WPI 2012
Lagrange vs Euler

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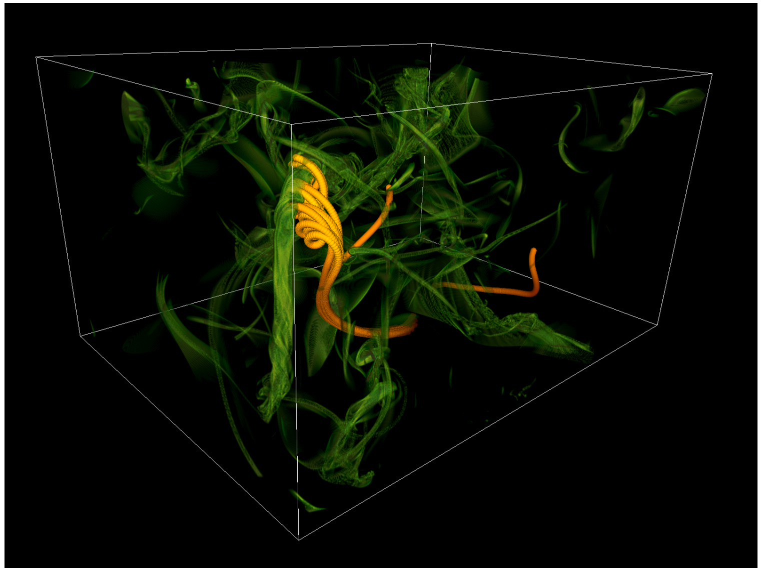
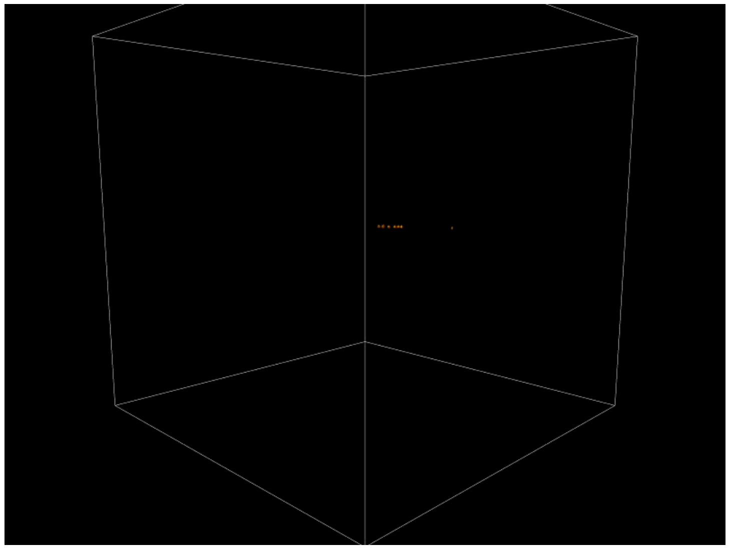


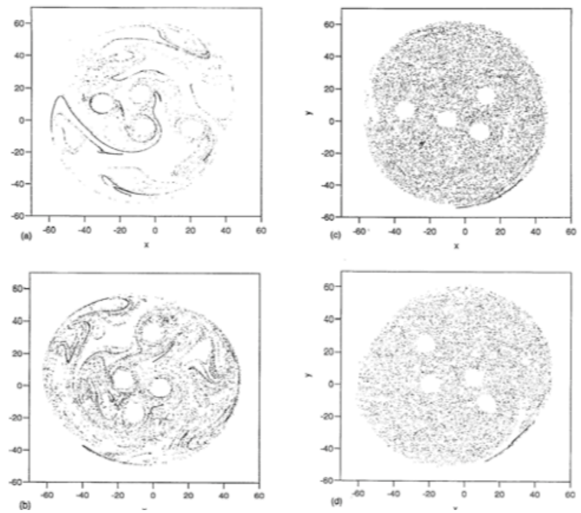
Lagrangian point sources

Eulerian



Lagrangian-Eulerian

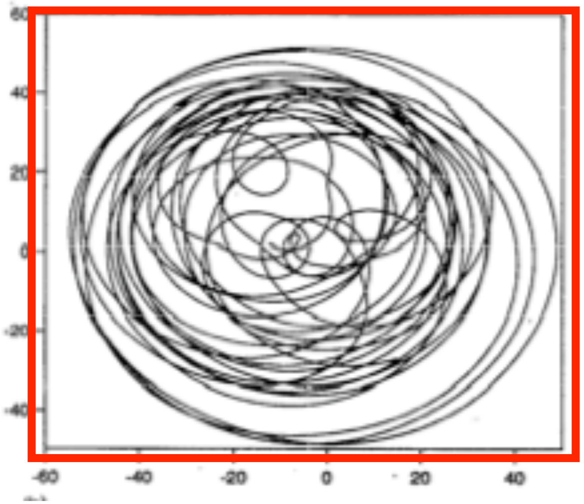
Point-vortices + tracers



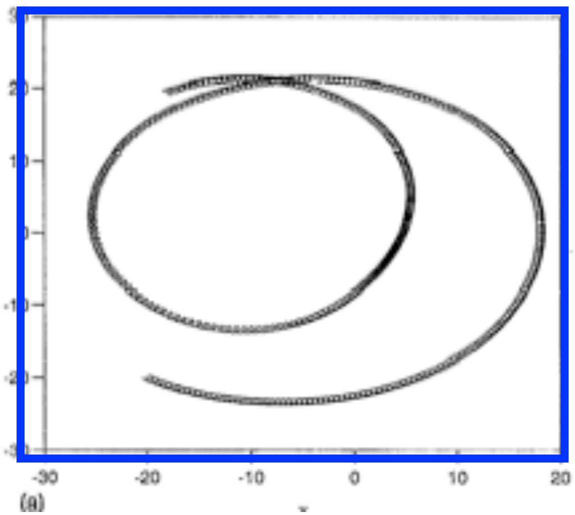
If N > 4 the system in not integrable

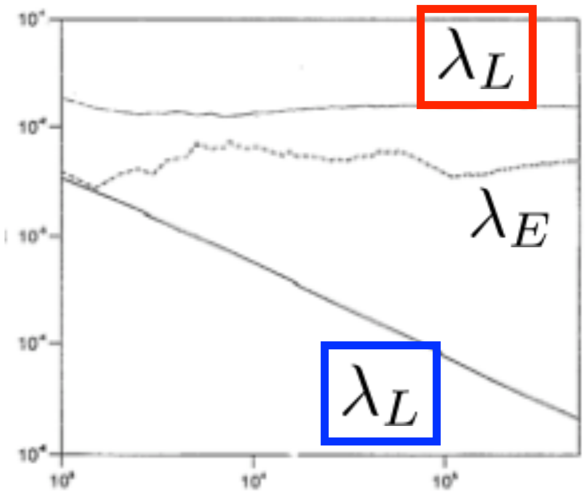
$$\lambda_E > 0$$

Tracers far from vortex



Tracers close to a vortex

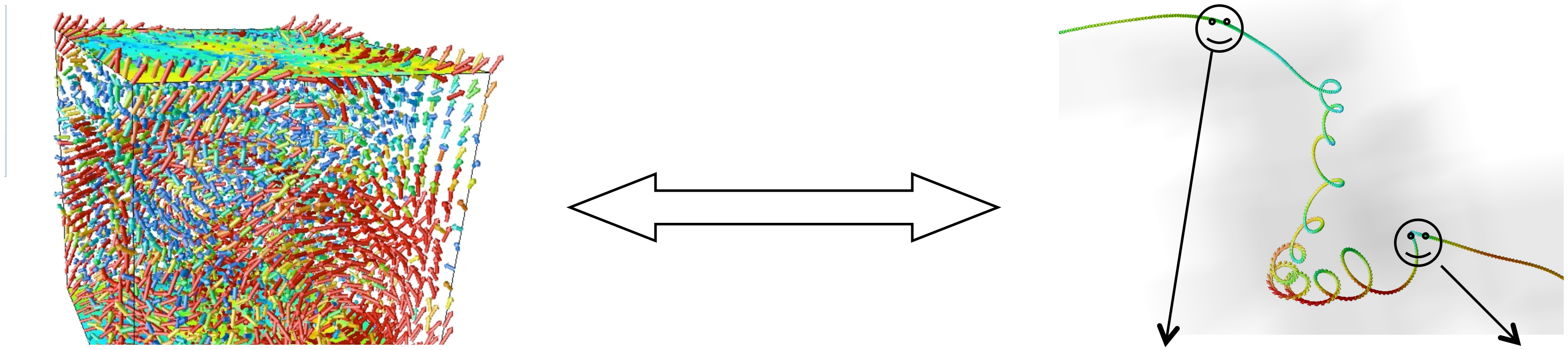




Lyapunov exponents

EULERIAN

LAGRANGIAN



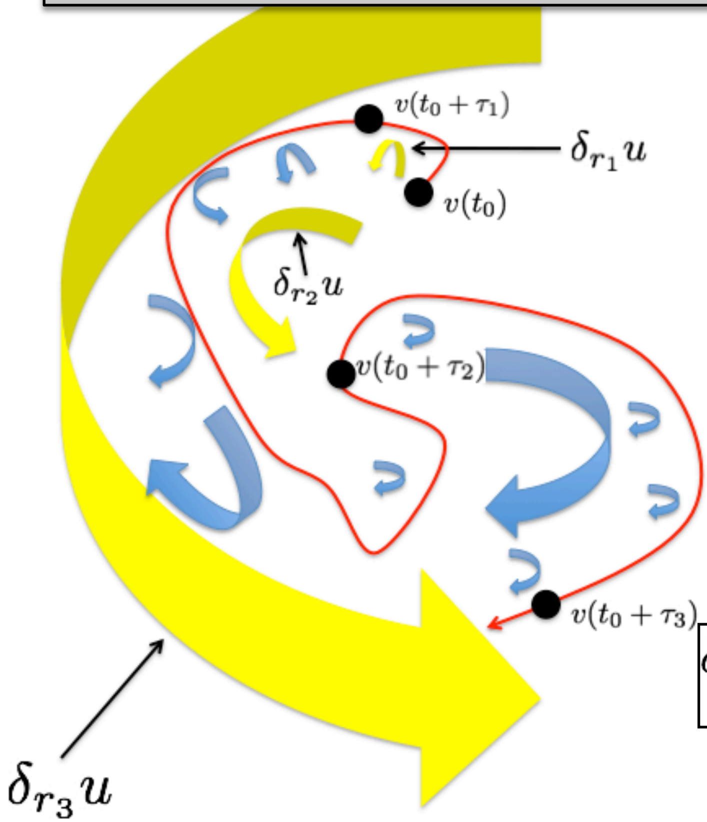
$$\delta_r \mathbf{u} = [\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})] \qquad \delta_\tau \mathbf{v} = [\mathbf{u}(\mathbf{x}(t+\tau)) - \mathbf{u}(\mathbf{x}(t))]$$

$$\begin{cases} \delta_r u_L = [\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})] \cdot \hat{\mathbf{r}} \\ \delta_r u_T = [\mathbf{u}(\mathbf{x}+\mathbf{x}) - \mathbf{u}(\mathbf{x})] \cdot \hat{\mathbf{n}} \end{cases}$$

OCCAM'S RAZOR

"Entia non sunt multiplicanda praeter necessitatem"

(Entities should not be multiplied unnecessarily)



BRIDGE RELATION

$$\delta_r u \sim \delta_\tau v$$

$$\tau \sim r/\delta_r u$$

$$\delta_\tau v = \sum_n \delta_{r_n} u \, e^{-\tau/\tau_{r_n}}$$

$$\tau \sim r^{2/3}$$

K41

$$\gamma(k) = k^{2/3}$$

- Locality of interactions (problem for 2D turbo)
- only one time scale (problem for MHD)

Intermittency - Eulerian

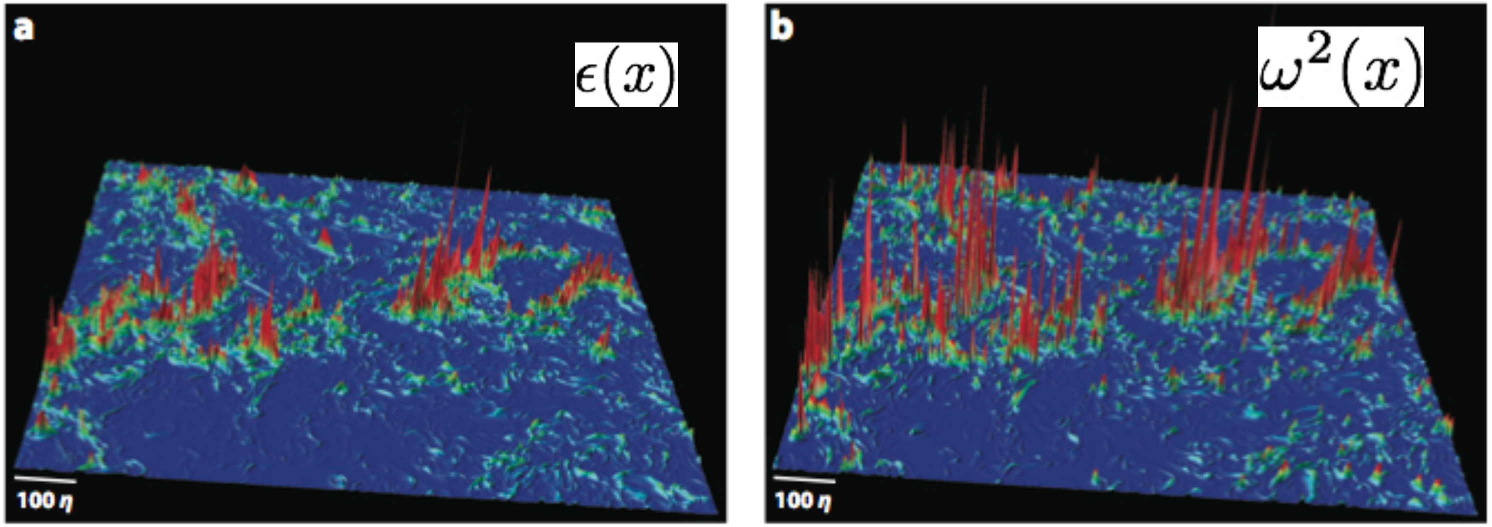
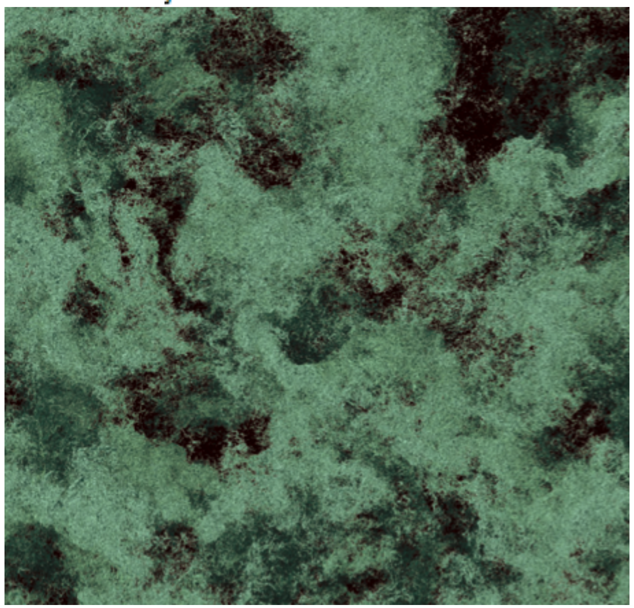


Figure 4

Snapshot of the intensity distributions of (*a*) the energy-dissipation rate $\tilde{\varepsilon} = \varepsilon/(2\nu)$ and (*b*) the enstrophy $\Omega = \omega^2/2$ on a cross section in DNS-ES at $R_\lambda = 675$ in arbitrary units.



Study of High–Reynolds Number Isotropic Turbulence by Direct Numerical Simulation

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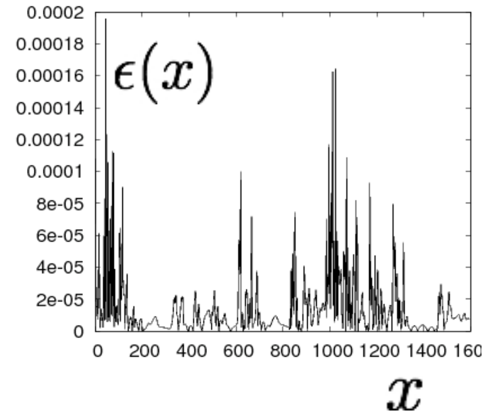
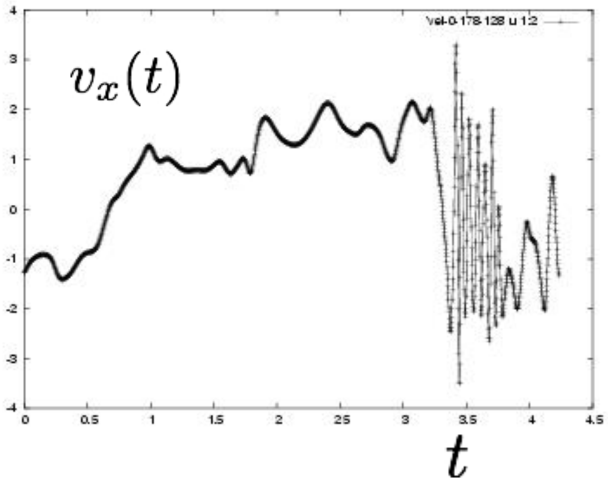
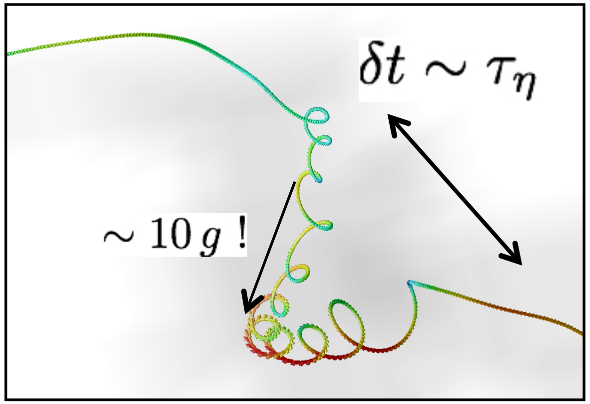
[2]Department of Scientific and Engineering Simulation, Graduate School of Engineering, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan



Intermittency - Lagrangian





vortex trapping

PHYSICS OF FLUIDS 17, 021701 (2005)

Particle trapping in three-dimensional fully developed turbulence

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Lagrangian Properties of Particles in Turbulence

