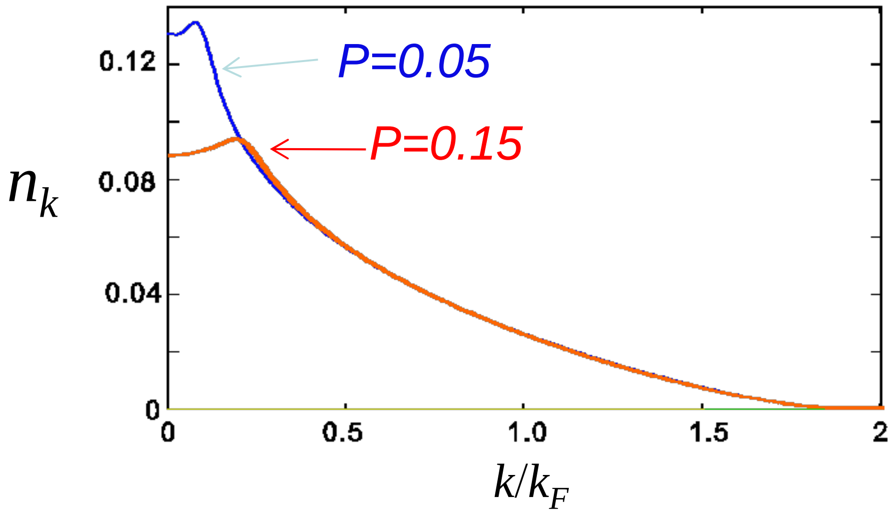
# Ground state: BdG vs. TEBD

$$N_\uparrow = 21, \quad N_\downarrow = 19$$



# Evidence of FFLO

Pair momentum distribution



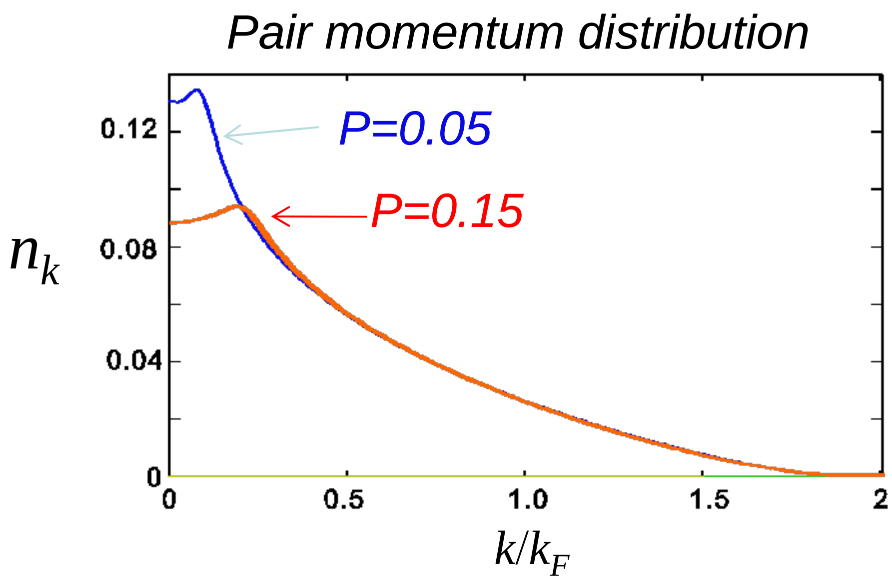

$N = N_\uparrow + N_\downarrow, \qquad P = (N_\uparrow - N_\downarrow)/N$

$g_{1D} = -8$

$$n_k = \frac{1}{L}\int\int dz dz' \, e^{ik(z-z')}\, O(z,z')$$

$$O(z,z') \equiv \langle \psi_\uparrow^\dagger(z)\psi_\downarrow^\dagger(z)\psi_\downarrow(z')\psi_\uparrow(z')\rangle$$

