BROADCASTING ON NETWORKS OF SENSORS COMMUNICATING THROUGH DIRECTIONAL ANTENNAS

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Abstract: In this paper we propose the use of random scaled sector graphs as the basis for a model for networks of sensors communicating through radio frequency using directional antennas. We propose two broadcasting algorithms and compare empirically their performance.

Keywords: Sensor networks, directional antennas, random scaled sector graphs, broadcasting.

1. Introduction

In future environment monitoring wireless networks of sensors will play an important role in sensing, collecting and disseminating information. In recent years, many papers have been devoted to network of sensors, see for example the surveys [Akyildiz et al., 2002], [A. Bharathidasas, 2002], and [Tilak et al., 2002]. The general setting is to have a large collection of wireless sensors randomly scattered in a remote or hazardous terrain, performing tasks of distributed sensing. The sensing information gathered by the sensors should be relaid to a base station. The sensor networks could be dynamic, where the sensors can move, or static where once a sensor falls in some place, it does not move. Sensor nodes are battery-operated, using non-rechargeable battery units, therefore the most important challenge in the design of communications protocols for networks of sensors is to minimize the consumption of energy. The lifetime of the system depends directly on the battery lifetime.

To communicate, among themselves or with the base, the sensors could use radio-frequency (RF) or optical communication. RF is the most widespread choice of communication for sensor networks. An omnidirectional transmission spreads the signal in a spherical region centered at the antenna. There are several inconveniences with omnidirectional radio broadcasting [Bao and Garcia-Luna, 2002], [Winters, 1999], a particular important issue is the signal interference. The usual way to deal with signal interference is to add to a network centralized control for managing synchronization and/or frequency assignment. However, a network of sensors cannot always afford to have a centralized control. An alternative is the use of directional antennas [Bao and Garcia-Luna, 2002], [Razavi et al., 1999]. Directional antennas have a focused beam which spans a sector of $\alpha$ degrees (see Figure 1 for a simplified draw). The beam angle can be as narrow as 5 or 10 degrees [Voipio and Vaanikainen, 1998]. In networks of sensors, directional antennas may have multiple advantages over omnidirectional antennas: less energy consumption, less fading area, furthermore as the transmission area is smaller the channel interference may have less influence.
Another way that sensors can communicate is using optical communication. Sensors send information using a laser (moving sideways about 40 degrees) and receive and decode the information by means of an (energy free) optical system of reflecting cubes and lenses. This system has some advantage over the radio-frequency communication: the sensors can be highly reduced in size; the system is basically interference-free; the system has less demands on energy consumption. On the other hand, an optical system needs free line of sight to communicate. Moreover, it seems more difficult to adaptate these kind of optical networks to dynamic conditions, the sensors must maintain communication while they move. The largest network of sensors using optical communication is the smart dust system developed at UC Berkeley [Kahn et al., 1999], [Warneke et al., 2001]. The smart dust system has been modeled using random sector graphs [Díaz et al., 2002],[Díaz et al., 2003].

In the present paper, we use the random sectors graphs to put forward a proposal for a model to design and analyze basic protocols on sensor networks with directional antenna communication.

Throughout the paper, we use standard terminology for directed and undirected graphs and for probability. The distance $d(x, y)$ between two points in the plane will be the Euclidean distance. Recall that a sequence of events holds with high probability (w.h.p.) if the probability of the $n$-th event is at least $1 - O(1/n^c)$ for some $c > 1$, and sufficiently large $n$. 

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2. Basic definitions

As in [Díaz et al., 2003], the objective of the network of sensors is to monitor a square region, divided into $s$ identical square cells, the size of this cells represents the sensing precision of the network. For sake of simplicity we assume that the region is the unit square $[0,1]^2$. Therefore, the cells have size $1/\sqrt{s} \times 1/\sqrt{s}$. We want to scatter uniformly at random a set of $n$ sensors, each one equipped with a RF system with a directional antenna. We assume that all sensors are equal and can broadcast in a sector of $\alpha$ degrees to a distance $r$. We also assume that the range of the sensors is at least one cell and at most a constant number of cells and therefore $r = g\sqrt{s}$ for some constant $g$. Observe that as $s$ grows, to keep $g$ constant, $r$ must decrease.

When sensor $i$ falls in $[0, 1]^2$, there will be a shift angle between the beam and the horizontal axis. We represent this angle as a random variable $b_i$ giving the “elevation” of the beam with respect to the horizontal direction. We represent the beam emitted by $i$ as the sector $S_i = S(x_i, b_i, \alpha, r)$, centered at $x_i$, with radius $r$, amplitude $\alpha$ and elevation $b_i$, see Figure 1. Every other sensor which falls inside of $S_i$ can potentially receive the signal emitted by $i$. We denote such a setting as the directed RF sensor system. In the present work, we disregard fading issues.

To model the communication of a directed RF sensor system we use a random model of geometric graphs. The following definition is from [Díaz et al., 2003].