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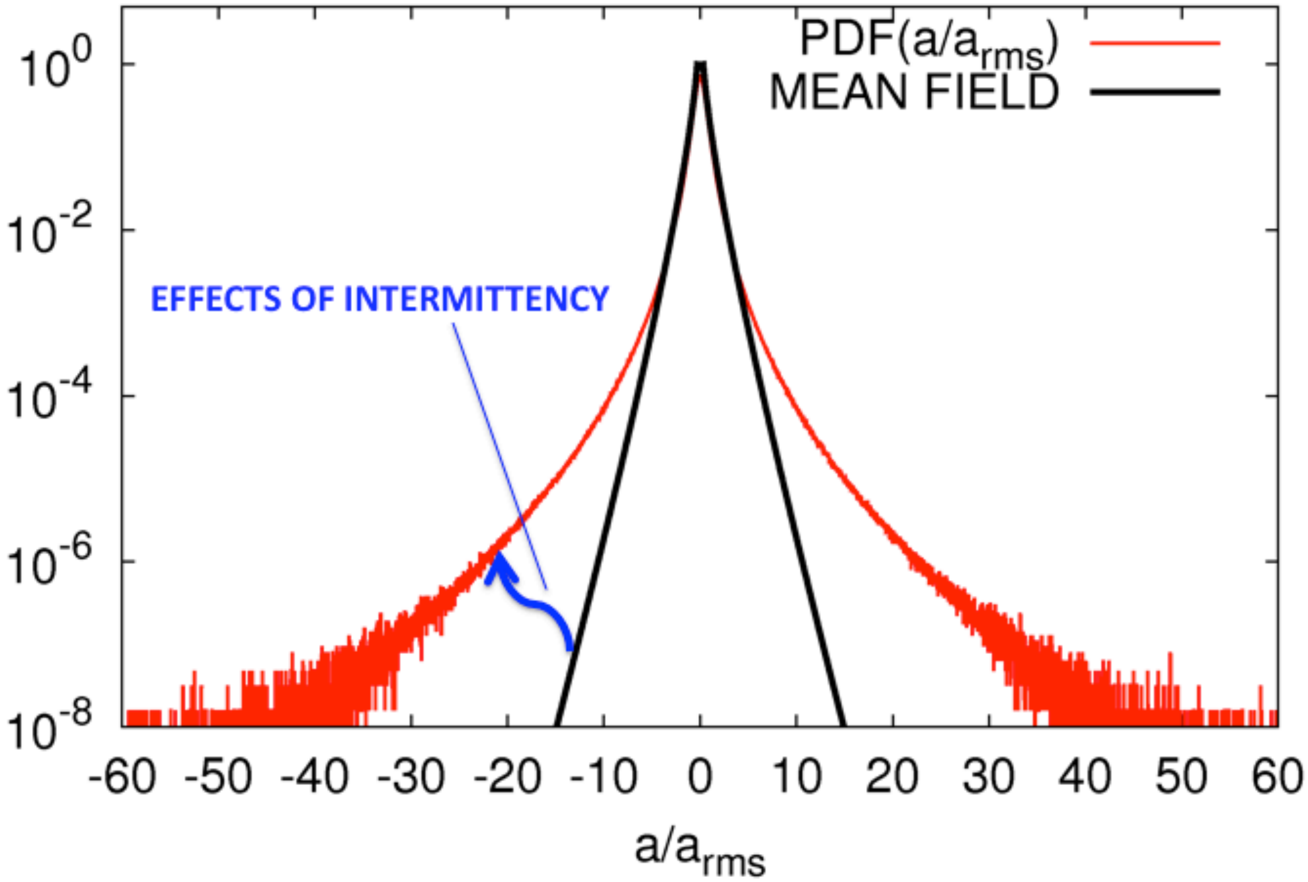
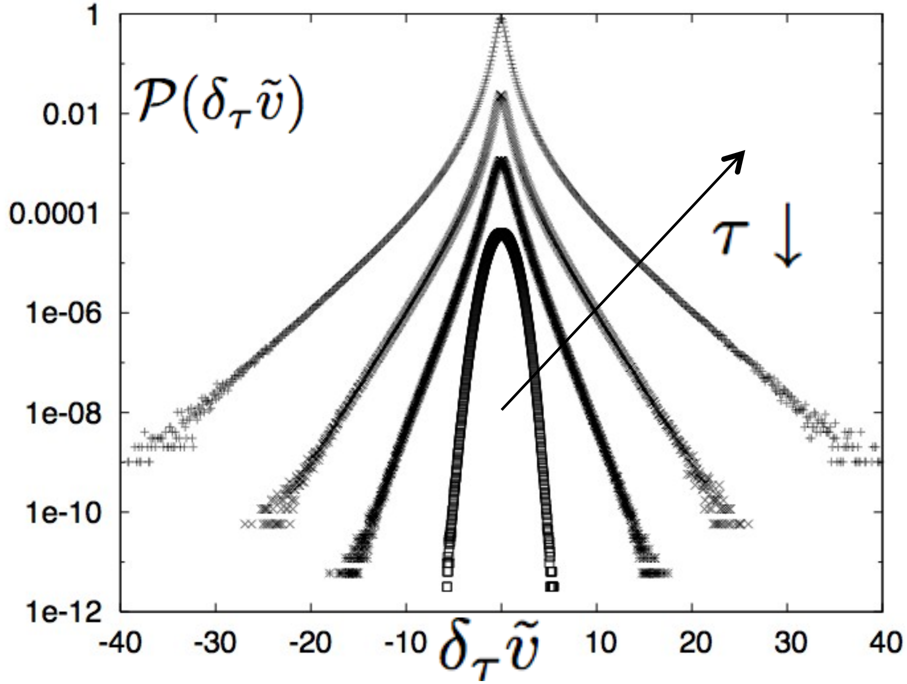
[2]Max Planck Institute for Dynamics and Self-Organization, D-37077 Goettingen, Germany; Laboratory of Atomic and Solid-State Physics and Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York 14853; Institute for Nonlinear Dynamics, University of Goettingen, D-37073 Goettingen, Germany; and International Collaboration for Turbulence Research



ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04]
COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

IF YOU WANT TO PREDICT EXTREME EVENTS YUO **CANNOT** NEGLECT INTERMITTENCY

EFFECTS OF INTERMITTENCY (II)



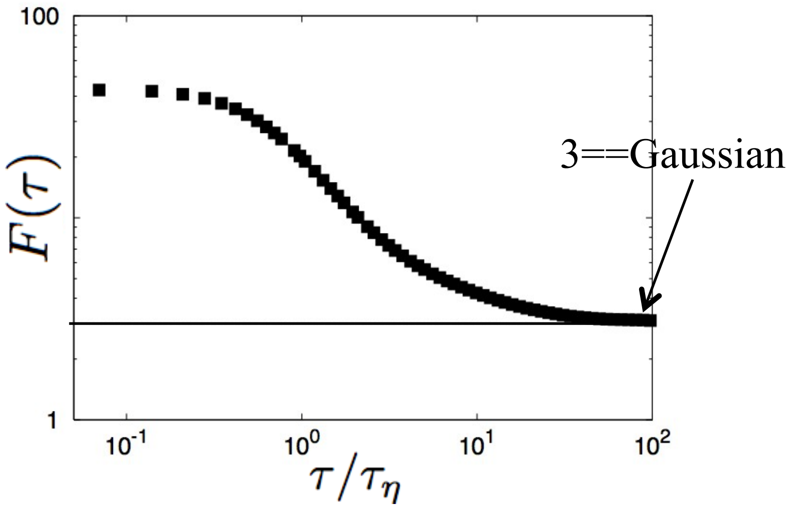
[DNS $Re_l = 600$; R. Benzi, L.B. R. Fischer, D. Lamb,L. Kadanoff, F. Toschi] PRL 100, 234503 2008.

$$\delta_\tau \tilde{v} = \frac{\delta_\tau v}{\langle (\delta_\tau v)^2 \rangle^{1/2}}$$

2048^3

16 Mega particles

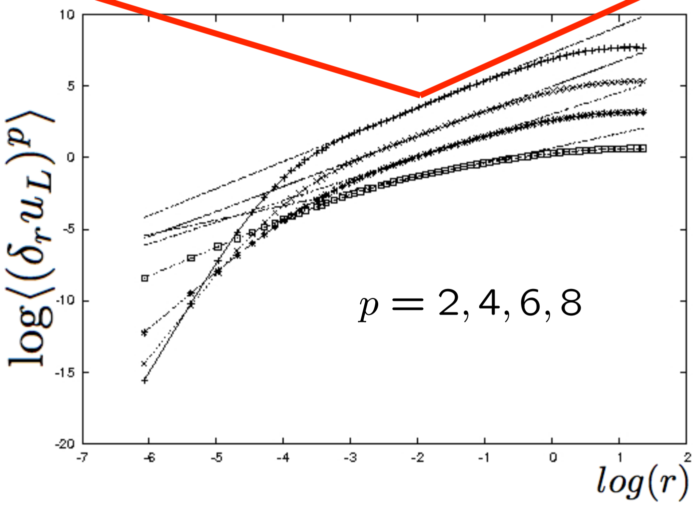
$$F(\tau) = \frac{\langle (\delta_\tau v)^4 \rangle}{\langle (\delta_\tau v)^2 \rangle^2}$$



EULERIAN

$$\zeta_L(p,r) \overset{\text{DEF}}{=} \frac{d\,\log\langle(\delta_r u_L)^p\rangle}{d\log r}$$

MAGNIFYING GLASS



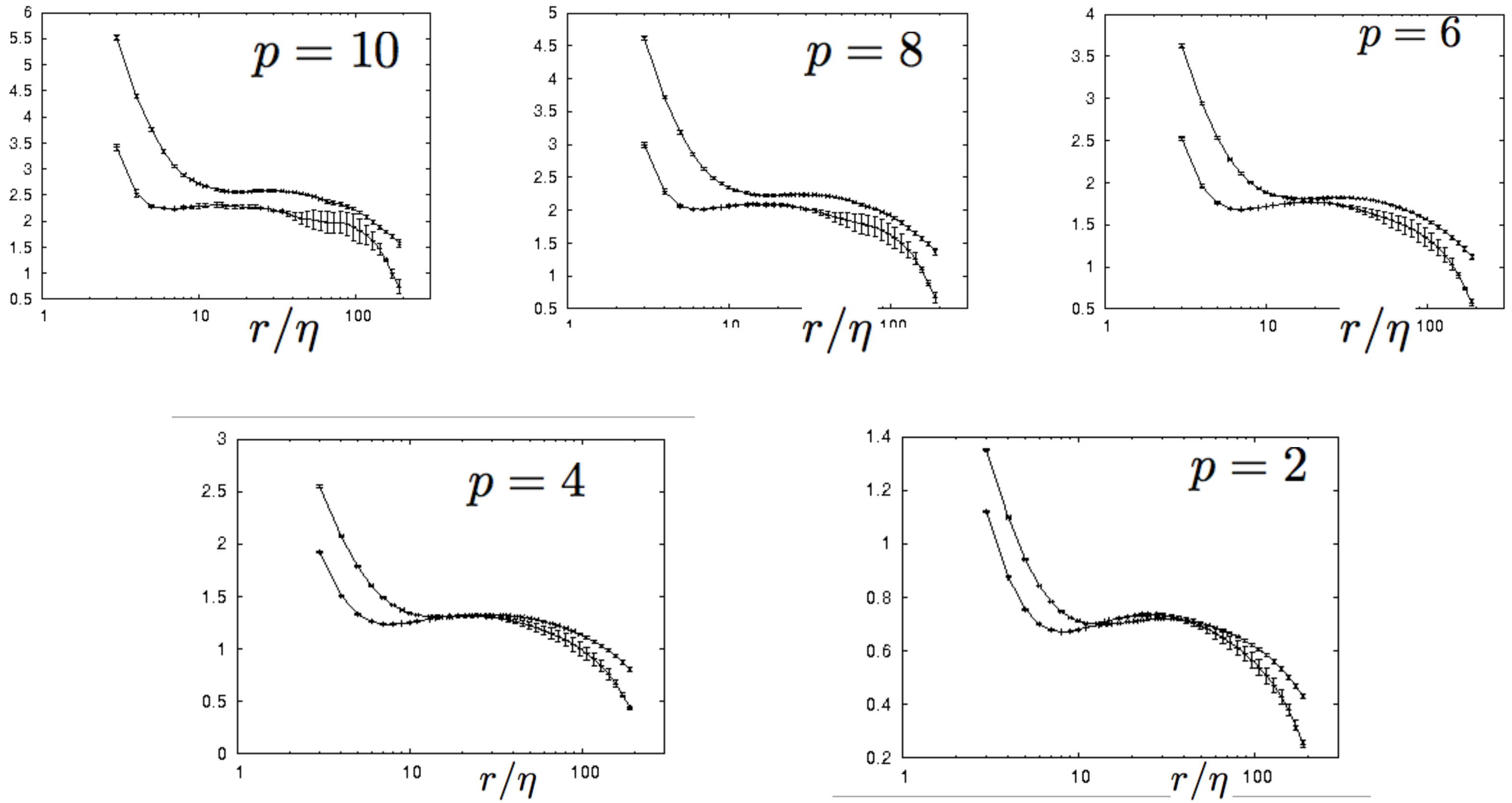
IN LOG-LOG ALL COWS ARE BLACK!

EULERIAN STATISTICS: LONGITUDINAL VS TRANSVERSE

$$S_L^{(p)}(r) = \langle (\delta_r u_L)^p \rangle \qquad S_T^{(p)}(r) = \langle (\delta_r u_T)^p \rangle$$

$$\zeta_L(p,r) = \frac{d \log \langle (\delta_r u_L)^p \rangle}{d \log r} \qquad \zeta_T(p,r) = \frac{d \log \langle (\delta_r u_T)^p \rangle}{d \log r}$$

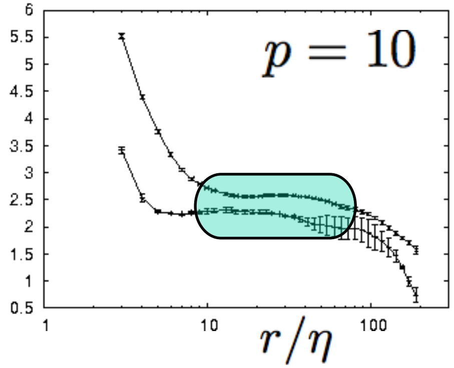
LOCAL SLOPES: LONGITUDINAL AND TRANSVERSE:

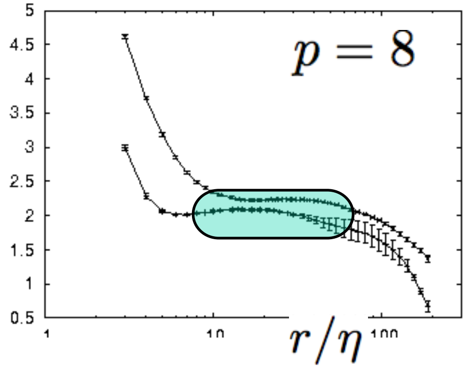


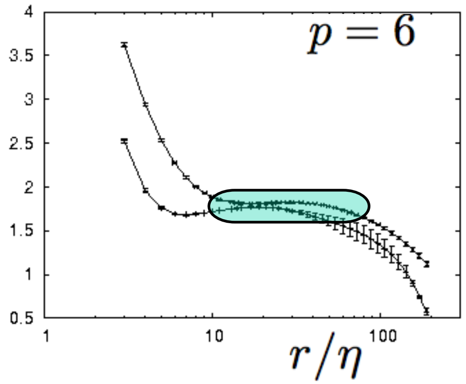
INERTIAL RANGE : LONGITUDINAL VS TRANSVERSE

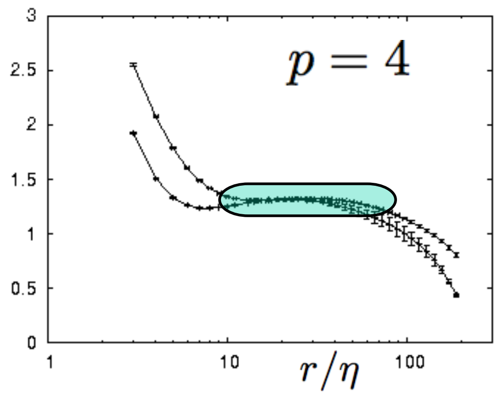
$$\zeta_L(p,r) \rightarrow \zeta_L(p) \qquad\qquad \zeta_T(p,r) \rightarrow \zeta_T(p)$$

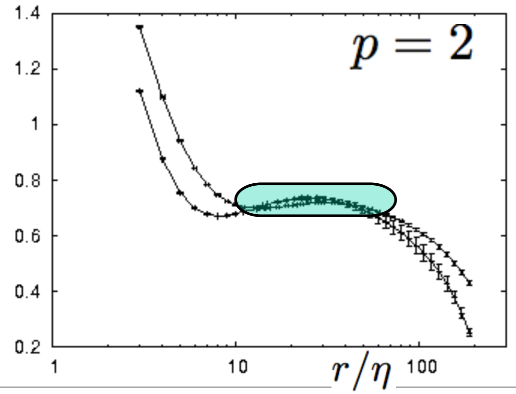
LOCAL SLOPES: LONGITUDINAL AND TRANSVERSE:











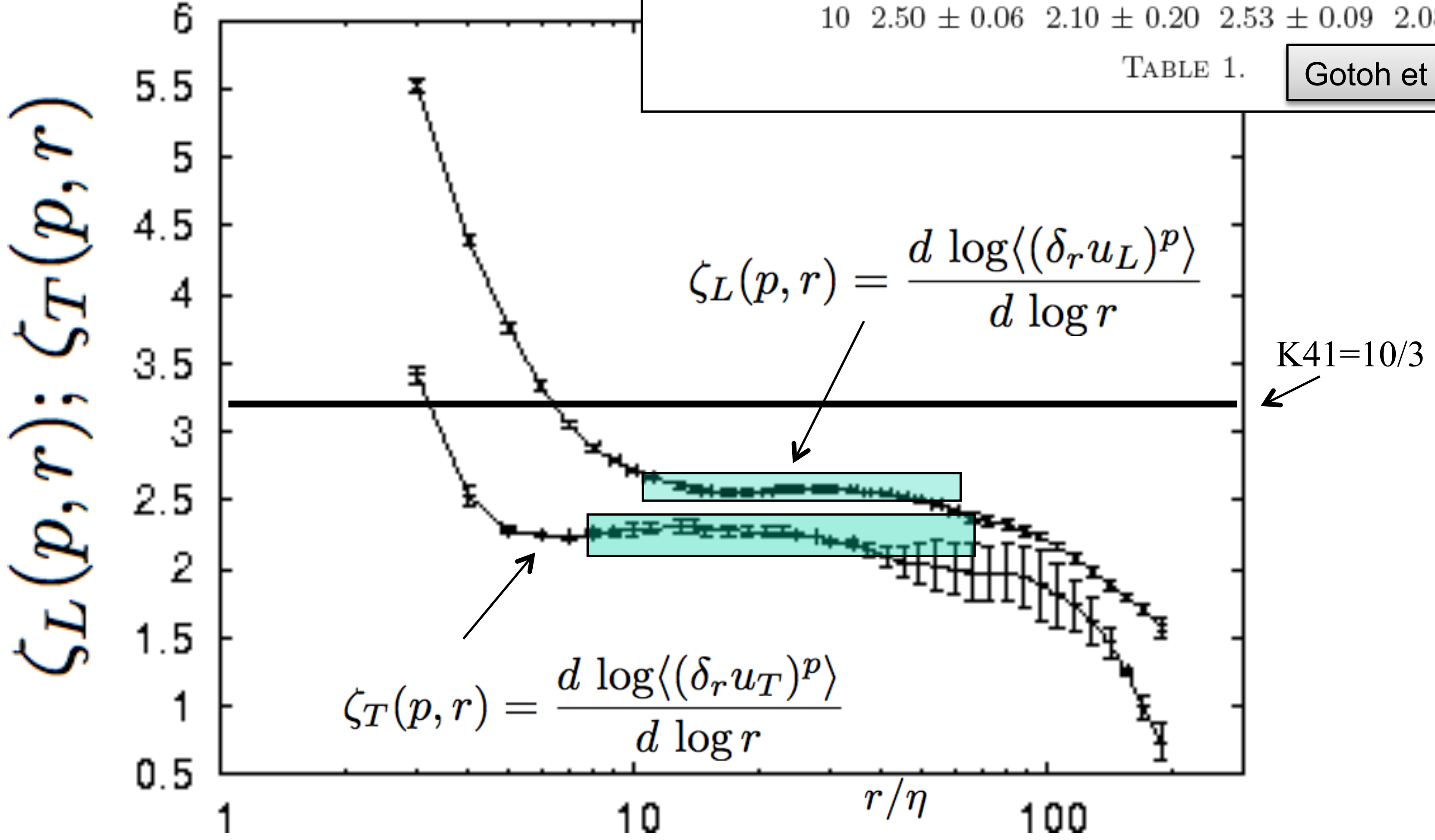
R. Benzi, L.B. R. Fischer, L. Kadanoff, D. Lamb, F. Toschi PRL 100, 234503 2008.