# Evidence of FFLO

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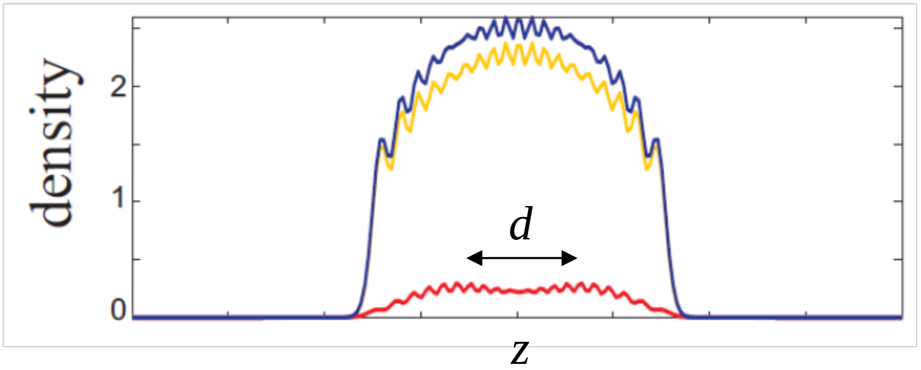
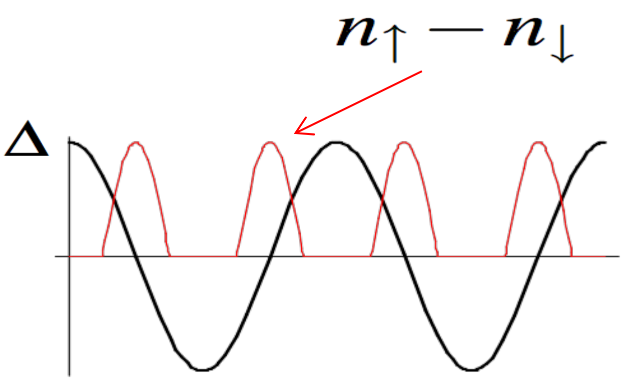
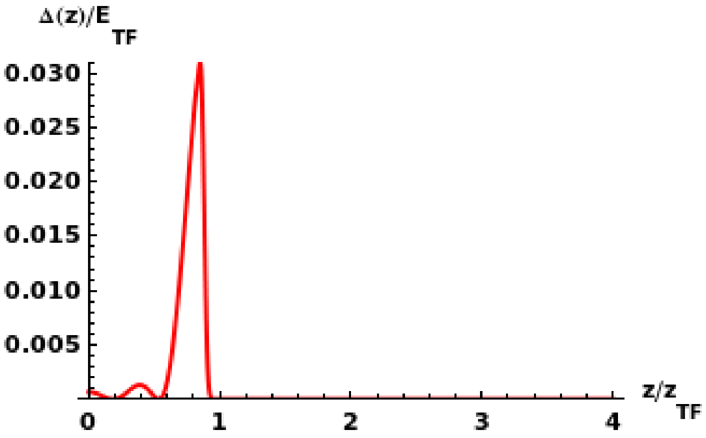