**Definition 2.1.** Assume that $\alpha$ is an angle. Let $X = (x_i)_{i \geq 1}$ be a sequence of independently and uniformly distributed (i.u.d.) random coordinates of points in $[0,1]^2$, let $B = (b_i)_{i \geq 1}$ be a sequence of i.u.d. angles and let $R = (r_i)_{i \geq 1}$ be a sequence of numbers in $[0,1]$. For any natural $n$, we write $X_n = \{x_1, \ldots, x_n\}$ and $B_n = \{b_1, \ldots, b_n\}$. We call the digraph $G_n = G(X_n, B_n, r_n, \alpha)$ the random scaled sector graph on $n$ nodes, where $V(G_n) = \{1, \ldots, n\}$ and the arcs are defined by: $(i, j) \in E(G_n)$ iff $x_j \in S_i = S(x_i, b_i, \alpha, r_i)$.

Due to the size of the motes, all the motes emit RF by a single channel, therefore if two or more signals arrive at the same time to a sensor, an interference is produced, and the sensor will be unable to unscramble the messages. As the sensors communicate through RF, we assume that there will never be a failure in the communication other than the produced by interference of simultaneous receiving, however as the sensors are spread in the terrain unattended some of them may be unable to operate due to the position in which they fell. We will assume a constant
probability \( p \) for a sensor to remain operative after landing. Now we can define formally our model.

**Definition 2.2.** Assume that \( \alpha \) is a fixed parameter of the sensors. Let \( X = (x_i)_{i \geq 1} \) be a sequence of independently and uniformly distributed (i.i.d.) random coordinates of points in \([0,1]^2\); let \( B = (b_i)_{i \geq 1} \) be a sequence of i.i.d. angles. A random directed antenna sensor system \( \mathcal{DA} \) is defined by the parameters \((X, B, \alpha, r, s, p, n)\). It has \( n \) sensors \( \{1, \ldots, n\} \), sensor \( i \) has coordinates \( x_i \), is operative with probability \( p \), and can send a message to another operative sensor \( j \) when \( x_j \in S_i \).

The first concern is to bound the number of sensor needed to cover the sensing grid. The following result was proved in [Pírez et al., 2003].

**Theorem 2.1.** Let \( \mathcal{DA} \) be a random directed antenna sensor system with \( n \) sensors. Then, if \( n = \Omega(s \log s) \), w.h.p., each of the \( s \) cells in the precision grid contain at least one operative sensor.

Furthermore, if for some constant \( c > c_0 \), \( n = \frac{s^2}{r^2} \log s \) and \( g = r \sqrt{s} \) is a constant, the subjacent random geometric graph, formed by connecting to vertices at Euclidean distance less than \( r \), is connected w.h.p [Penrose, 1999]. Moreover, w.h.p., for any pair of sensors \( u, v \), that are at distance at least \( r \) of the border, there is a directed path from \( u \) to \( v \) and from \( v \) to \( u \) [Pírez et al., 2003]. We will restrict ourselves in this paper to random directed antenna sensor systems for which the above condition holds.

We will use the term **interior sensor** for a sensor that is at distance at least \( r \) of the border. Given an interior sensor \( u \) and a sensor \( v \), the probability that \( x_v \in S_u \) is \( \frac{1}{\alpha r} \). Using Chernoff's bound it is straightforward to prove the following result.

**Lemma 2.1.** Let \( \mathcal{DA} \) be a random directed antenna sensor system. Let \( Z \) be a random variable counting the number of sensors that fall in the sector \( S_u \) associated to an interior sensor \( u \). Then, \( E[Z] = \alpha r^2 n \). Moreover, the variable \( Z \) is concentrated around its mean value.

### 3. Broadcasting protocols

The first basic step in a sensor network is to develop an efficient protocol for broadcasting. We will have one interior sensor that wants to broadcast a message to all other interior sensors a message. We will measure efficiency through two components: the energy used by a sensor (in listening and sending) and the overall time needed for the protocol to finish.

We propose two broadcasting protocols, the first one the **deter-bro** protocol corresponds to a classical flooding algorithm. The second one,
the random-bro protocol, is a randomized protocol of the type medium access control protocol [Woo and Culler, 2001], [Ye et al., 2003].

For both protocols, at the beginning, the sensors are in a listening mode. After receiving the message they will switch to an active or a waiting mode being ready to emit the message in a future step. After emitting they will change to a sleeping mode, that will last until the end of the protocol.

To perform the broadcasting some synchronization is needed. The first broadcasting wave must synchronize the local clocks of the sensors. To do so, the message must incorporate the actual time in it, the number of the phase that is being performed and the total number of phases to finish the protocol. Let us describe our first broadcasting protocol.

<table>
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<th>dener-bro protocol</th>
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<td>Phase 1 (t = 0)</td>
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<td>The sensor u which wants to broadcast information, becomes active, broadcasts the message and changes to a sleeping state. All sensors that receive the message became active. (In case all sensors in $S_u$ become active as $u$ is the unique sensor transmitting.)</td>
</tr>
<tr>
<td>Phase 2 (1 ≤ t ≤ maxb) All active sensors broadcast the message and change to a sleeping state. All sensors that receive the message become active.</td>
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</table>

The theoretical analysis of the dener-bro protocol seems difficult. The first step to do is computing the expected increase in the number of active sensors from $t$ to $t+1$. The distribution of the sensors that become active in a phase, due to interference, is not uniform, and seems to oscillate.
heavily between consecutive phases, these facts introduce complicated variations in the analysis. Observe, that this is an important difference with the analysis of protocols on the random sector graphs arising as model for smart dust systems [Diaz et al., 2003]. However we conjecture that the number of phases will be at most twice the network’s diameter.