R. Benzi, L. B., R. Fisher D. Lamb and F. Toschi, JFM 653, p. 221 (2010).

p	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$ ref. []	$\zeta_T^{(p)}$ ref. []
2	0.71 ± 0.02	0.71 ± 0.02	0.70 ± 0.01	0.71 ± 0.01
4	1.29 ± 0.03	1.27 ± 0.05	1.29 ± 0.03	1.26 ± 0.02
6	1.78 ± 0.04	1.68 ± 0.06	1.77 ± 0.04	1.67 ± 0.04
8	2.18 ± 0.05	1.92 ± 0.10	2.17 ± 0.07	1.93 ± 0.09
10	2.50 ± 0.06	2.10 ± 0.20	2.53 ± 0.09	2.08 ± 0.18

TABLE 1.

Gotoh et al. (PoF 2002)



LONGITUDINAL AND TRANSVERSE SCALE DIFFERENTLY

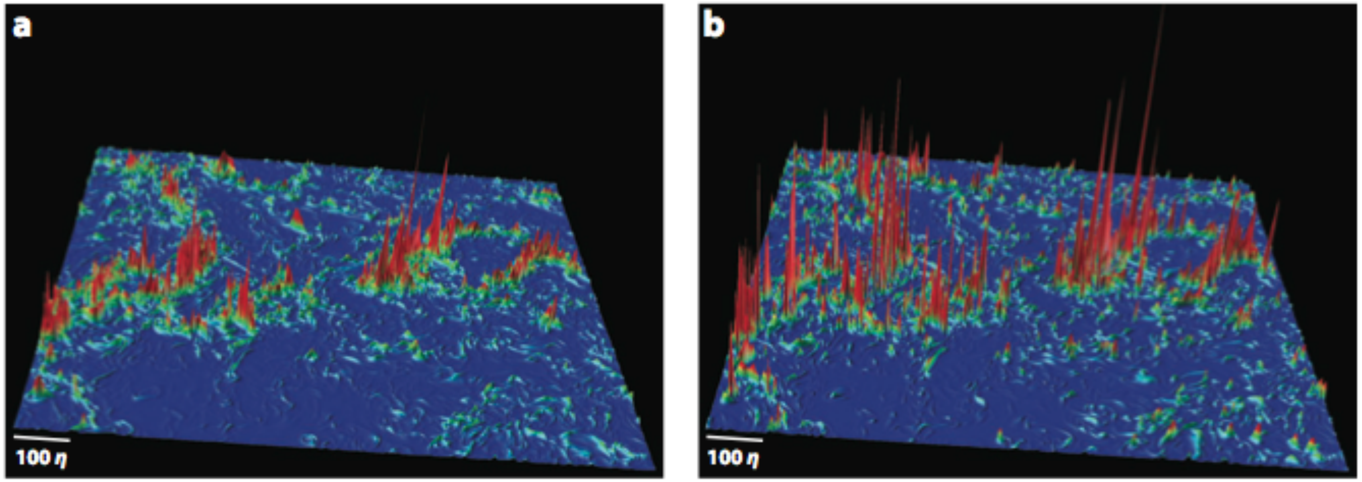
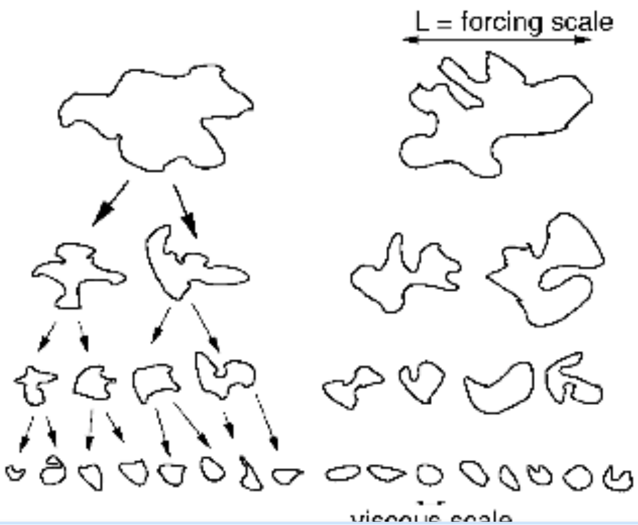


Figure 4

Snapshot of the intensity distributions of (*a*) the energy-dissipation rate $\bar{\varepsilon} = \varepsilon/(2\nu)$ and (*b*) the enstrophy $\Omega = \omega^2/2$ on a cross section in DNS-ES at $R_\lambda = 675$ in arbitrary units.



CASCADE PROCESS -> LARGE DEVIATIONS -> MULTIFRACTAL MEASURE OR MULTIAFFINE SIGNALS

EULERIAN

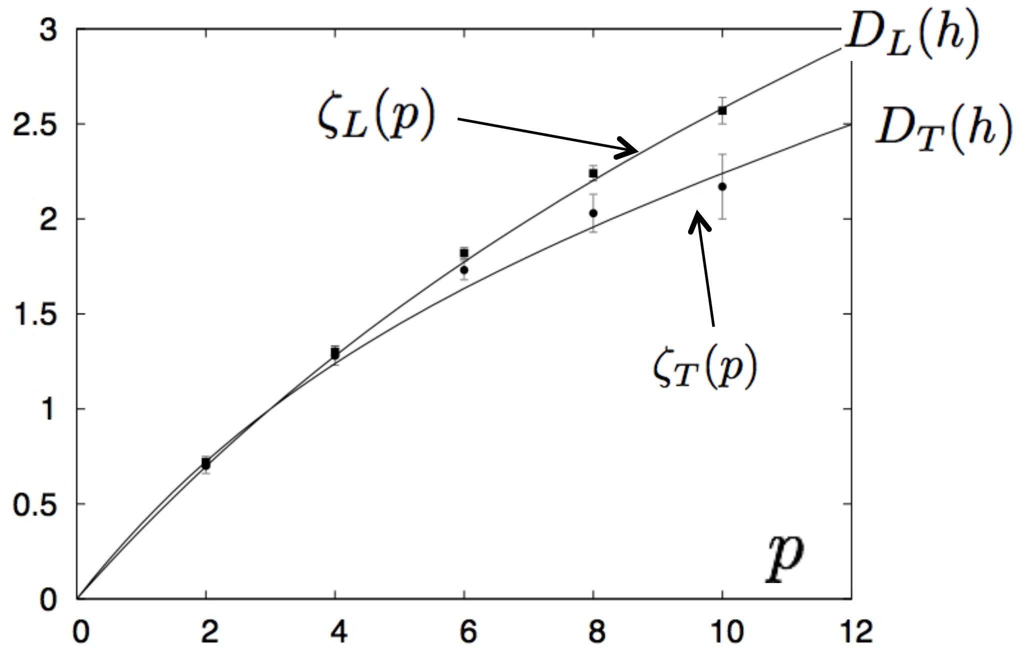
USE TWO DIFFERENT MULTIFRACTAL D(H) TO FIT SEPARATELY
LONGITUDINAL AND TRANSVERSE EULERIAN SCALING

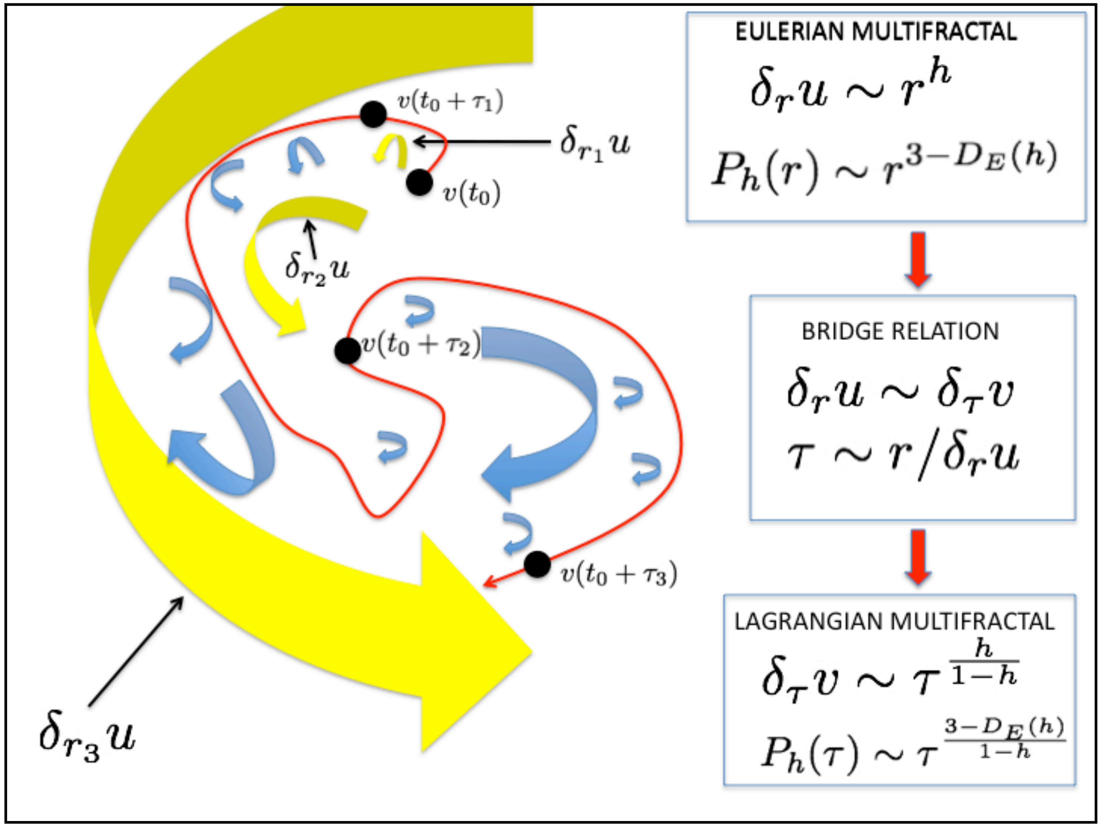
$$\delta_r u_L \sim r^h; \mathcal{P}_h(r) \sim r^{3-D_L(h)}$$
$$S_L^{(p)} \sim r^{\zeta_L(p)}$$

$$\delta_r u_T \sim r^h; \mathcal{P}_h(r) \sim r^{3-D_T(h)}$$
$$S_T^{(p)} \sim r^{\zeta_T(p)}$$

$$\zeta_{L,T}(p) = \min_h [ph + 3 - D_{L,T}(h)]$$

Parisi & Frisch (1983)





Borgas (1993)

$$\langle(\delta_r u)^3\rangle \propto \epsilon r \quad \longleftrightarrow \quad \langle(\delta_\tau v)^2\rangle \propto \epsilon \tau$$

$$\begin{cases} S_L^{(p)}(r) = \langle (\delta_r u_L)^p \rangle \\ S_T^{(p)}(r) = \langle (\delta_r u_T)^p \rangle \end{cases}$$

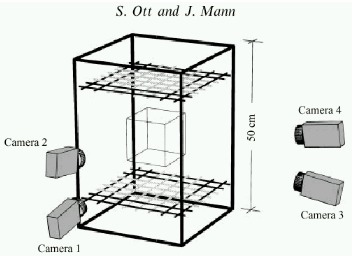
$$\zeta_{L,T}(p) = \min_h [ph + 3 - D_{L,T}(h)]$$

SAME!!!!!

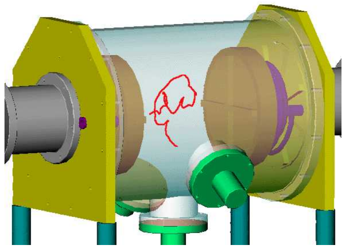
$$S_{Lag}^{(p)}(\tau) = \langle (\delta_\tau v)^p \rangle \sim \tau^{\zeta_{Lag}(p)}$$

$$\zeta_{Lag}(p) = \min_h [\frac{ph + 3 - D_{L,T}(h)}{1 - h}]$$

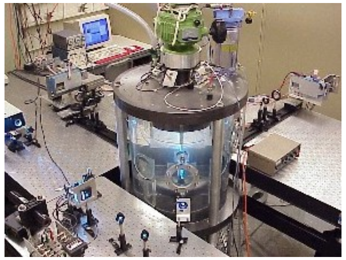
LAGRANGIAN



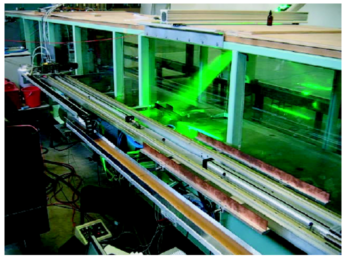
Ott and Mann experiment at Risø
conventional 3D PTV –
Re~100-300



Pinton et al ENSL
Acoustic/Laser Doppler tracking -
Re ~800 (single particle tracking)



Bodenschatz et al at Cornell-MPI
silicon strip detectors (now also CCD) Re ~ 1000-1500



Warhaft et al experiment at Cornell
Fast moving camera
Re~ 300

Luthi, Tsinober et al
3D PTV and 3D scanning PTV for velocity gradients

and many others….
F. Toschi & E. Bodenschatz ARFM **41**, 375 (2009)

non intrusive tracking down to

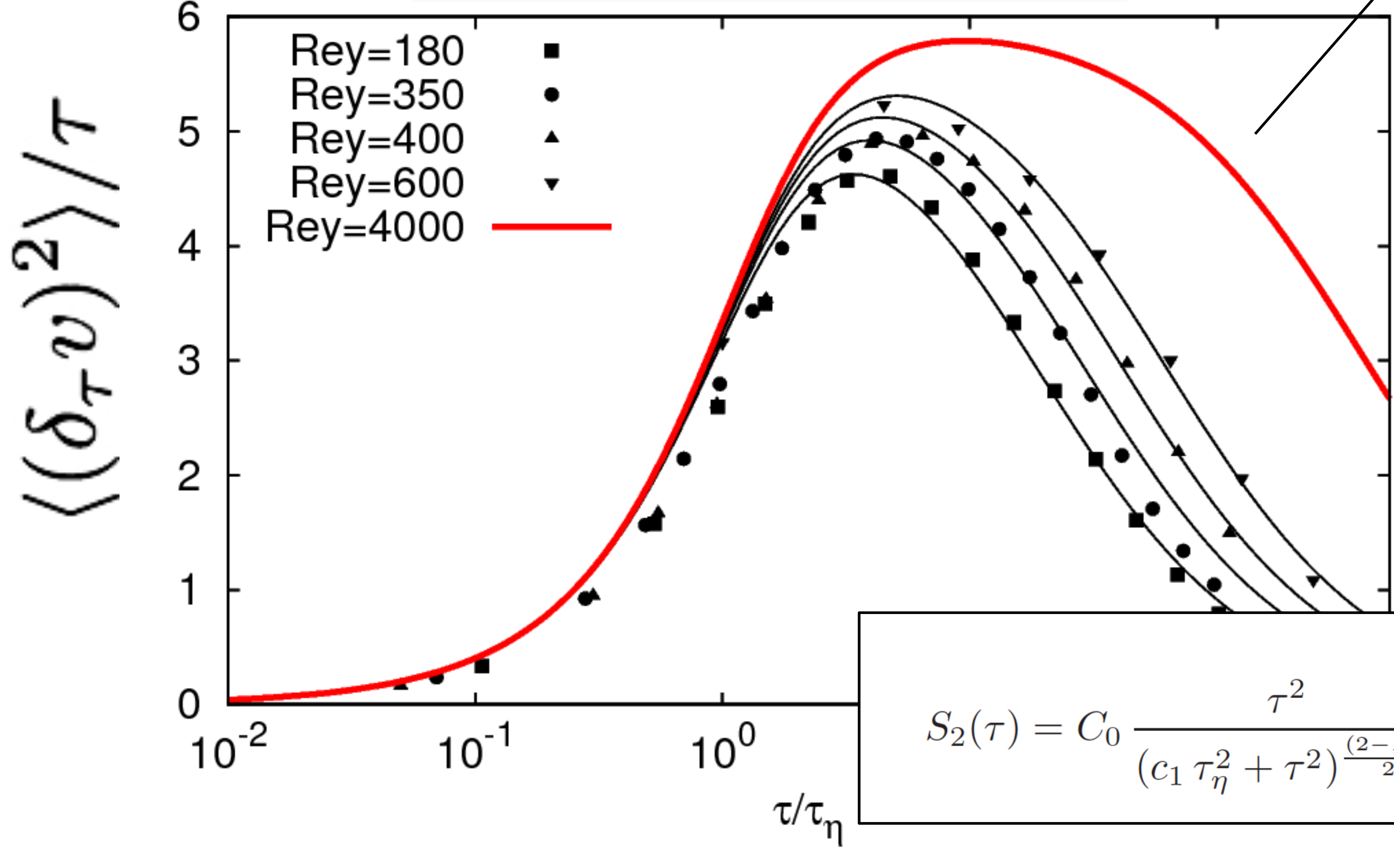
$$\tau \sim \tau_\eta$$

LAGRANGIAN

$$\langle (\delta_\tau v)^p \rangle \sim \epsilon^{p/2} \tau^{p/2}$$

Batchelor parametrization

L.B. and A. Lanotte NSF-ITP-11-103 (2001)