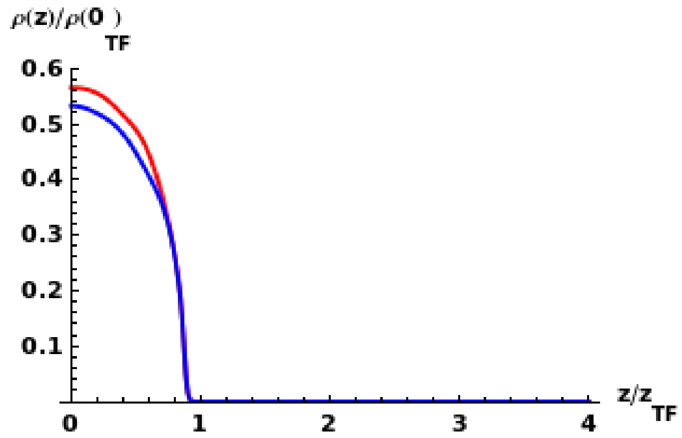




The peak momentum $q$ matches well with $\pi/d$

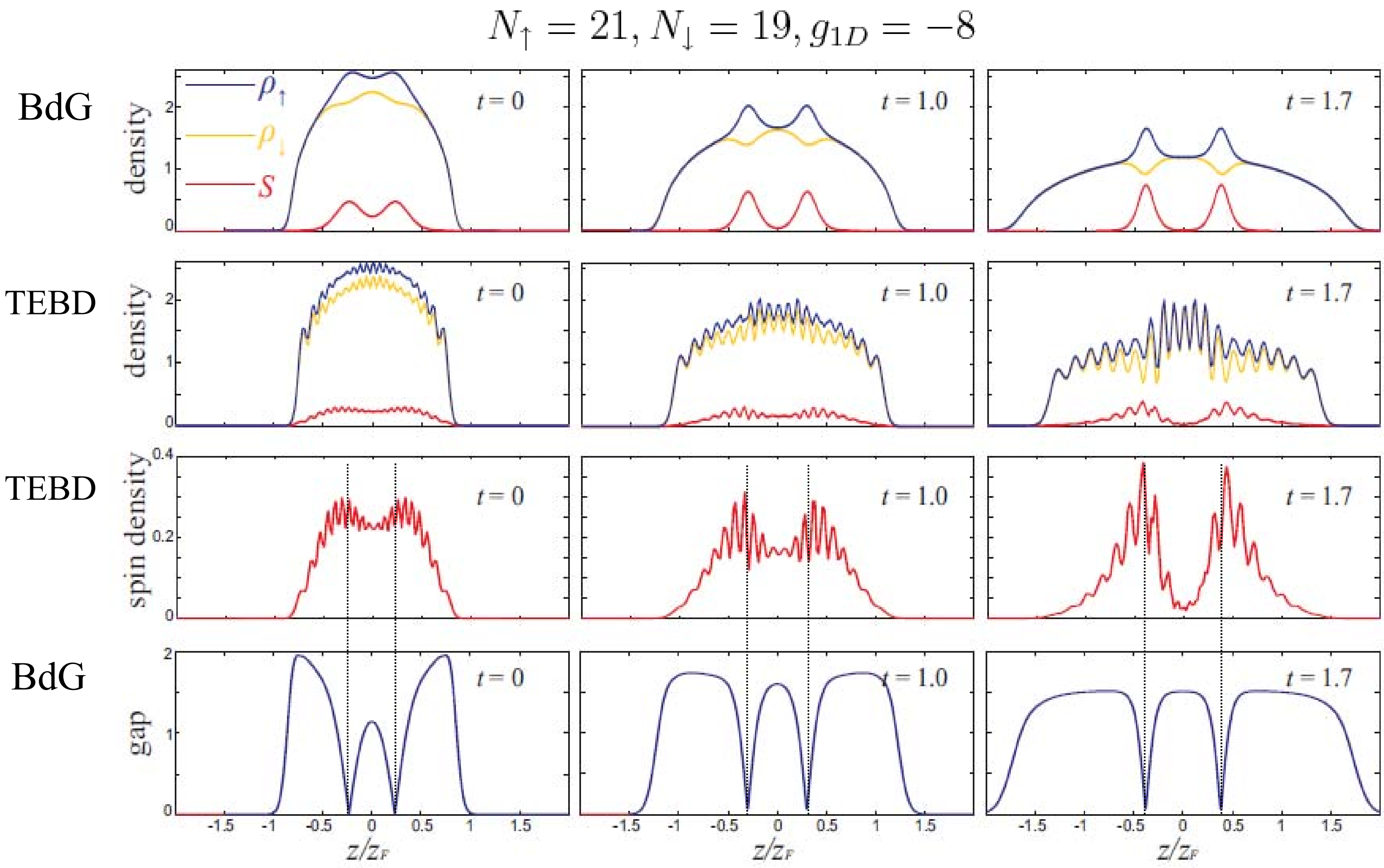
# Seeing FFLO during expansion

BdG



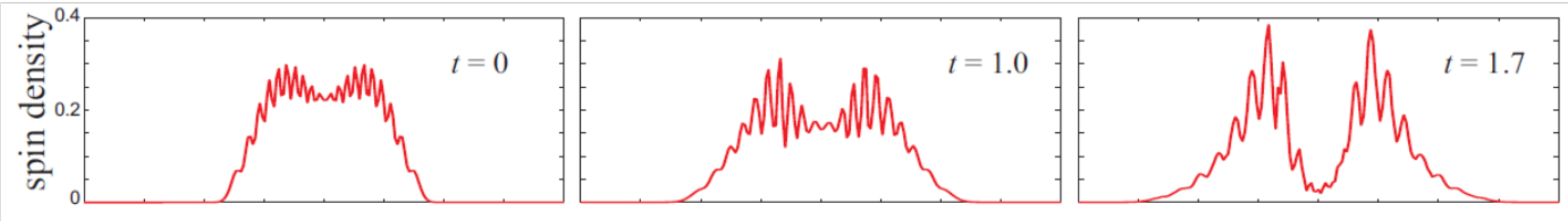



# Seeing FFLO during expansion




Lu, Baksmaty, Bolech and Pu, PRL **108**, 225302 (2012)

# Seeing FFLO during expansion




Spin density modulation becomes more pronounced during expansion!

In 1D, low density means stronger (relative) interaction.

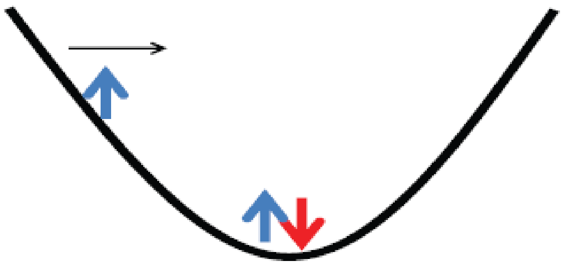
$$E_{\text{int}} \sim \rho_{1D}$$
$$E_{\text{kin}} \sim d^{-2} \sim \rho_{1D}^{2}$$
$$E_{\text{int}} / E_{\text{kin}} \sim \rho_{1D}^{-1}$$

