Exercise Sheet 5, Proseminar Stochastic Processes
Winter Semester 2016-17, 250069 PS

Exercise 1 *The Taqqus of the Planet Koozebane each have $K \in \mathbb{N}$ offspring before they evaporate. For each Taqqu, K is independent of everything else and has distribution $\mathbb{P}(K = 0) = \mathbb{P}(K = 1) = \frac{1}{4}$, $\mathbb{P}(K = 2) = \frac{1}{2}$.*

1. *Compute the generating function and the moment generating function of K.*
2. *Assume that X_n is the total population of Taqqus, starting with $X_0 = 1$. Compute the probability that the Taqqus go extinct.*
3. *Show that, provided the Taqqus survive for n steps, that the probability that they die out decreases to zero, exponentially in n.*

Exercise 2 *Let $N_t \simeq Pois(\lambda t)$. Show that $\mathbb{P}(N_{t+h} = 0) = (1 - \lambda h - o(\lambda h))\mathbb{P}(N_t = 0)$. If we write $p(t) = \mathbb{P}(N_t = 0)$, show that $p(t)$ satisfies the differential equation $p'(t) = -\lambda p(t)$. Solve this equation.*

Exercise 3 *Telephone calls arrive at the station according to a Poisson process with an hourly rate λ.*

1. *The phone equipment is not entirely functioning: a phonecall is not properly connected with probability q. Show that the number of properly received calls has distribution $Pois(\lambda(1 - q)t)$ for time unit an hour.*
2. *A second stream of phone calls comes in with hourly rate μ. Find the distribution of total number of incoming calls. After putting down the phone, how much time does the operator have to wait on average for the next call?*

Exercise 4 *Let $N_t \simeq Pois(\lambda t)$. Show that $\mathbb{P}(N_t \text{ is even}) = e^{-\lambda t}\cosh(\lambda t)$ and $\mathbb{P}(N_t \text{ is odd}) = e^{-\lambda t}\sinh(\lambda t)$.*