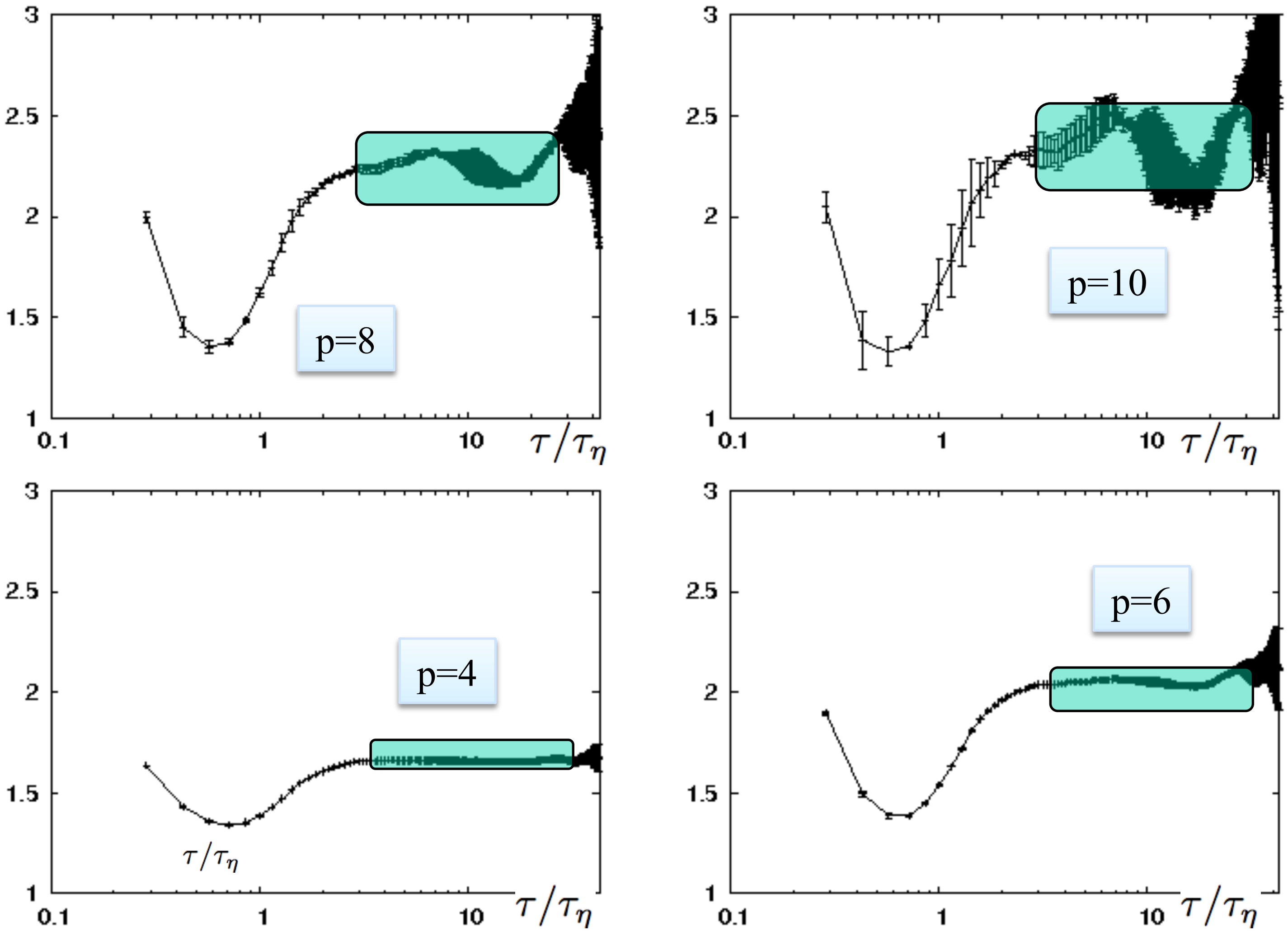


$$S_2(\tau) = C_0 \, \frac{\tau^2}{(c_1 \, \tau_\eta^2 + \tau^2)^{\frac{(2-z_2)}{2}}} (1 + c_3 \tau / T_L)^{-z_2} \, ,$$

B. Sawford and P.K. Yeung PoF **23**, 091704 (2011) & NEXT TALK by Eberhard

R. Benzi, L. B., R. Fisher D. Lamb and F. Toschi, JFM 653, p. 221 (2010).

$$\chi_\tau(p) = \frac{d\,\log\langle(\delta_\tau v)^p\rangle}{d\,\log\langle(\delta_\tau v)^2\rangle} \qquad \chi_\tau(p) \rightarrow \chi(p)$$



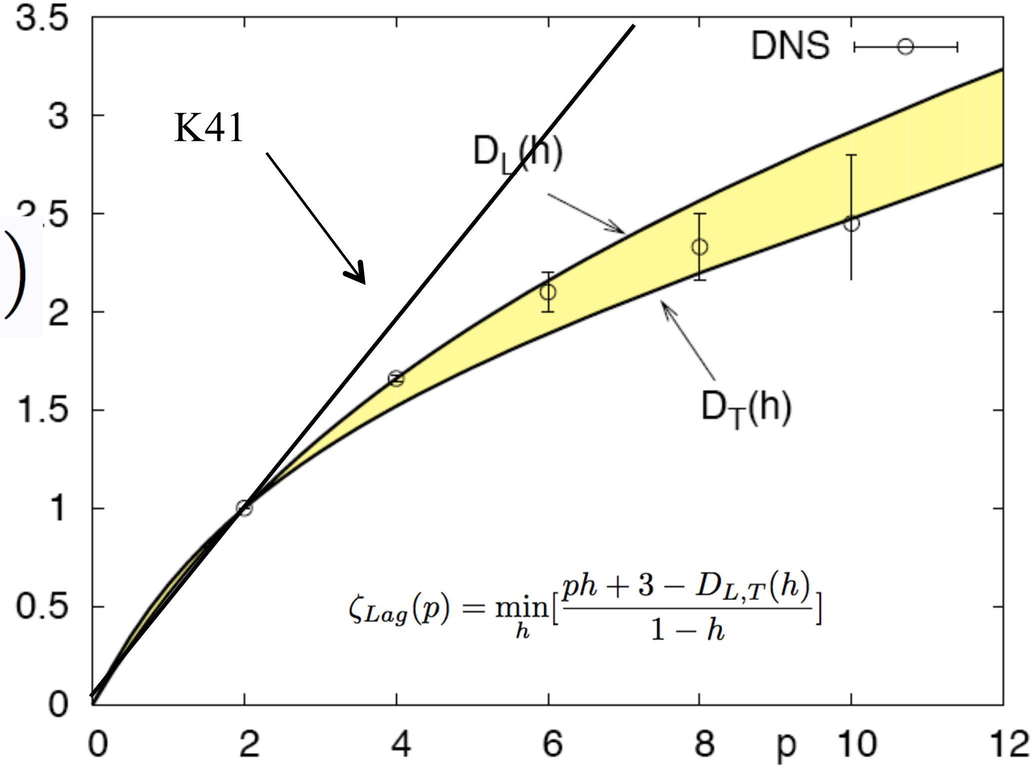
INFINITELY-MANY ANOMALOUS SCALING EXPONENTS
(MULTIFRACTAL FIELD, Parisi & Frisch, 1983)

$$S_p(\tau) = \langle (v(t+\tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)}$$

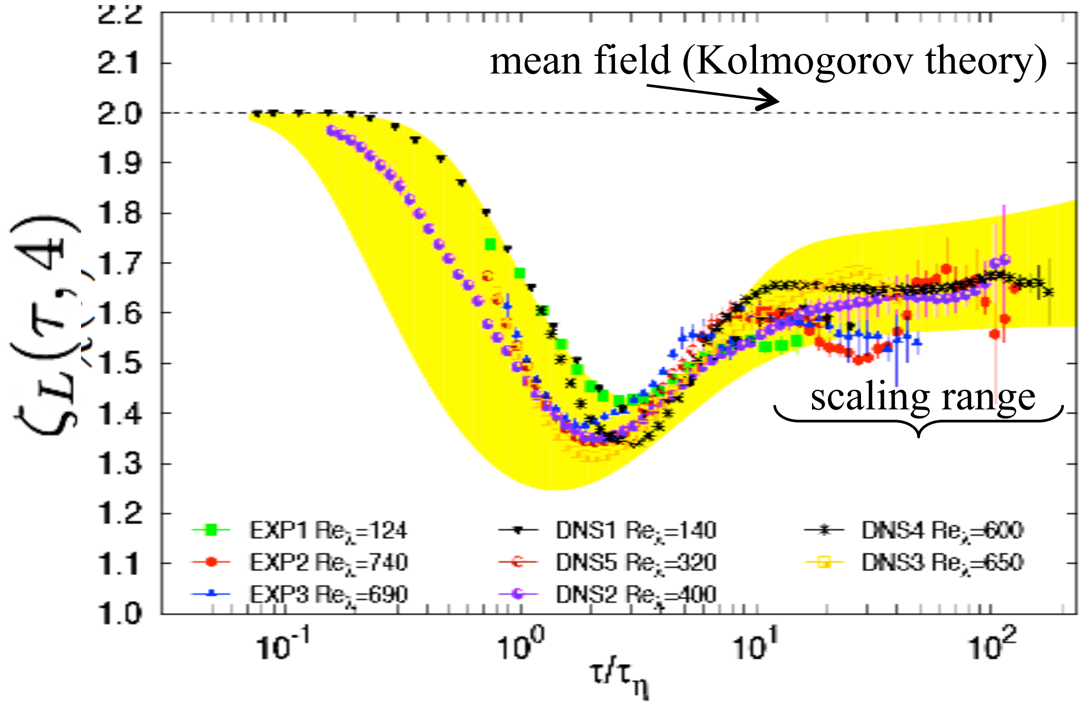
in the scaling range:

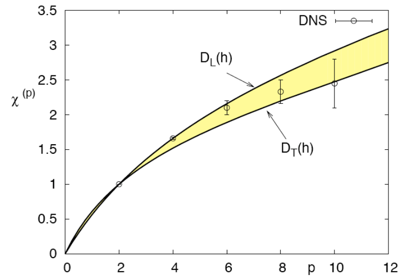
$$\zeta_L(\tau, p) \longrightarrow \zeta_L(p)$$

$\zeta_L(p)$



$$F(\tau) = \langle(\delta_\tau v)^2\rangle^{(\zeta_L(\tau,4)-2)}$$





International Collaboration for Turbulence Research, A. Arneodo,[1] J. Berg,[2] R. Benzi,[3] L. Biferale,[3] E. Bodenschatz,[4] A. Busse,[5] E. Calzavarini,[6] B. Castaing,[1] M. Cencini,[7] L. Chevillard,[1] R. Fisher,[8] R. Grauer,[9] H. Homann,[9] D. Lamb,[8] A.S. Lanotte,[10] E. Leveque,[1] B. Lüthi,[11] J. Mann,[2] N. Mordant,[12] W.-C. Müller,[5] S. Ott,[2] N. Oullette,[13] J.-F. Pinton,[1] S.B. Pope,[14] S.G. Roux,[1] F. Toschi,[15,16] H. Xu,[4] and P.K. Yeung[17]

[Phys. Rev. Lett 100, 254504 2008]

WHAT HAPPENS AROUND DISSIPATIVE TIME?

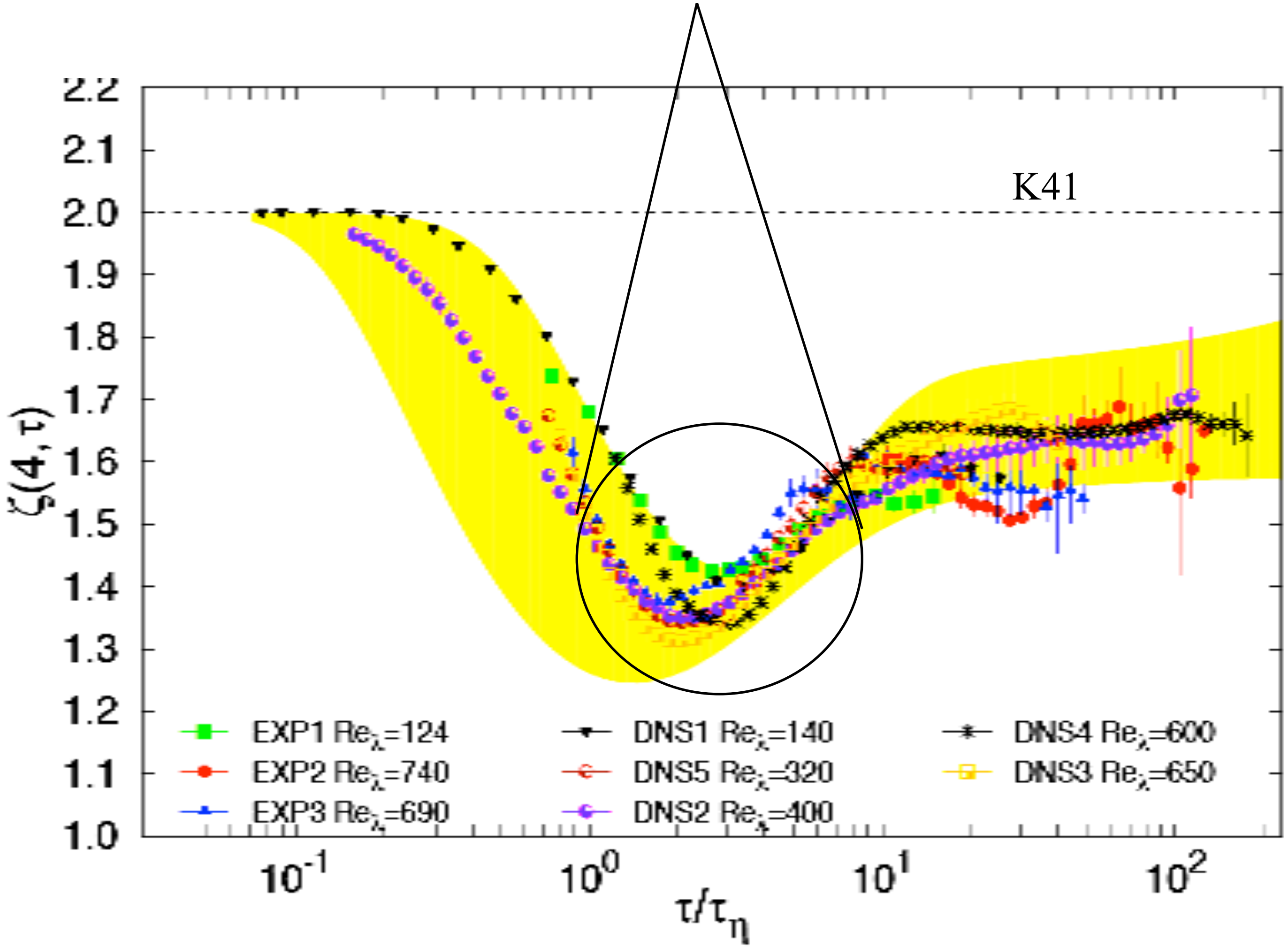


FIG. 1: Log-Lin plot of the local exponent for the fourth moment, $\zeta(4,\tau)$, averaged over the three velocity components, as a function of the normalised time lag τ/τ_η. Data sets come from three experiments (EXP) (see table 1) and five direct numerical simulations (DNS) (see table 2). Error bars are estimated out of the spread between the three components, but for EXP1 and EXP3 where only two components have been considered because of large systematic anisotropic effects in the third one. Each data set is plotted only in the time range where the known experimental/numerical limitations are certainly not affecting the results. In particular, for each data set, the largest time lag always satisfies $\tau < T_L$. The minimal time lag is set by the highest fully resolved frequency. The shaded area displays the prediction obtained by the MF model by using $D_L(h)$ or $D_T(h)$, with

BATCHELOR-MENEVEAU -> LAGRANGIAN

[CHEVILLARD ET AL PRL 2003]

$$\delta_\tau v = v_0 \frac{\tau/T_L}{\left[(\frac{\tau}{T_L})^\beta + (\frac{\tau_\eta}{T_L})^\beta\right]^{\frac{1-2h}{\beta(1-h)}}}$$

$$\delta_\tau v \sim \left(\frac{\tau}{T_L}\right)^{\frac{h}{1-h}}$$

$$\tau \gg \tau_\eta$$

$$\delta_\tau v \sim \tau \frac{\delta_{\tau_\eta} v}{\tau_\eta} \sim a\tau$$

$$\tau \ll \tau_\eta$$

$$\mathcal{P}_h(\tau, \tau_\eta) \sim \left[(\frac{\tau}{T_L})^\beta + (\frac{\tau}{T_L})^\beta\right]^{\frac{3-D(h)}{\beta(1-h)}}$$

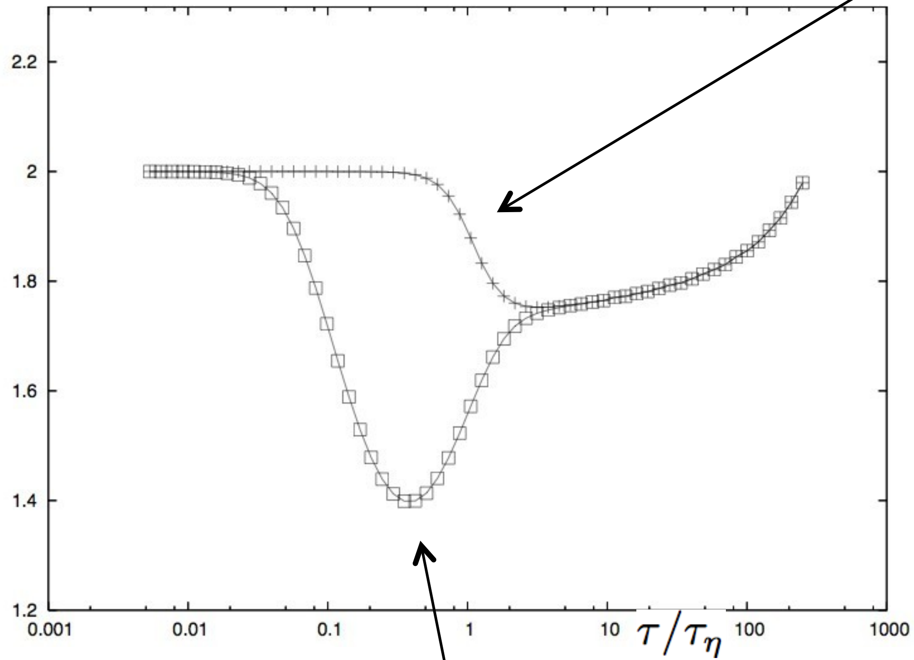
start from Eulerian $D(h)$

but: dissipative time fluctuates (as the dissipative scale): $\tau_\eta = \frac{\eta}{\delta_\eta v}$