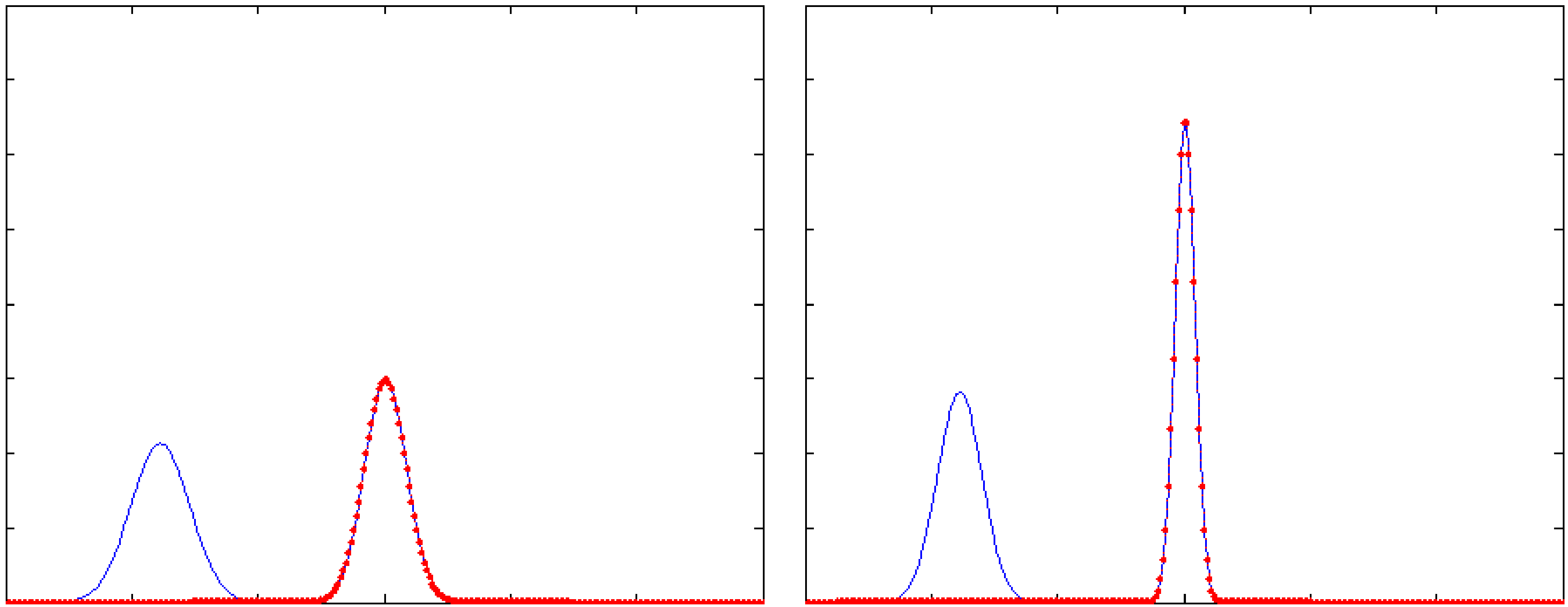
1D

$$E_{\text{int}} \sim \rho_{3D}$$
$$E_{\text{kin}} \sim d^{-2} \sim \rho_{3D}^{2/3}$$
$$E_{\text{int}} / E_{\text{kin}} \sim \rho_{1D}^{1/3}$$

3D

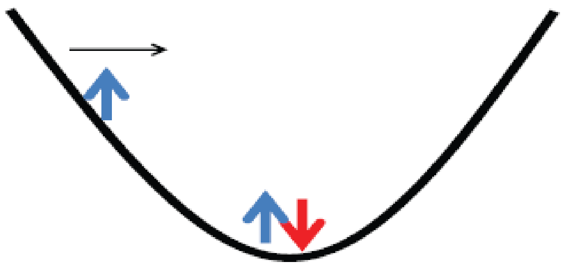
# Atom-pair interaction: a spin transport probe