The energy consumption is limited to at worst the consumption due to listening duration, at most, the total number of phases, plus the energy required to emit once.

Experimental results show that for a system with $\alpha = 40$ degrees, $r = 0.1$ and $n = 2000$ after just 18 phases of the deter-bro protocol, 92% of the sensors got the message, see Figure 2. It is a bit surprising that experimentally, this protocol behaves so well, one will expect that due to the high interference the number of sensors not receiving the message would be higher. However this message reaches almost all sensors quite quickly. In Figure 3 we show the average number of phases, over up to 50 executions, necessary to complete the protocol.

Our second protocol incorporates randomization, in order to eliminate interference in one phase of the deter-bro protocol, at the expense of increasing the total duration of the protocol.
Figure 4. Percentage of sensors covered per phase by the deter-bro protocol, for $\alpha = 40$ degrees and $r = 0.1$

<table>
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<th>random-bro protocol</th>
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<tbody>
<tr>
<td>Phase 1 ($t = 0$)</td>
<td>The sensor $u$ which wants to broadcast information, became active, broadcast the message and changes to a sleeping state.</td>
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</table>
| Phase $l$ ($1 \leq l \leq \text{max}$) | Each active sensor $v$ computes $k_v := \text{rand} (1, \log^2 n)$ and change to a waiting state. From $i = 1$ to $\log^2 n$  
Each waiting sensors $v$ with $k_v = i$, $v$ broadcast the message and switches to a sleeping state. Any sensor that receives the message become active. |

So in this protocol, each mote which receives the message, uses randomization to select a time step, inside the following $\log^2 n$ steps, to emit the message. Using a classical hashing argument, the expected number of sensors which emit at the same time step is constant.

Using a balls and bins argument, together with Chernoff's bounds, Boole's inequality, and some results from random scaled graphs, we get

**Theorem 3.1.** Under the random-bro protocol, w.h.p., at each step almost all listening neighbors of an emitting sensor will receive the message.

Therefore the expected number of phases will be the network's diameter. As in the deter-bro protocol, each sensor emits only once. However, the time in which a mote is listening is increased by a factor of $O(\log^2 n)$. The only advantage of the random-bro protocol is that it guarantees that w.h.p all motes will receive the broadcasted message.

In Figure 5, we can see that most of the sensors get the message after 800 steps. In Figure 6, we show the expected number of phases that the
Figure 5. The trace of number sensors that get the message using the random-bro protocol, when $\alpha = 40$ and $r = 0.1$

Figure 6. Number of phases with respect to number of nodes for the random-bro protocol, for $\alpha = 40$ degrees and $r = 0.1$

The random-bro protocol performs on up to 50 executions for several inputs with up to 10000 nodes. In all cases 100% of the sensors received the message, see Figure 7.

4. Conclusions

In this paper we presented a model for network of sensors communicating through radio frequency using a directional antenna, with a single channel of frequency. The big issue to analyze is the effect of the
interference in the broadcasting schema. The random model proposed is grounded on the random scaled sector graphs introduced in [Díaz et al., 2002]. We deal only with the basic problem of broadcasting a message from a single source.

We have presented two broadcasting protocols. Empirically we have shown that the first protocol, the deter-bro protocol based on flooding, even producing more interference, can cover a huge portion of the network, in a relatively short time. The second protocol, the random-bro protocol a MAC type protocol, covers completely all the sensors, w.h.p. but at the cost of a greater number of iteration. Assuming the sensors remain in a listening mode until they get activated by hearing a signal (with slight consumption of energy), once activated they either broadcast the message at the next step or compute the future step to send. After emitting they shut-off (to spend 0 energy). The energy consumption is bigger in the random-bro protocol, as more sensors remain in the listening state for a longer period of time. However, the coverage is smaller in the deter-bro.

Many interesting problems remain open. The first one is to formally perform the analysis of the deter-protocol. Moreover, we have focused in this paper only in the broadcasting issue. It seems to us, that other tasks like designing localization or route establishment protocols must be considered. Recall that the objective of localization protocol is that all sensors know approximately their position in the terrain. In the route establishment the final objective is to establish a schedule in which the sensed information must be sent through to the base station. The ideas
presented for solving the problems on optical smart dust networks [Diaz et al., 2003], could be a basis to design and evaluate protocol for networks of sensors communicating through beams of RF. However, interference play a significant different role and lots of details should be worked out.

Other model variations should also be investigated: For example, when the sensors have a constant number of different frequency channels, or we have different sets of sensors with different or adjustable or reconfigurable technical characteristics, like variable range of emission or variable beams that span sectors with different angles.
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