$$\tau_\eta(h) \sim Re_\lambda^{\frac{2(h-1)}{1+h}}$$

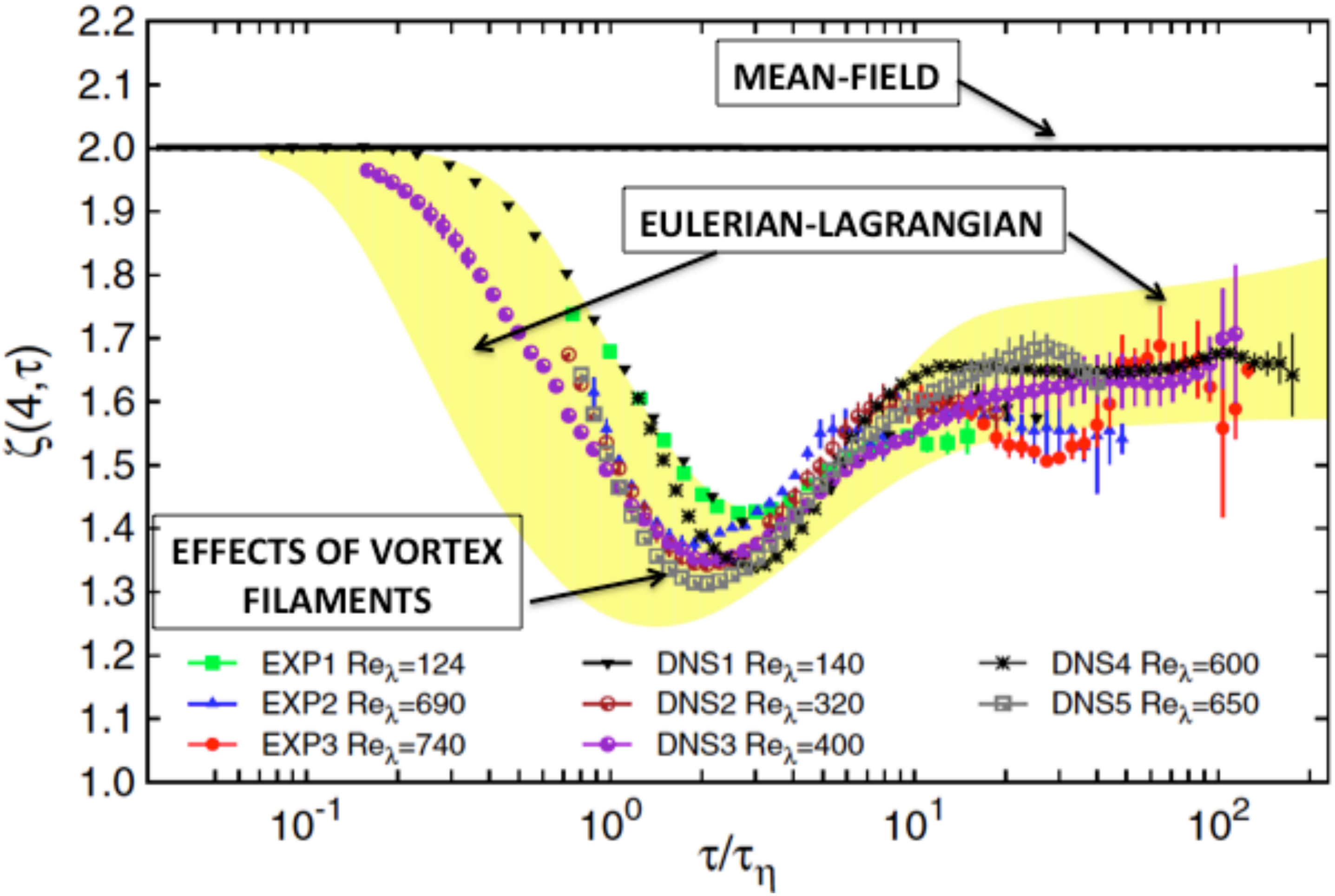
MultiFractal WITHOUT DISSIPATIVE FLUCTUATING $\tau_\eta(h)$

$$\zeta_p(\tau) = \frac{\mathrm{d}\log(S_p(\tau))}{\mathrm{d}\log(S_2(\tau))}$$



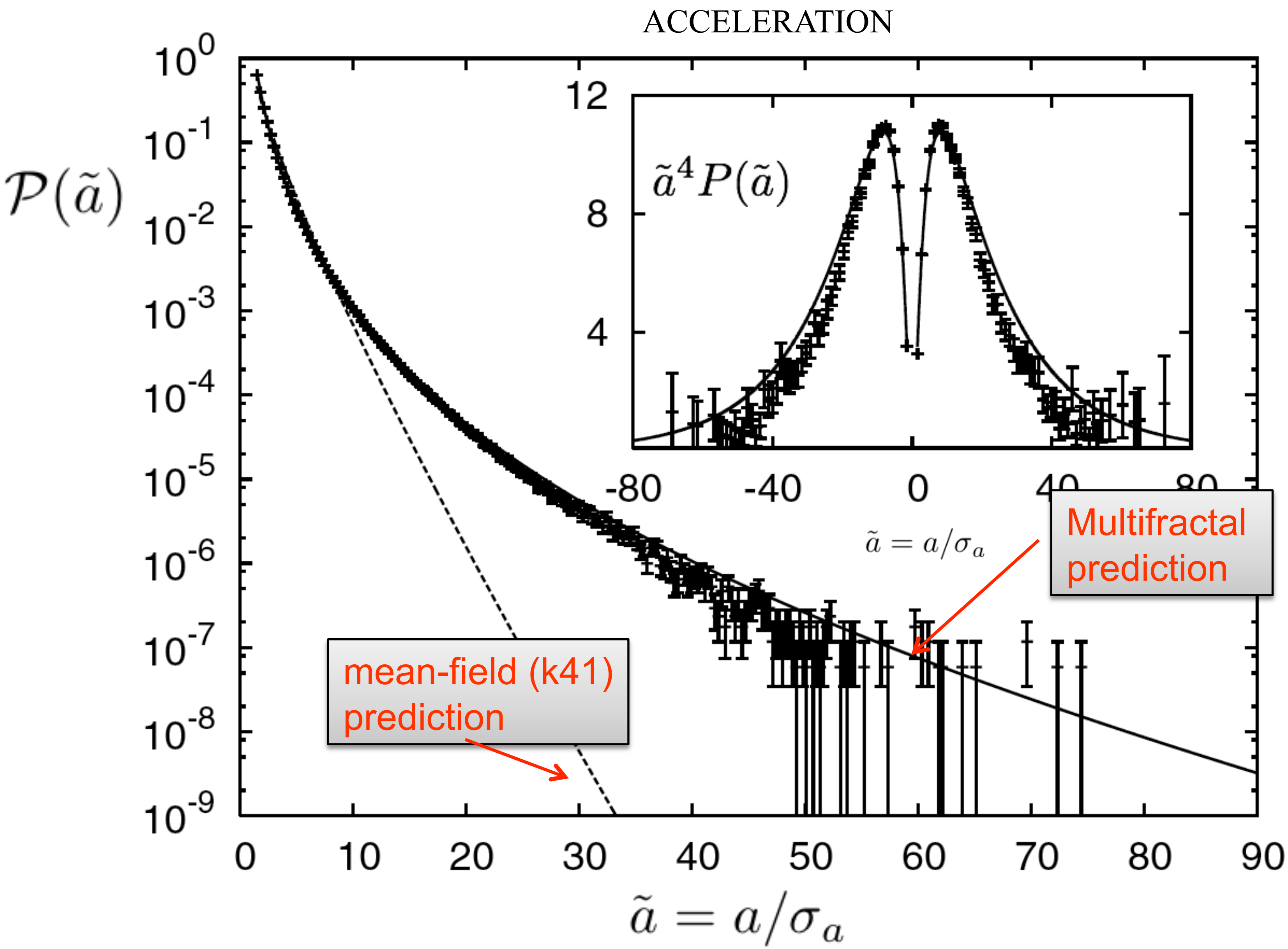
MultiFractal WITH DISSIPATIVE FLUCTUATING $\tau_\eta(h)$

BOTTLENECK IS A DISSIPATIVE EFFECT



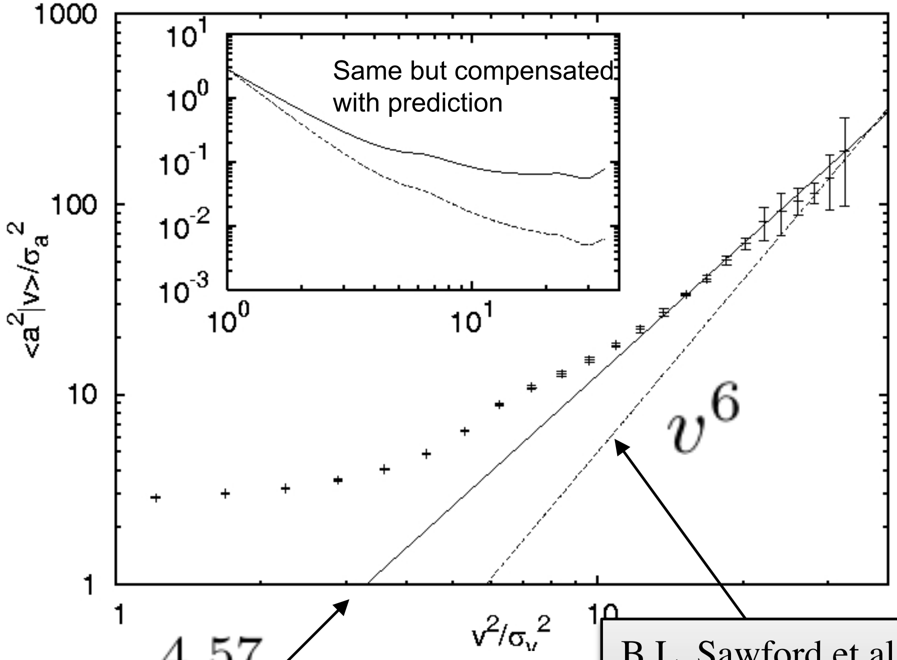
MEAN-FIELD
EULERIAN-LAGRANGIAN
EFFECTS OF VORTEX FILAMENTS
EXP1 Re$_\lambda$=124
EXP2 Re$_\lambda$=690
EXP3 Re$_\lambda$=740
DNS1 Re$_\lambda$=140
DNS2 Re$_\lambda$=320
DNS3 Re$_\lambda$=400
DNS4 Re$_\lambda$=600
DNS5 Re$_\lambda$=650
$\zeta(4,\tau)$
τ/τ_η
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
10^{-1}
10^{0}
10^{1}
10^{2}

L. B., G. Boffetta, A. Celani, B. Devenish, A. Lanotte and F. Toschi *Phys. Rev. Lett.* **93**, 064502, 2004



$$\mathcal{P}(a) \sim \int_{h\in I} \mathrm{d}h\, a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \quad \exp\left(- \frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2} \right)$$

$$\langle a^2|v^2\rangle$$



$v^{4.57}$

Multifractal prediction

B.L. Sawford et al., Phys. Fluids **15**, 3478 (2003).

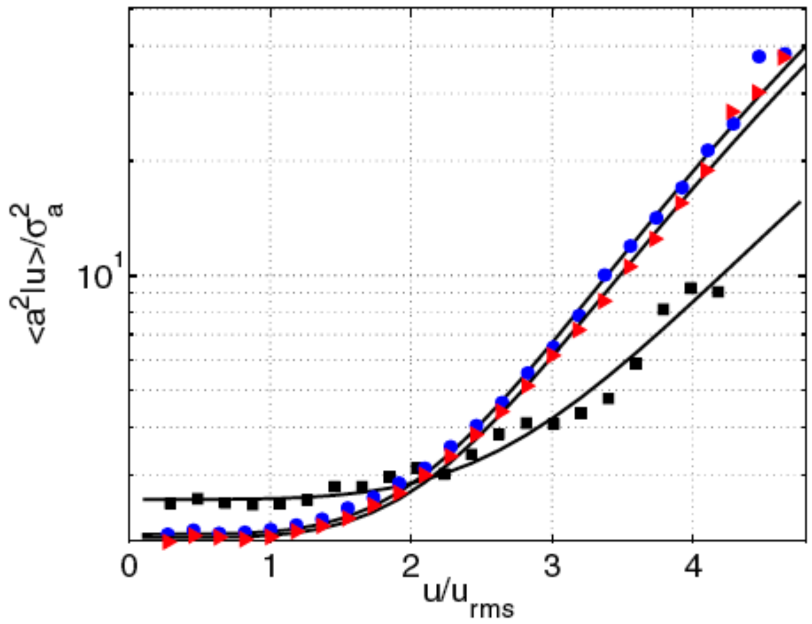
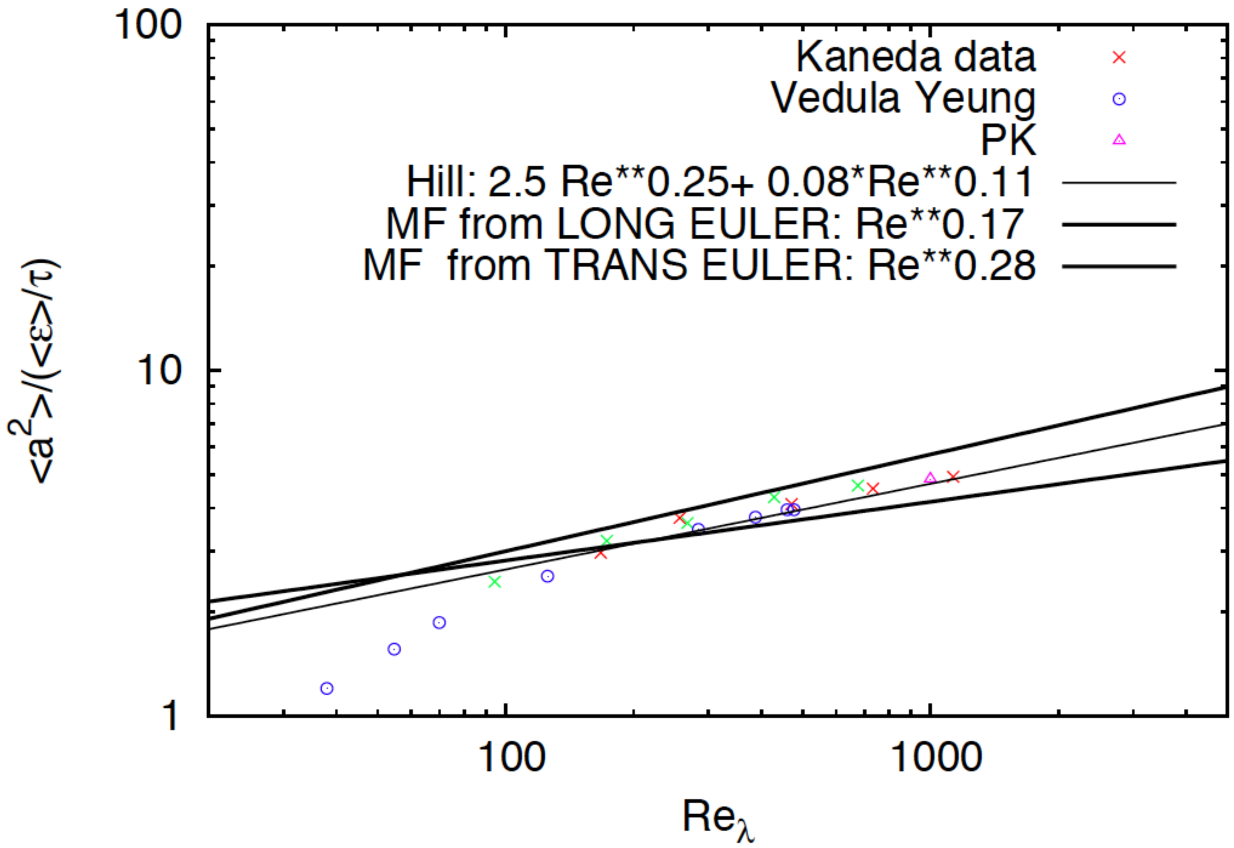


FIG. 4 (color online). Normalized conditional acceleration variance $\langle a^2|u\rangle/\sigma_a^2$ for $R_\lambda = 690$, 485, 285, circles, triangles, and squares, respectively. Solid lines are the fit (3).

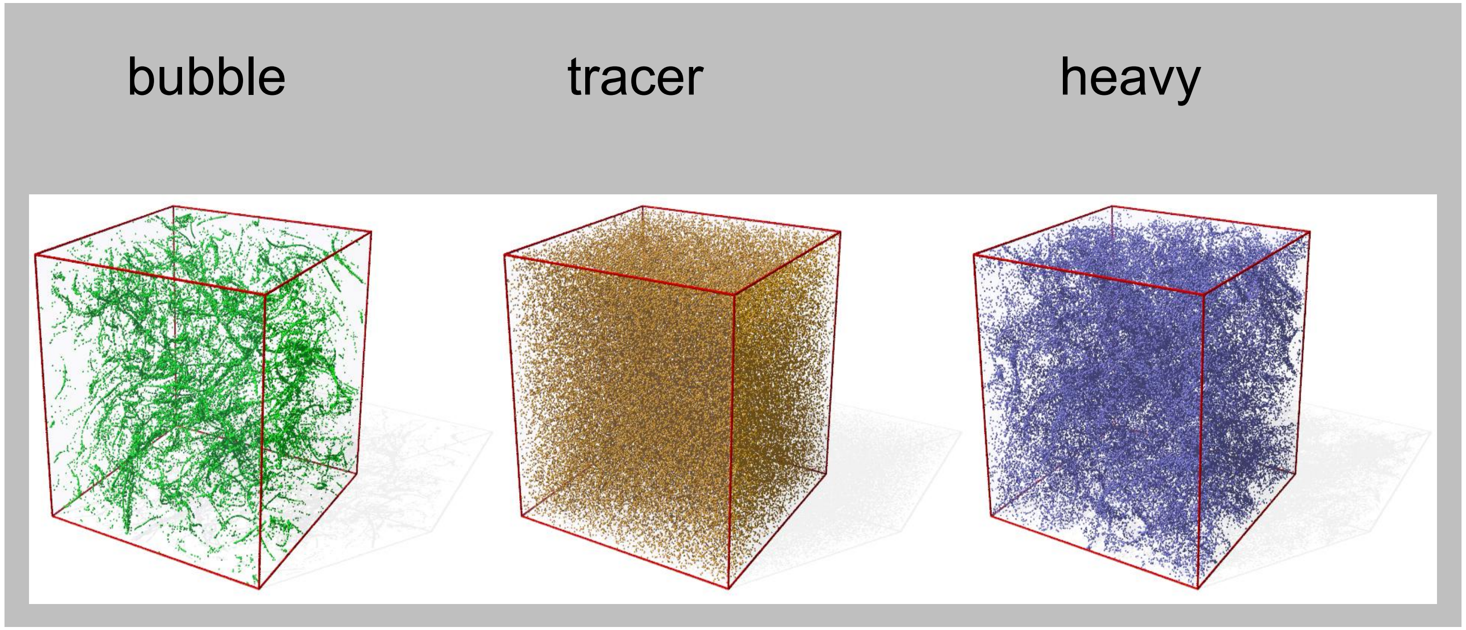
Joint Statistics of the Lagrangian Acceleration and Velocity in Fully Developed Turbulence. Crawford, Mordant, and Bodenschatz PRL 94, 024501 (2005)

L. B., G. Boffetta, A. Celani, B. Devenish, A. Lanotte and F. Toschi *Phys. Rev. Lett.* **93**, 064502, (2004)



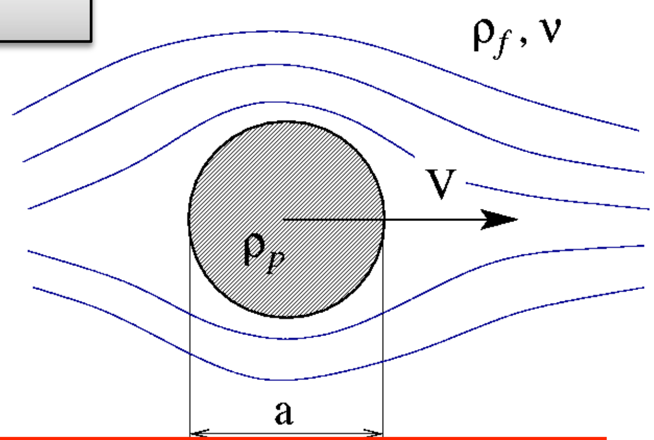
Kaneda data
Vedula Yeung
PK
Hill: 2.5 Re**0.25+ 0.08*Re**0.11
MF from LONG EULER: Re**0.17
MF from TRANS EULER: Re**0.28
100
10
1
100
1000
$\langle a^2\rangle/(\langle\varepsilon\rangle/\tau)$
Re_λ