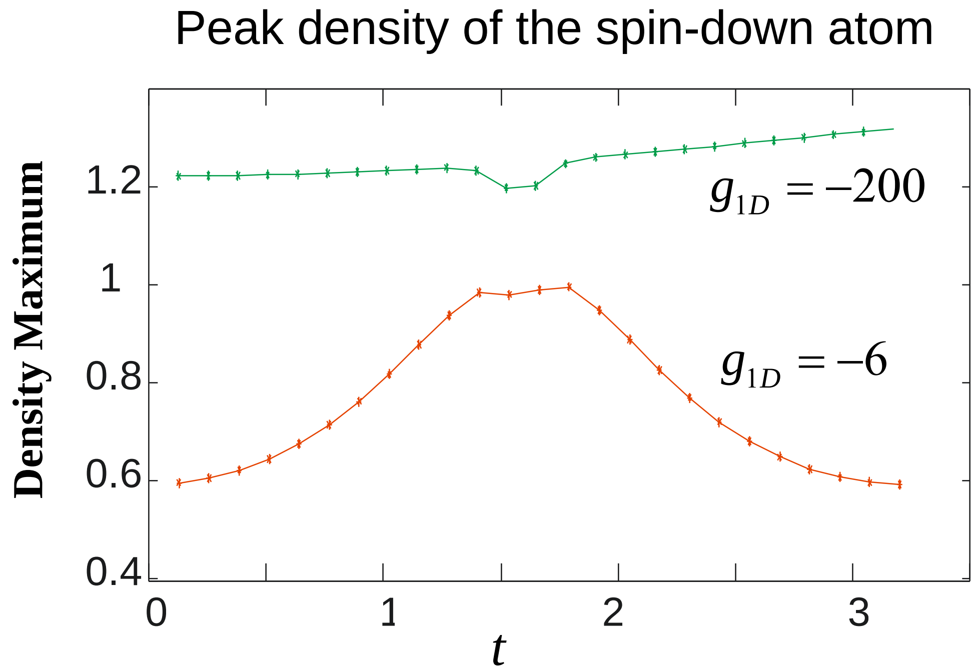




$g_{1D} = -6$ $g_{1D} = -200$

# Atom-pair interaction: a spin transport probe



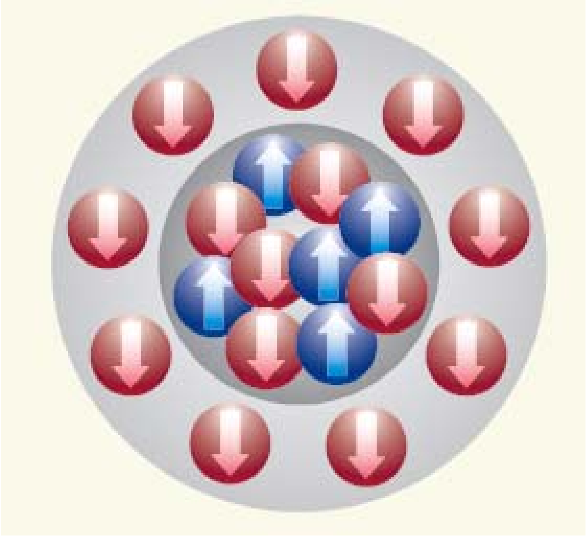



**Weakly interacting case: enhanced attractive interaction between the incoming** spin$-\uparrow$ **and the** spin$-\downarrow$ **particle.**

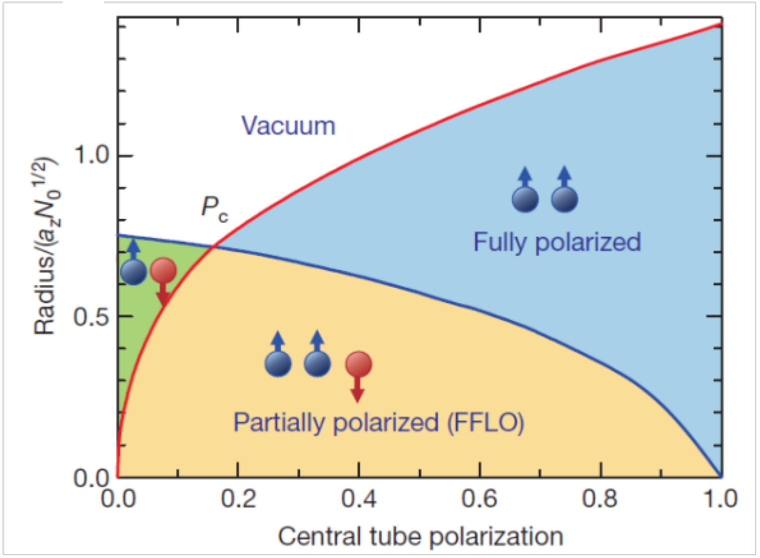
**Strongly interacting case: weakly repulsive interaction is between the incoming** spin$-\uparrow$ **and the bosonic molecule.**

