# Time average on the numerical integration of non-autonomous differential equations

Sergio Blanes

IMM, Universitat Politècnica de València, serblaza@imm.upv.es

**Abstract**

Given the non-autonomous differential equations

$$x' = f(t, x), \qquad x(0) = x_0 \in \mathbb{C}^d, \tag{1}$$

in this talk we show how to obtain an associated equation

$$y' = \widetilde{f}(t, y), \qquad y(0) = x_0, \tag{2}$$

where $\widetilde{f}(t, y)$ is a polynomial function of $t$ of degree $s-1$ such that

$$\|x(h) - y(h)\| = \mathcal{O}(h^{2s+1}), \tag{3}$$

and the coefficients of the polynomial depend linearly on $f(c_i h, y)$ where $c_i,\ i = 1, \ldots, \hat{s}$ are the nodes of any desired quadrature rule of order $p \geq 2s$. Eq. (4) has the same algebraic structure as eq. (1), so $y(t)$ shares most qualitative properties with $x(t)$ [1] and, for many problems, its numerical integration can be carried more efficiently. We also show how to obtain an autonomous equation

$$y' = \widehat{f}(y), \qquad y(0) = x_0, \tag{4}$$

such that the solution of (4) at $t = h$ satisfies (3), or the sequence

$$z^{[i]\prime} = \widehat{f}_i(z^{[i]}), \qquad z^{[i]}(0) = z^{[i-1]}(h), \tag{5}$$

$i = 1, \ldots, k$, with $z^{[0]}(h) = x_0$, $\widehat{f}_i(z^{[i]})$ is a linear combination of $f(c_i h, y)$, and such that $z^{[k]}(h) = x(h) + \mathcal{O}(h^{2s+1})$ (commutator-free quasi-Magnus integrators [2]). We show how to use these techniques to numerically solve the linear and non-linear Schrödinger equations with explicitly time dependent Hamiltonian.

## References

[1] S. Blanes and F. Casas, *A Concise Introduction to Geometric Numerical Integration*, CRC Press, Boca Raton, 2016.

[2] S. Blanes, F. Casas, M. Thalhammer, High-order CFQM exponential integrators for non-autonomous linear evolution equations. Submitted.