OPEN QUESTIONS (FAILURES): Effects of Inertia



Eqs of motion for a single particle

Maxey & Riley Phys. Fluids 26, 883 (1983)



- •Small particles
- •Small Reynolds numbers (on the particle radiud)
- •Undeformable
- •Small volume fraction
- •collisionless

$$\frac{a(u-V)}{\nu} \ll 1 \qquad a \ll \eta$$

$$m_p \frac{dV_i}{dt} = (m_p - m_f) g_i + m_f \left.\frac{Du_i}{Dt}\right|_{\boldsymbol{X}(t)}$$

Buoyoancy + fluid acceleration

$$-6\pi a\mu \left[V_i(t) - u_i(\boldsymbol{X}(t),t) - \frac{1}{6} a^2 \left.\nabla^2 u_i\right|_{\boldsymbol{X}(t)} \right]$$

Stokes drag

$$-\frac{m_f}{2}\frac{d}{dt}\left[V_i(t) - u_i(\boldsymbol{X}(t),t) - \frac{1}{10} a^2 \left.\nabla^2 u_i\right|_{\boldsymbol{X}(t)} \right]$$

Added mass

$$-6\pi a\mu \int_0^t ds \left(\frac{d/ds \left[V_i(s) - u_i(\boldsymbol{X}(s),s) - \frac{1}{6} a^2 \left.\nabla^2 u_i\right|_{\boldsymbol{X}(s)} \right]}{\sqrt{\pi\nu(t-s)}} \right)$$

Basset-history terms

Simplified limit

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\tau_p = \frac{a^2}{3\nu\beta}$$

$$\frac{d\boldsymbol{V}}{dt} = \beta\frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t)-\boldsymbol{V}}{\tau_p} + (1-\beta)\boldsymbol{g}$$

Three-parameters problem

τ_f Fluid characteristic time

τ_p Particle's characteristic time

$$\rho_p \gg \rho_f \rightarrow \beta = 0$$ HEAVY

$$\rho_f = \rho_p \rightarrow \beta = 1$$ TRACERS

$$\rho_f \gg \rho_p \rightarrow \beta = 3$$ LIGHT

Stokes number

$$St = \frac{\tau_p}{\tau_f}$$

Density contrast

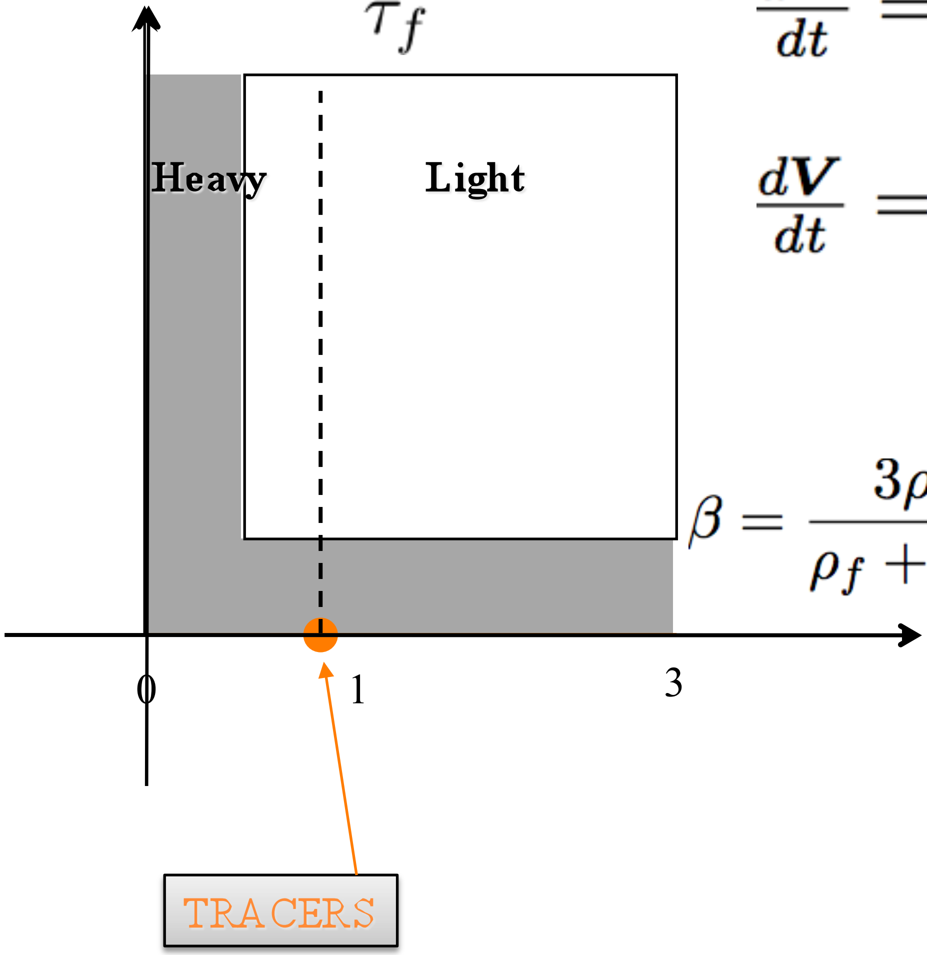
$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

Reynolds

$$Re = \frac{UL}{\nu}$$

Validity of assumption $a/\eta < 1$

$$St = \frac{\tau_p}{\tau_f}$$

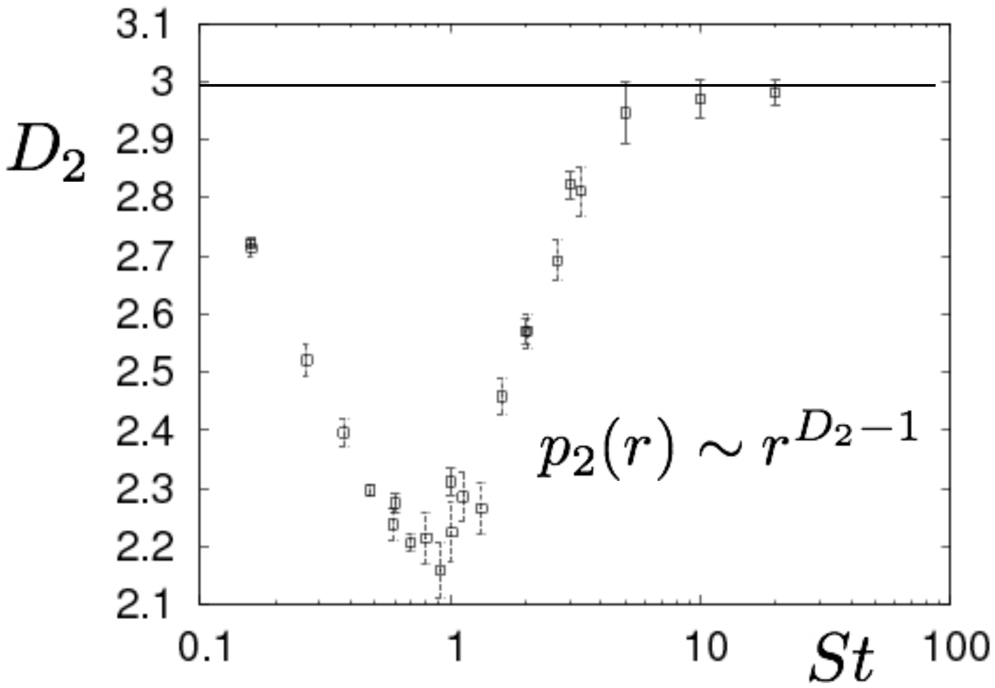


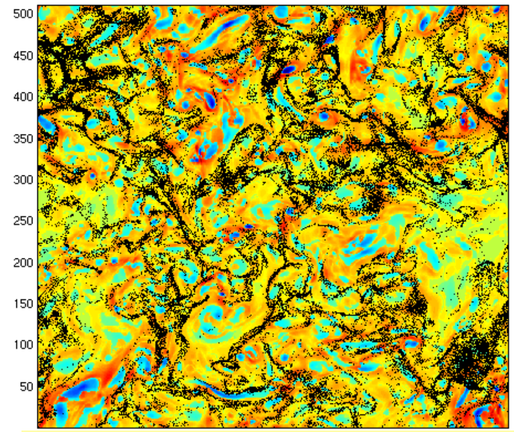
$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\frac{d\boldsymbol{V}}{dt} = \beta\frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t)-\boldsymbol{V}}{St}$$

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

Preferential Concentration



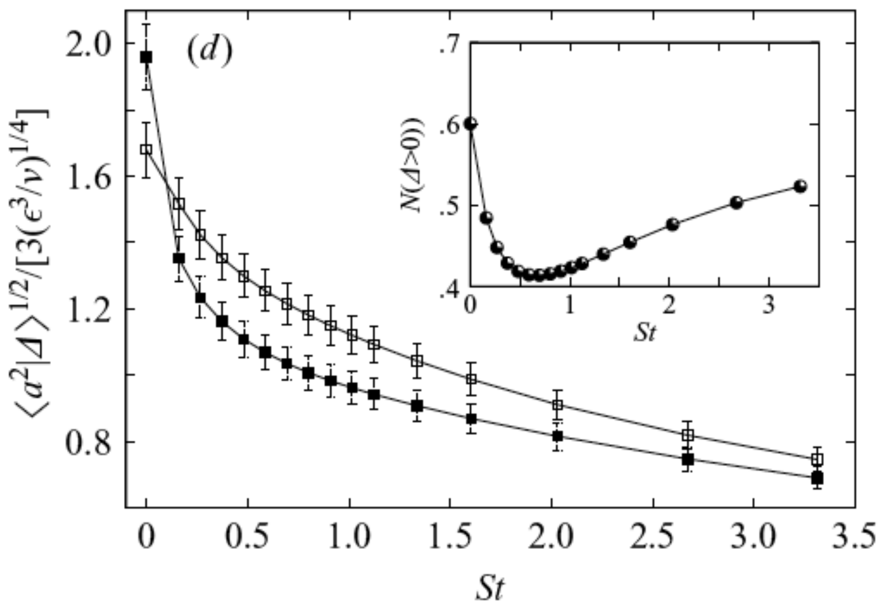


■ Rotating regions Δ > 0
□ Strain regions Δ < 0

$$\Delta = \left(\frac{\det[\hat{\boldsymbol{\sigma}}]}{2}\right)^2 - \left(\frac{\mathrm{Tr}[\hat{\boldsymbol{\sigma}}^2]}{6}\right)^3 \quad \Delta \le 0 \quad \Delta > 0$$

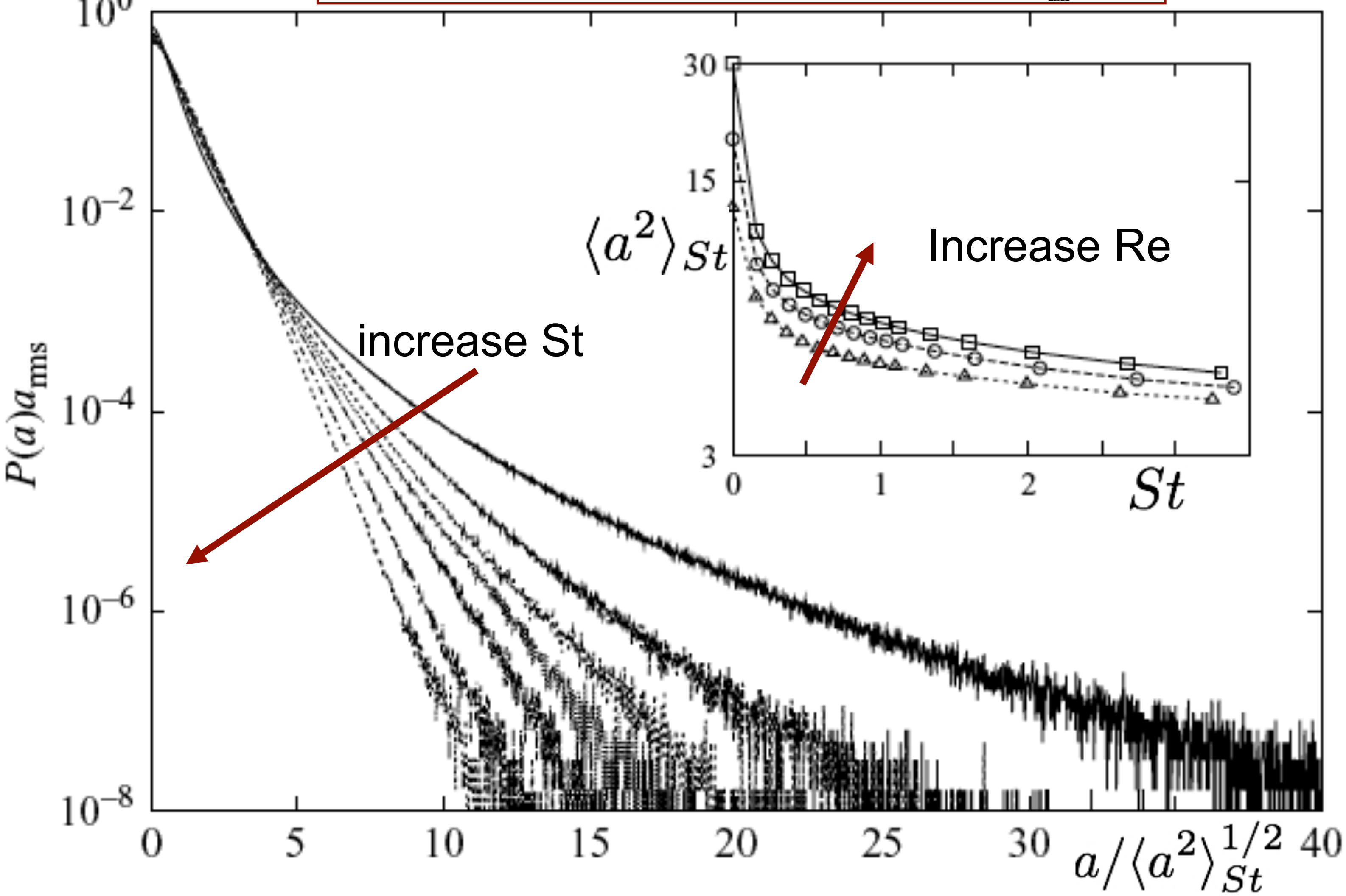
Okubo–Weiss parameter Q
is the determinant of the strain matrix

$$\sigma_{ij} = \frac{\partial u_i}{\partial x_j}$$



Acceleration: pdf(a) vs. St

St=0, 0.16, 0.37, 0.58, 1.01, 2.03, 3.33 at Re=185



Q: how to include inertia in Multifractal phenomenology? Nobody knows

J. Bec, L.B., G. Boffetta, M. Cencini, A. Lanotte, S. Musacchio & F. Toschi JFM 550, p 349, 2006

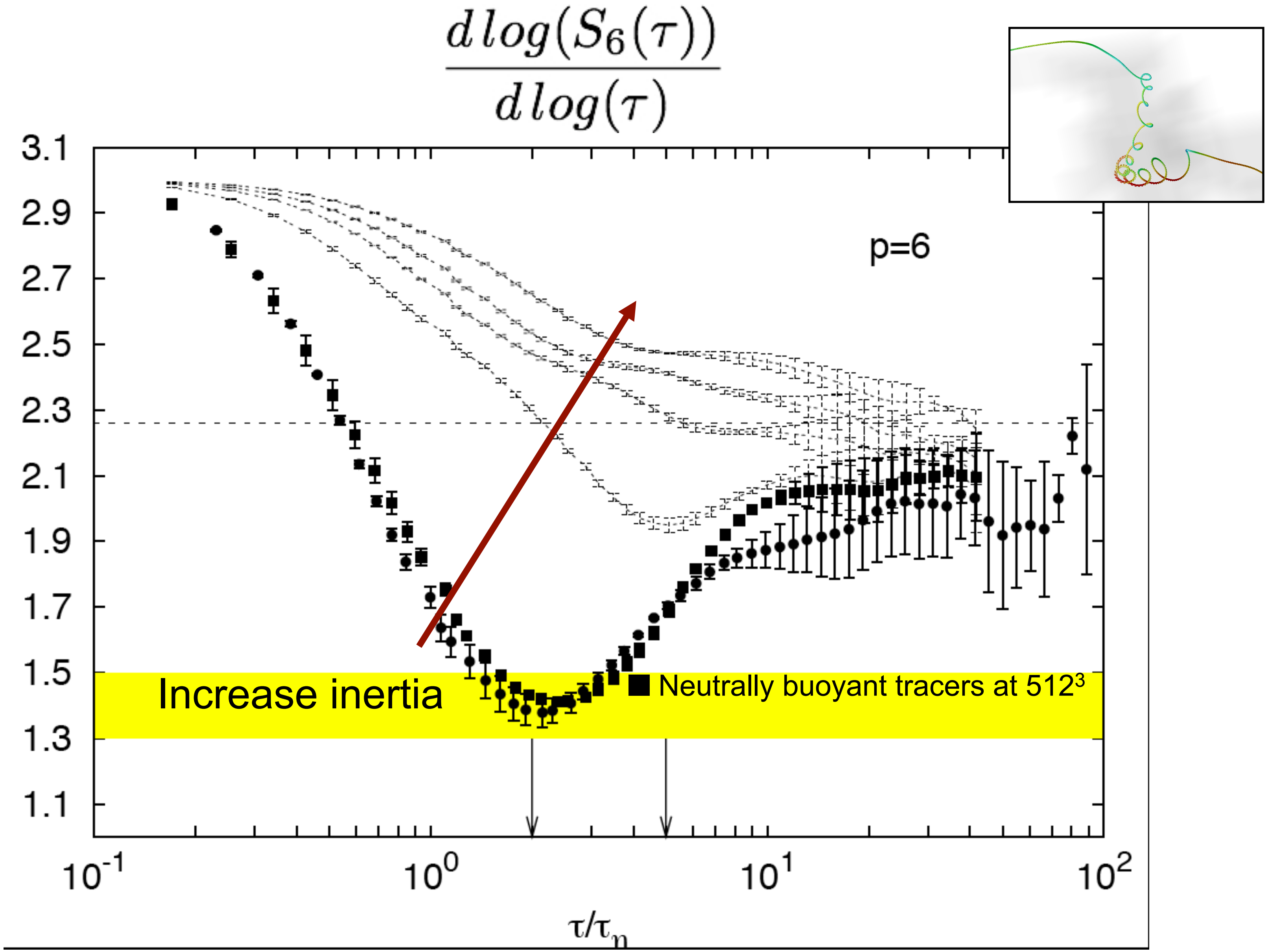
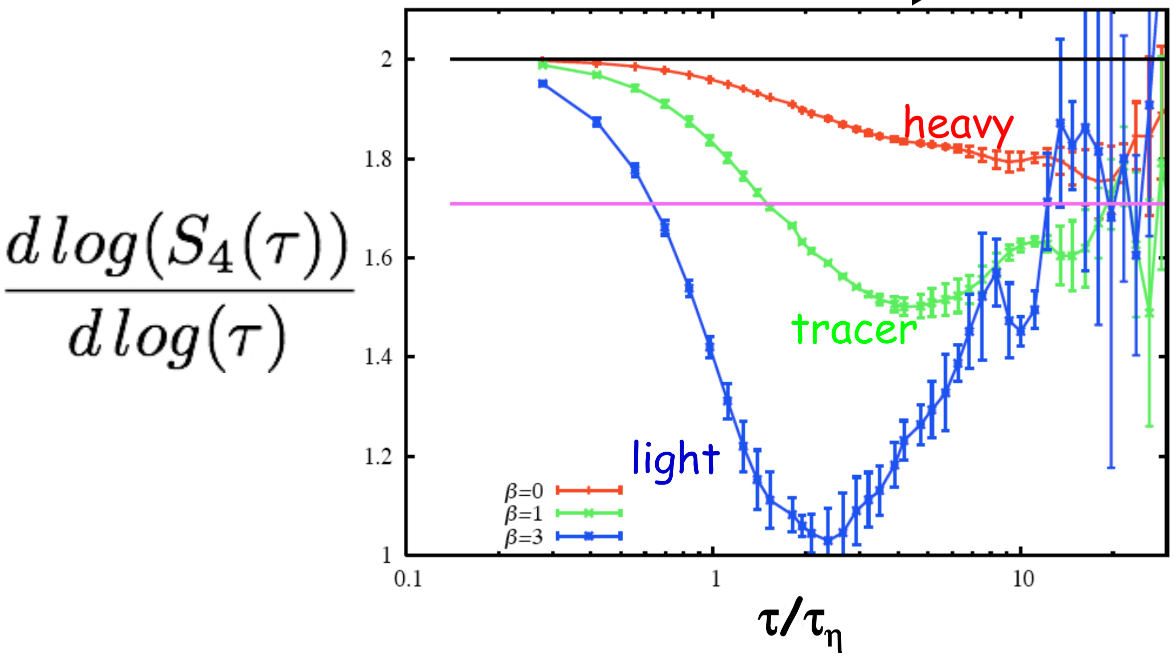


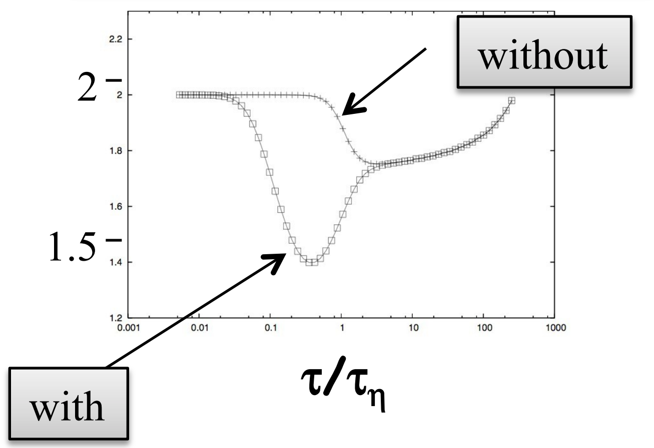
Figure from: On the effects of vortex trapping on the velocity statistics of tracers and heavy particle in turbulent flows
J. Bec, L. B., M. Cencini, A. S. Lanotte, and F. Toschi, PoF 18, 081702, 2006.