# From 1D to 3D: a BdG study

*3D*

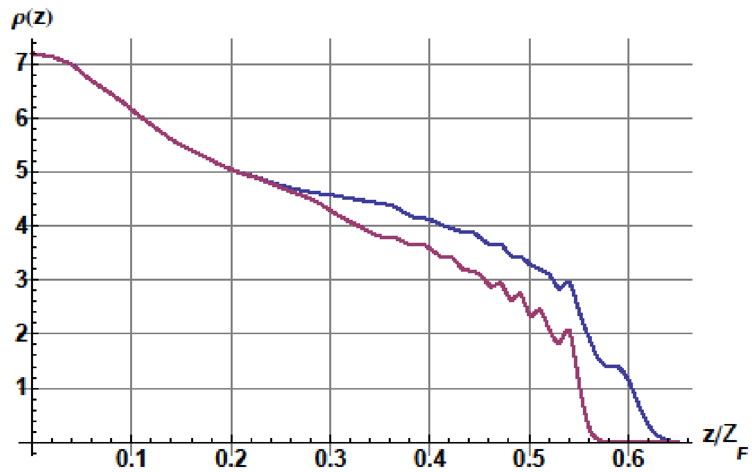


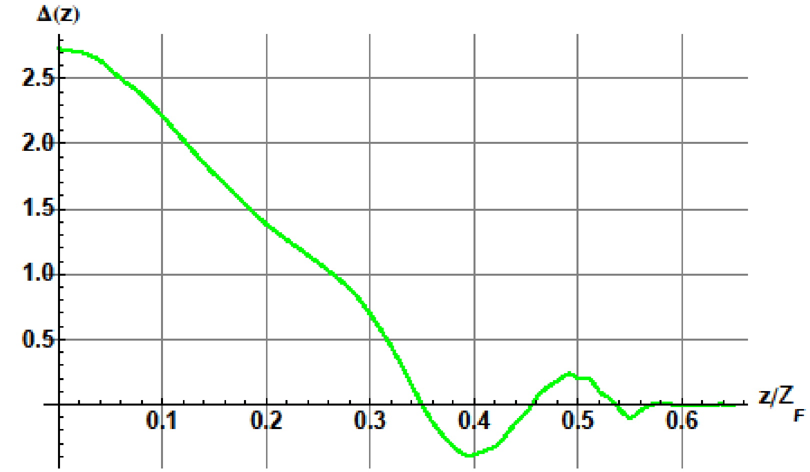
*1D*



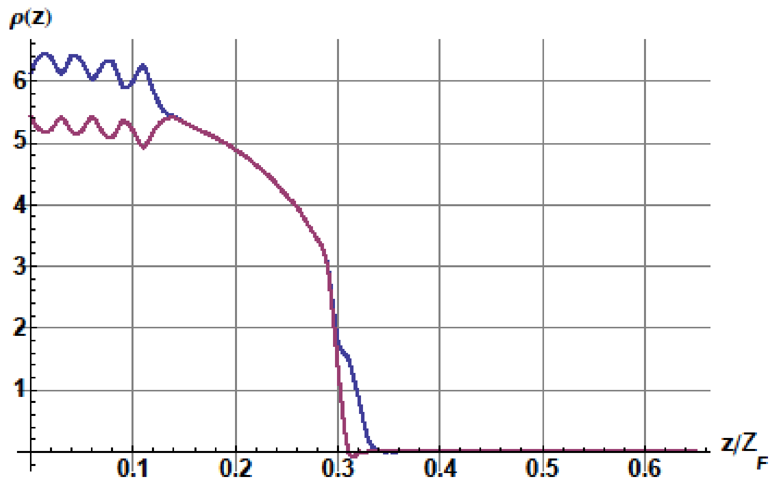
# From 1D to 3D: a BdG study

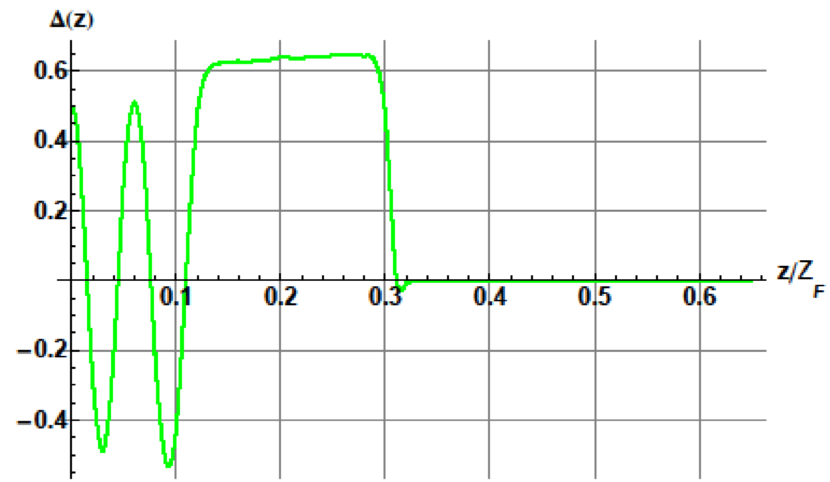
3D




1D

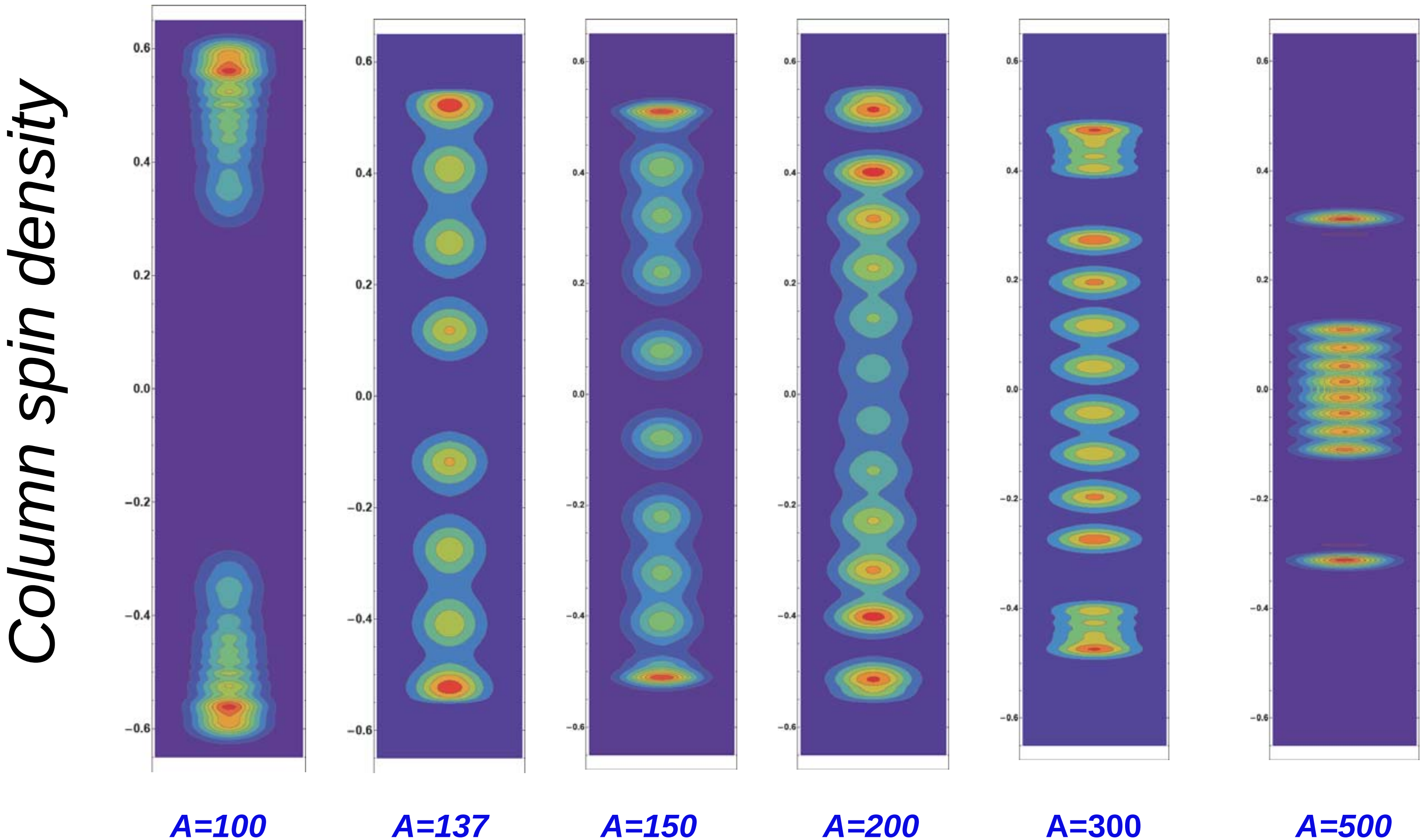



***Density Profiles***

***Gap***

# From 1D to 3D: a BdG study




$$V(r,z)=\frac{1}{2}m\left(\omega_r^2(x^2+y^2)+\omega_z^2 z^2\right)$$

$$A=\omega_r/\omega_z$$

# Summary

- Cold atoms are ideal platform to study dimensional effects
- Lower spatial dimensions → exotic quantum phases
- Equilibrium and non-equilibrium physics can be probed

Mean-field theory may provide qualitatively correct answers even at low D.

Fermions are interesting.

# Acknowledgement


- Leslie Baksmaty
- Hong Lu
- Carlos Bolech
- Randy Hulet