Viscoelasticity and Growth

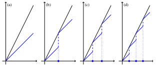
Andrea Chiesa

Oberwolfach Workshop
Singularities in Discrete Systems

07/05/2025

Research interests

Torino



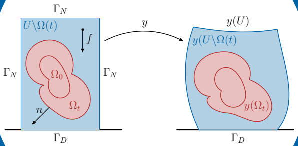
StrD

M. Morandotti

Prague

M. Krúžik

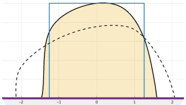
Vienna



Viscoelastic growth

U. Stefanelli

Kyoto



MCF

K. Takasao

Tokyo

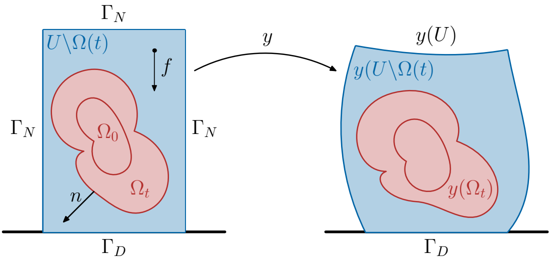
K. Švadlenka

Deformation

$y\colon U \to \mathbb{R}^d$

Accretion

$\Omega(t) := \{\theta(x) < t\}$



$$-\operatorname{div}\left(\partial_F W^a(\nabla y)+\partial_{\dot F} R^a(\nabla y, \nabla \dot y)-\operatorname{div} \mathrm{D}H(\nabla^2 y)\right) = f \qquad \text{in } [0,T]\times\Omega(t)$$

$$-\operatorname{div}\left(\partial_F W^r(\nabla y)+\partial_{\dot F} R^r(\nabla y, \nabla \dot y)-\operatorname{div} \mathrm{D}H(\nabla^2 y)\right) = f \qquad \text{in } [0,T]\times(U \setminus \overline{\Omega(t)})$$

$$\gamma\big(y(\theta(x),x), \nabla y(\theta(x),x)\big)|\nabla\theta(x)| = 1 \qquad \text{in } U \setminus \overline{\Omega_0}$$

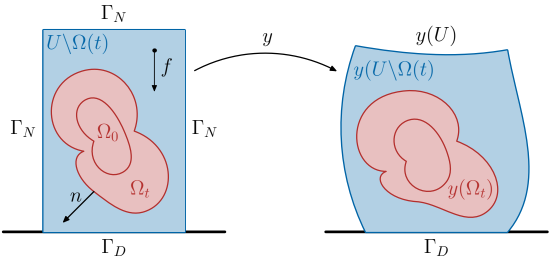
The model

Deformation

$y\colon U \to \mathbb{R}^d$

Accretion

$\Omega(t) := \{\theta(x) < t\}$



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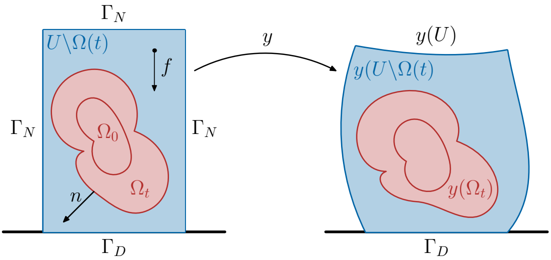
The model

Deformation

$y\colon U \to \mathbb{R}^d$

Accretion

$\Omega(t) \coloneqq \{\theta(x) < t\}$



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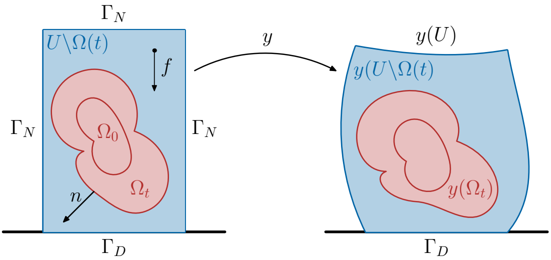

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Existence result

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Theorem (C.-Stefanelli, 2025)

There exist a solution $(y,\theta)\in C^1([0,T]\times U;\mathbb{R}^d)\times C^{0,1}(U)$ *to the system satisfying the Energy Equality*

$$\int_U \left(W^a(\nabla y)\mathbb{1}_{\Omega(t)}+W^r(\nabla y)\mathbb{1}_{U\setminus\Omega(t)}+H(\nabla^2 y)-f\cdot y\right)\mathrm{d}x - E_0$$

$$= -\int_0^t\int_U 2R^a(\nabla y,\nabla\dot y)\,\mathbb{1}_{\Omega(t)}+2R^r(\nabla y,\nabla\dot y)\,\mathbb{1}_{U\setminus\Omega(t)}+\dot f\cdot y\,\mathrm{d}x\,\mathrm{d}s$$

$$+\int_0^t\int_{\{\theta=s\}}\frac{W^a(\nabla y)-W^r(\nabla y)}{|\nabla\theta|}\mathrm{d}\mathcal{H}^{d-1}\mathrm{d}s.$$

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- A. Chiesa, U. Stefanelli. Viscoelasticity and accretive phase-change at finite strains. *Z. Angew. Math. Phys.* 76, 53 (2025)

Thank you for your attention!