Role of vortex filaments *vs* fluctuations of dissipative scales

Light/heavy particles are trapped/ejected inside/outside vortex filaments: they fill more/less the fluctuations of the dissipative scale.



Mutifractal AND fluctuations of dissipative scale



Universality of inertial range with respect to 'huge' smal-scale effects

- Kraichnan et al: superposition of random vortex filaments: k41 scaling with longitudinal=transverse scaling.
- Belin, Maurer, Tabeling & Willaime: filaments transition (statistical instability) at Re ~ 700
- Chorin: collection of sel-avoiding vortex filaments -> fractal structure
- Passot Politano et al: influence of vortex filaments on the energy spectrum
- Migdal: loop turbulence, statistics driven by velocity circulation

Frisch: Turbulence, Cambridge Univ. Press, 1995

8.9.2 Statistical signature of vortex filaments: dog or tail?

Having identified 'simple' geometric objects, the vortex filaments, in turbulent flows, it is natural to ask if any of the known statistical properties of turbulence can be thus explained. Are the vortex filaments the *dog* or the *tail*? In the former case, they would be essential to explain the energetics and the scaling properties of high-Reynolds-number flow. In the latter case, they would have only marginal signatures, for example on the tails of p.d.f.s of various small-scale quantities and on the exponents ζ_p for large *p*s.

IS FORWARD ENERGY CASCADE THE END OF THE STORY IN 3D?

CAN WE DISENTANGLE DIFFERENT PHYSICAL MECHANISM LEADING TO FORWARD/BACKWARD ENERGY TRASNFER IN 3D NS?

The nature of triad interactions in homogeneous turbulence

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(Received 24 July 1991; accepted 22 October 1991)

$$\boldsymbol{u}(\boldsymbol{k}) = u^+(\boldsymbol{k})\boldsymbol{h}^+(\boldsymbol{k}) + u^-(\boldsymbol{k})\boldsymbol{h}^-(\boldsymbol{k})$$

$$\boldsymbol{h}^\pm = \hat{\boldsymbol{\nu}} \times \hat{\boldsymbol{k}} \pm i\hat{\boldsymbol{\nu}}$$

$$\hat{\boldsymbol{\nu}} = \boldsymbol{z} \times \boldsymbol{k}/||\boldsymbol{z} \times \boldsymbol{k}||$$

$$i\boldsymbol{k} \times \boldsymbol{h}^\pm = \pm k\boldsymbol{h}^\pm$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

$$u^{s_k}(\mathbf{k},t) \quad (s_k = \pm 1)$$

$$\frac{d}{dt}u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}}(s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^*. \qquad (15)$$

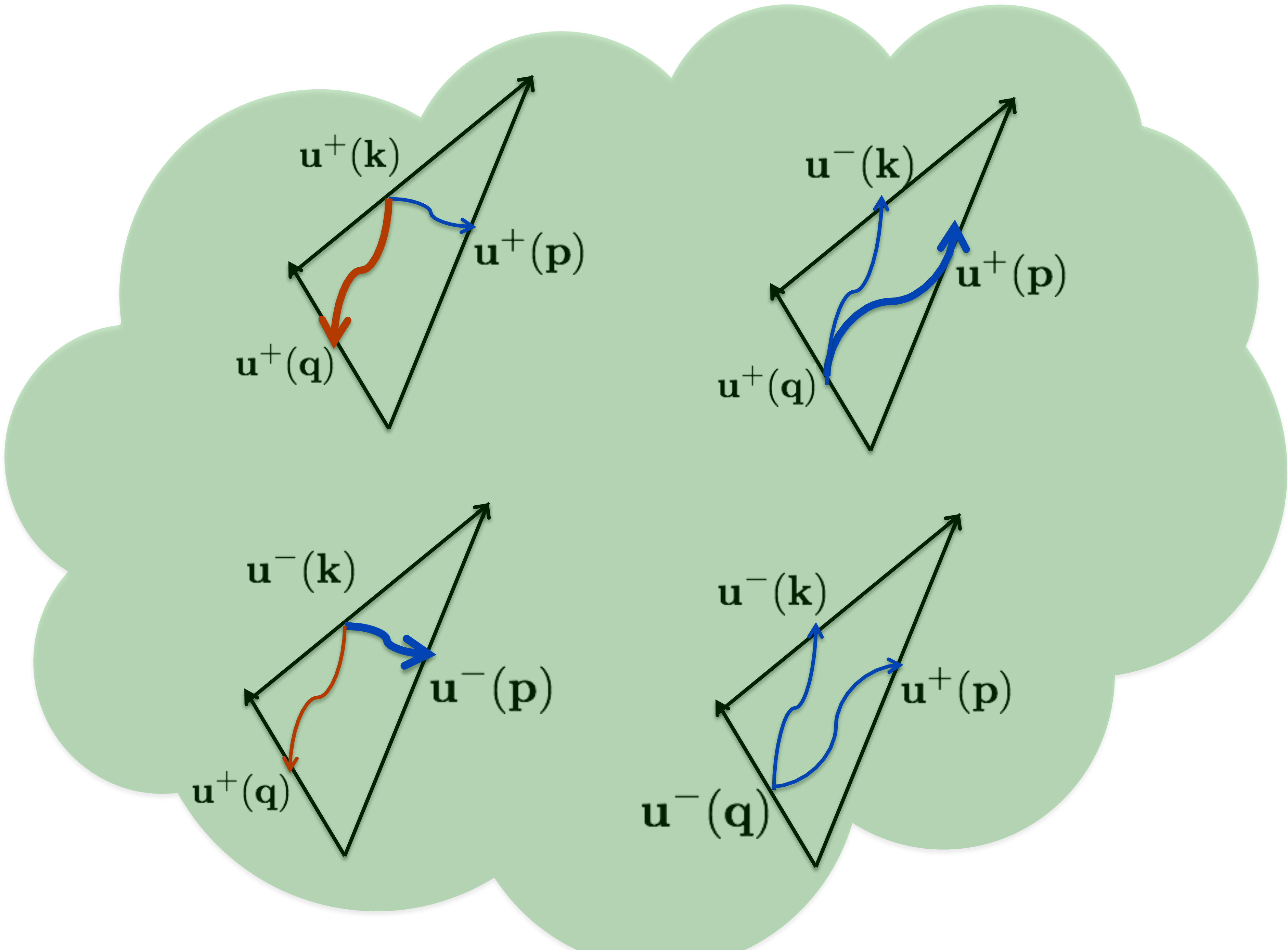
Eight different types of interaction between three modes $u^{s_k}(\mathbf{k})$, $u^{s_p}(\mathbf{p})$, and $u^{s_q}(\mathbf{q})$ with $|\mathbf{k}|<|\mathbf{p}|<|\mathbf{q}|$ are allowed according to the value of the triplet (s_k, s_p, s_q)

$$\dot{u}^{s_k} = r(s_p p - s_q q)\frac{s_k k + s_p p + s_q q}{p}(u^{s_p} u^{s_q})^*,$$

$$\dot{u}^{s_p} = r(s_q q - s_k k)\frac{s_k k + s_p p + s_q q}{p}(u^{s_q} u^{s_k})^*,$$

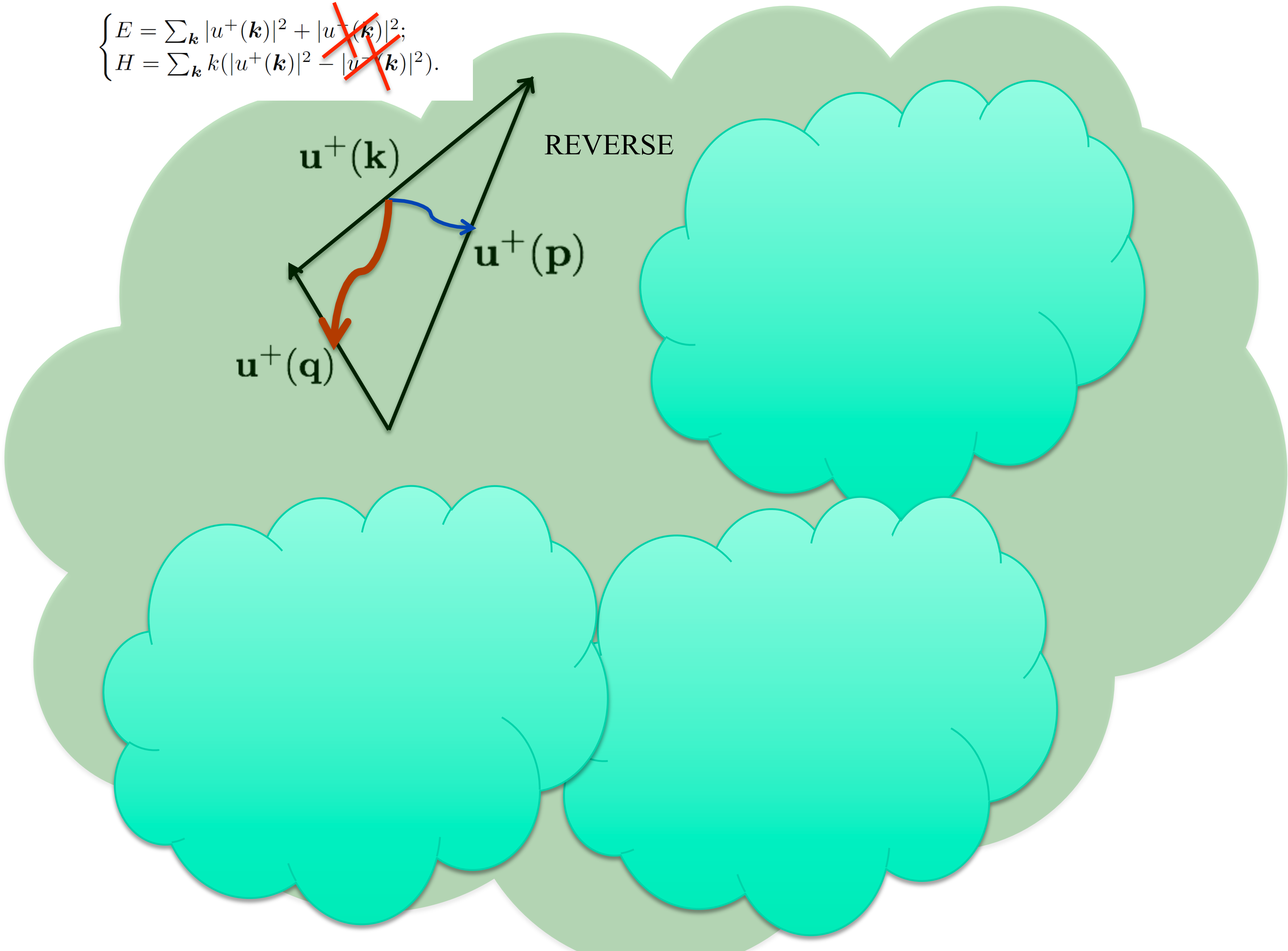
$$\dot{u}^{s_q} = r(s_k k - s_p p)\frac{s_k k + s_p p + s_q q}{p}(u^{s_k} u^{s_p})^*.$$

TRIADIC INTERACTION IN WHOLE NAVIER_STOKES EQS

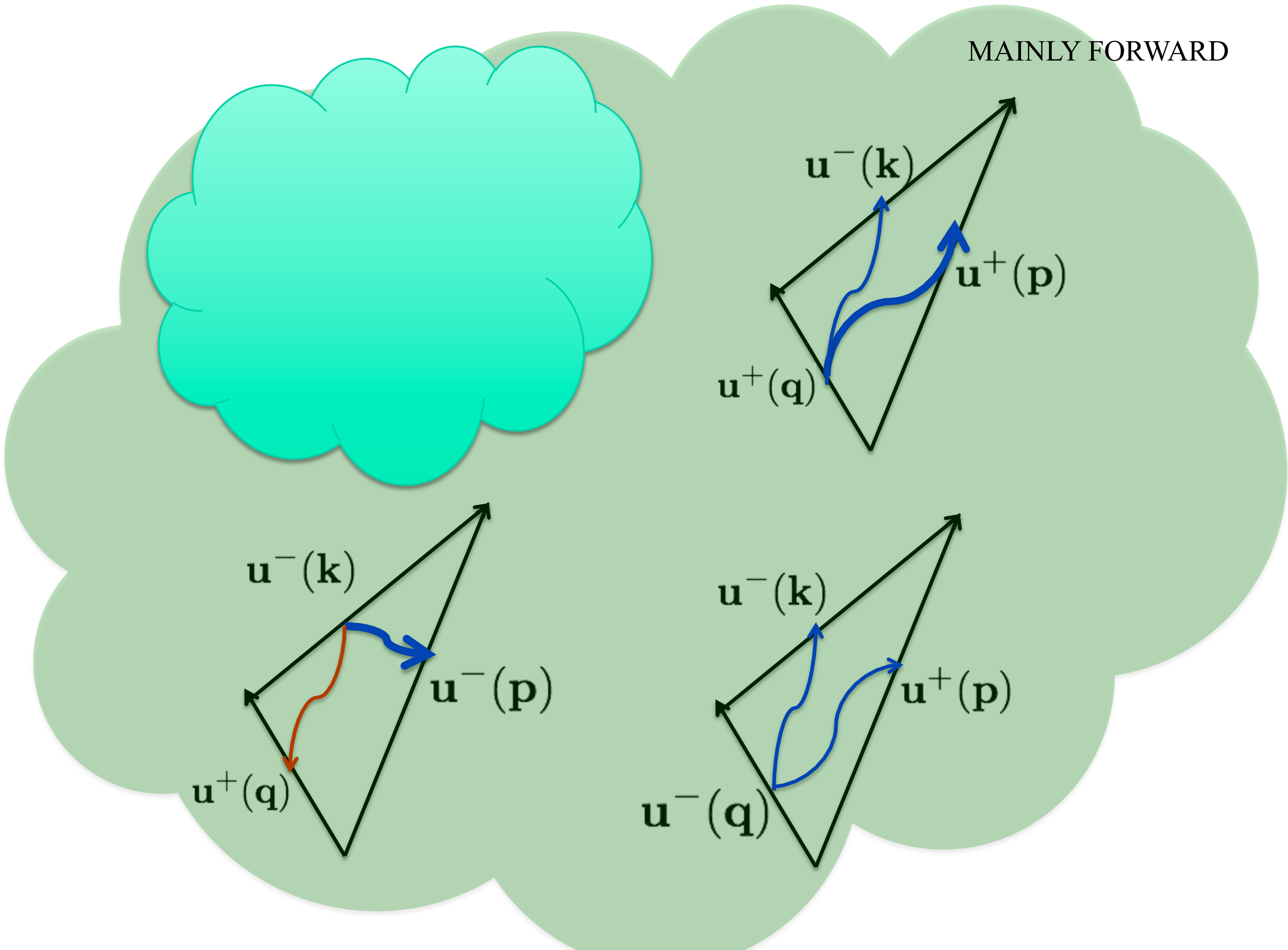


TRIADIC INTERACTION IN DECIMATED NAVIER_STOKES EQS

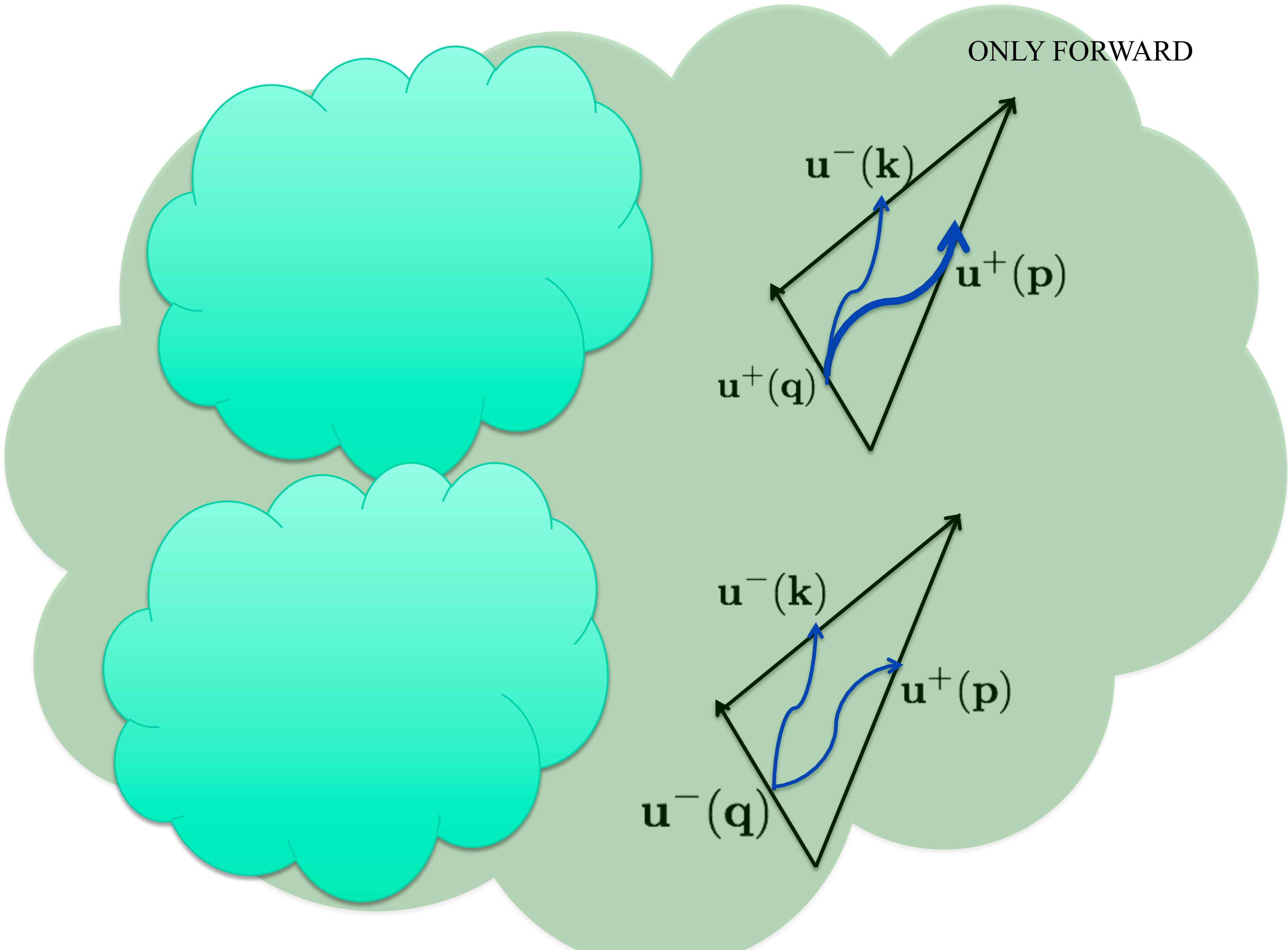
$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + \cancel{|u^-(\boldsymbol{k})|^2}; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - \cancel{|u^-(\boldsymbol{k})|^2}). \end{cases}$$



TRIADIC INTERACTION IN WHOLE NAVIER_STOKES EQS



TRIADIC INTERACTION IN WHOLE NAVIER_STOKES EQS



ONLY REVERSE

$$\mathcal{P}^{\pm} \equiv \frac{\boldsymbol{h}^{\pm} \otimes \overline{\boldsymbol{h}^{\pm}}}{\overline{\boldsymbol{h}^{\pm}} \cdot \boldsymbol{h}^{\pm}}. \qquad \boldsymbol{v}^{\pm}(\boldsymbol{x}) \equiv \sum_{\boldsymbol{k}} \mathcal{P}^{\pm} \boldsymbol{u}(\boldsymbol{k});$$

$$\boxed{\boldsymbol{u}(\boldsymbol{k}) = u^{+}(\boldsymbol{k})\boldsymbol{h}^{+}(\boldsymbol{k}) + u^{-}(\boldsymbol{k})\boldsymbol{h}^{-}(\boldsymbol{k})}$$

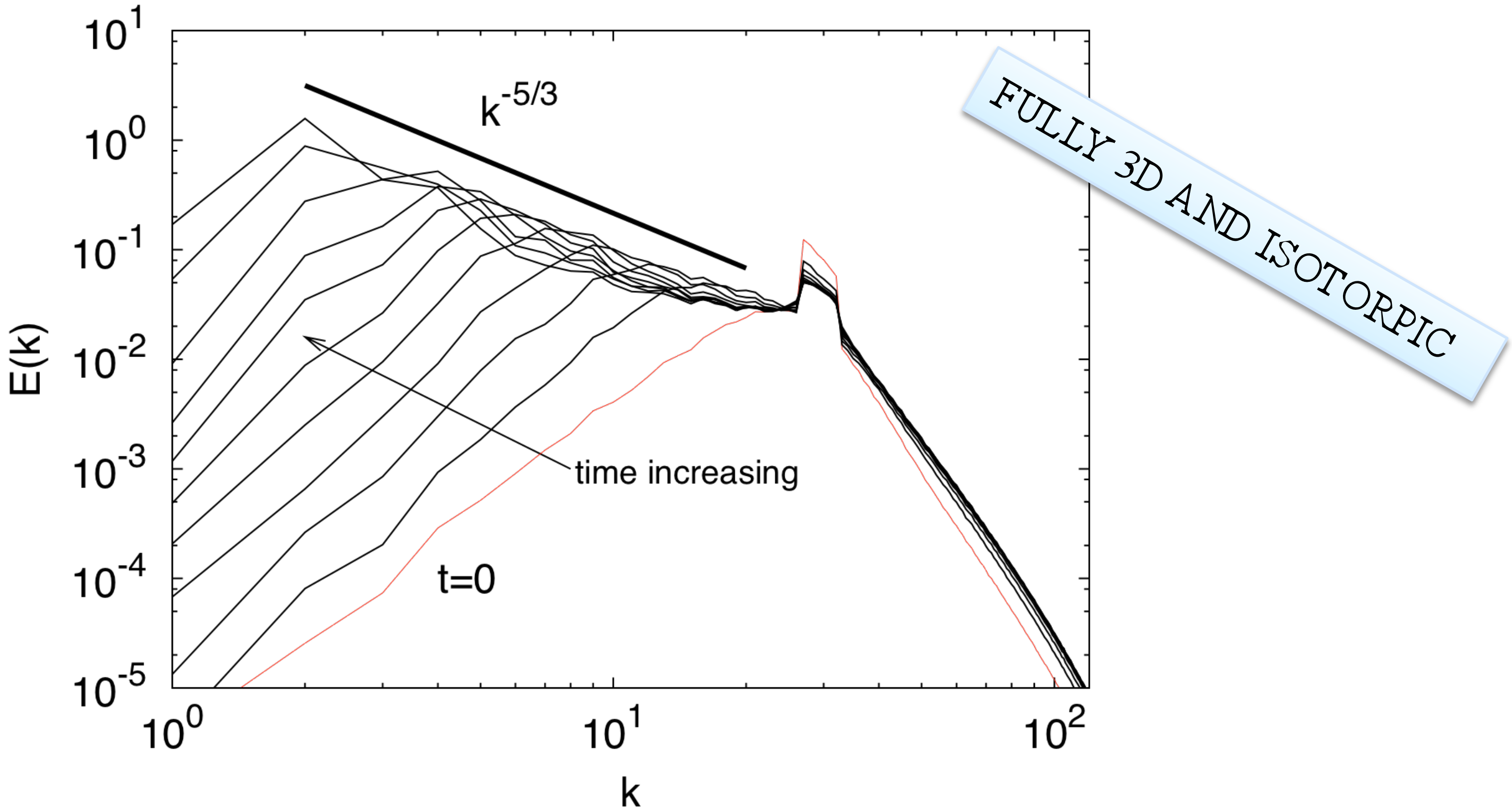
LOCAL BELTRAMIZATION (IN FOURIER)

$$\partial_t v^{+} + \mathcal{P}^{+} B[v^{+}, v^{+}] = \nu \Delta v^{+} + \mathbf{f}^{+}$$

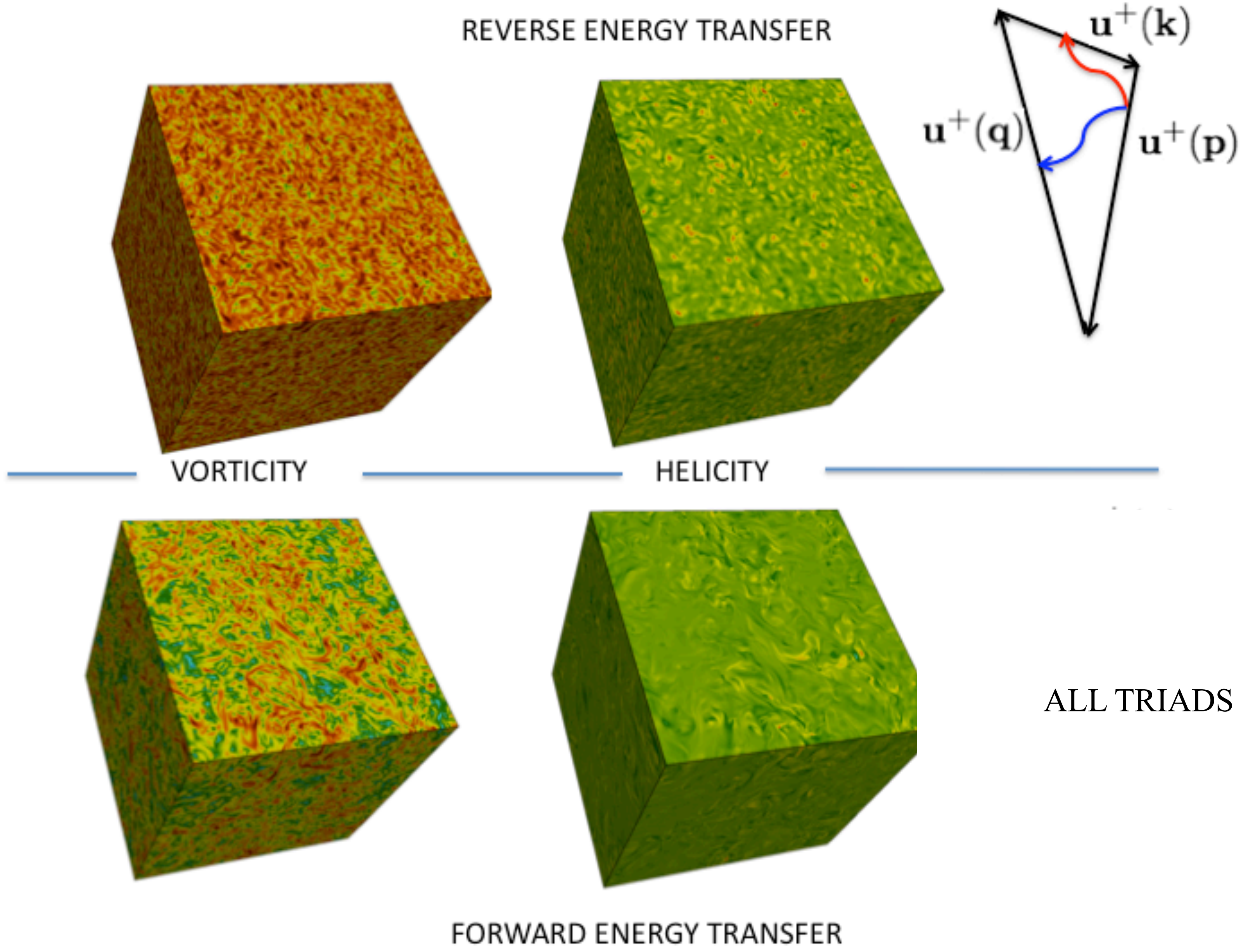
$$\frac{d}{dt} u^{s_k}(\mathbf{k}) + \nu k^2 u^{s_k}(\mathbf{k}) = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} g_{\mathbf{k},\mathbf{p},\mathbf{q}} (s_p p - s_q q) \times [u^{s_p}(\mathbf{p}) u^{s_q}(\mathbf{q})]^*. \qquad (15)$$

$$\boxed{s_p = s_q = s_k = +}$$

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$



L.B., S. MUSACCHIO & F. TOSCHI Phys. Rev. Lett. **108** 164501, 2012


REVERSE ENERGY TRANSFER
$\mathbf{u}^+(\mathbf{k})$
$\mathbf{u}^+(\mathbf{q})$
$\mathbf{u}^+(\mathbf{p})$
VORTICITY
HELICITY
ALL TRIADS
FORWARD ENERGY TRANSFER

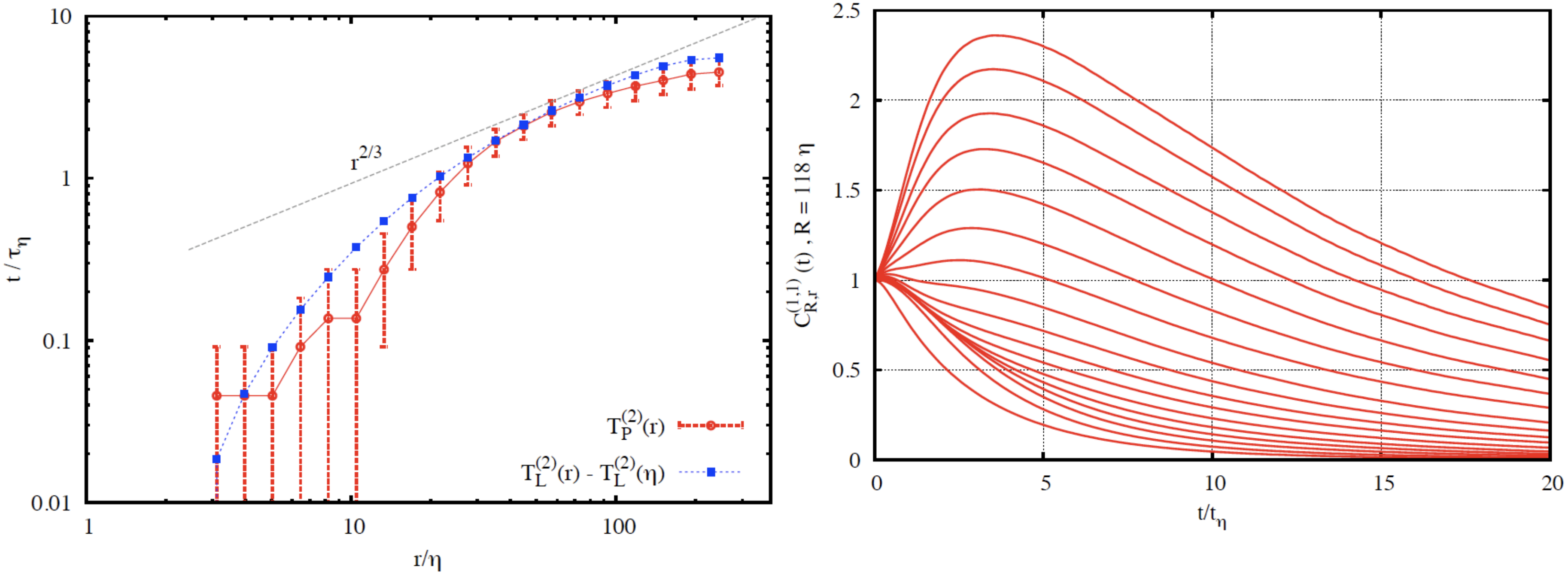
MULTI-TIME MULTI-SCALE CORRELATION FUNCTIONS

$$C(R,r|\tau) = \langle|\delta_r u(x(t+\tau),t+\tau))||\delta_R u(x(t),t))|\rangle$$

$$C_{p,q}(R,r|\tau) = \langle|\delta_r u(x(t+\tau),t+\tau))|^p|\delta_R u(x(t),t))|^q\rangle$$

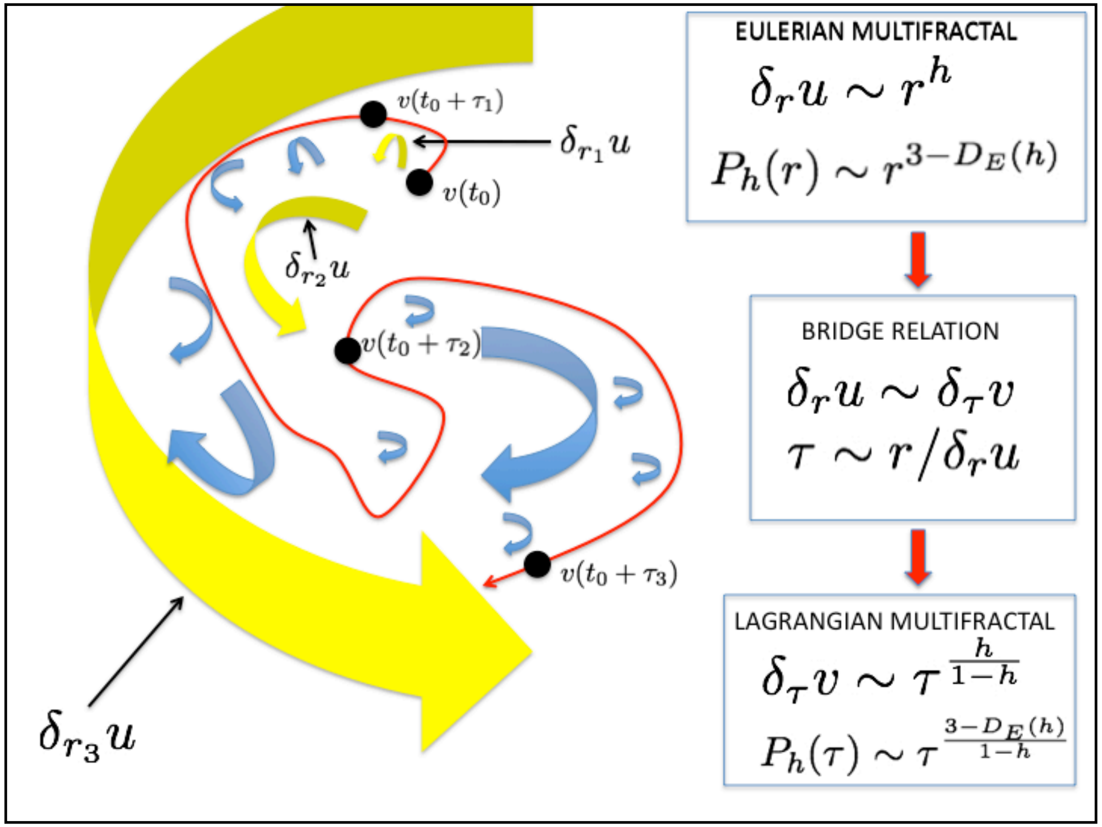
$$T(R) = \int_0^\infty d\tau C(R,R|\tau)/C(R,R|0)$$

$$T_{p,q}(R) = \int_0^\infty d\tau C_{p,q}(R,R|\tau)/C_{p,q}(R,R|0) \sim R^{2p/3+\delta_{p,q}}$$



L. B., E. Calzavarini, F. Toschi "Multi-time multi-scale correlation functions in hydrodynamical turbulence." Phys. Fluids 23 085107, 2011.

1. EULERIAN-LAGRANGIAN BASED ON THE SIMPLEST (OCCAM'S RASOR) PRINCIPLE IS GOOD ALSO FOR INTENSE FLUCTUATIONS (NOT KNOWN FOR VERY INTENSE ONES)
2. IS IT THE END OF THE STORY: NO (2D, MHD, SHEAR, ETC…)
3. WHY LAGRANGIAN SCALING IS SO POOR (ONLY ESS UP TO NOW)
4. WHAT HAPPENS WHEN TOPOLOGY PLAYS A KEY ROLE: INERTIAL PARTICLES
5. WHAT ABOUT MULTI-TIME MULTI-SCALE



Thanks to (order of appearance)

M. Cencini
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G. Falkovich
A. Pumir