BGG Sequences and Geometric Overdetermined Systems

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- The machinery of BGG sequences provides a systematic construction of invariant differential operators for these geometries.
- The first operators in each BGG sequence define an overdetermined system, and the construction provides a partial prolongation of this system.
- This can be extended to a full prolongation for the BGG operators and often also for semi–linear systems with the same principal symbol.

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Structure

parabolic subalgebras

Parabolic subgroups form a special class of subgroups in semi–simple Lie groups. In the complex case, $P \subset G$ is parabolic if and only if G/P is compact. These subgroups can be completely classified in terms of structure theory. A simple description valid over $\mathbb{R}$ and $\mathbb{C}$ is via $|k|$–gradings.

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Let $\mathfrak{g}$ be a semisimple Lie algebra. A subalgebra $\mathfrak{p} \subset \mathfrak{g}$ is called *parabolic* iff there is a grading $\mathfrak{g} = \mathfrak{g}_{-k} \oplus \cdots \oplus \mathfrak{g}_k$ of $\mathfrak{g}$ such that

- no simple ideal of $\mathfrak{g}$ is contained in $\mathfrak{g}_0$
- $\mathfrak{g}_- := \mathfrak{g}_{-k} \oplus \cdots \oplus \mathfrak{g}_{-1}$ is generated by $\mathfrak{g}_{-1}$
- $\mathfrak{p} = \mathfrak{g}_0 \oplus \cdots \oplus \mathfrak{g}_k$

parabolic subgroups; generalized flag varieties

A subgroup P in a semisimple Lie group G is called *parabolic* iff its Lie algebra $\mathfrak{p}$ is parabolic in $\mathfrak{g}$. Then G/P is compact, and there are natural subgroups $G_0, P_+ \subset P$ such that

- G_0 has Lie algebra $\mathfrak{g}_0$ and is reductive
- P_+ has Lie algebra $\mathfrak{p}_+ := \mathfrak{g}_1 \oplus \cdots \oplus \mathfrak{g}_k$, it is nilpotent and normal in P, and $\exp : \mathfrak{p}_+ \to P_+$ is a diffeomorphism
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Homogeneous spaces of the form G/P are called *generalized flag varieties*. A crucial example to keep in mind is $G = SO(n+1,1)$, $P \subset G$ is the stabilizer of an isotropic line, $G/P \cong S^n$ and G is the group of conformal isometries of S^n. Here $k = 1$, $P_+ \cong \mathbb{R}^n$ is Abelian and $G_0 \cong CO(n)$.

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- Among these there are conformal, projective, almost Grassmannian and almost quaternionic structures.
- More general examples include hypersurface–type CR structures, path geometries, quaternionic contact structures, and several types of generic distributions in low dimensions.

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The structure of P and hence its representation theory are complicated. Since $G_0 \cong P/P_+$, any representation of G_0 gives rise to a representation of P. These are exactly the completely reducible representations, and they correspond to the "usual" geometric objects.

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In the discussion of invariant operators, there is a fundamental distinction into regular and singular infinitesimal character, referring to weights in the interior or in the boundary of a Weyl chamber.

BGG sequences

In regular infinitesimal character, BGG sequences provide a construction of all standard invariant operators on G/P as well as distinguished curved analogs of these operators. The method was introduced by A.C., J. Slovák, V. Souček (Ann. of Math., 2001) and improved by D. Calderbank and T. Diemer (Crelle, 2001).

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Let $\mathbb{V}$ be a representation of G. Restricting to P, this gives rise to a natural bundle ("tractor bundle") VM on any manifold M endowed with a parabolic geometry $(p : \mathcal{P} \to M, \omega)$ of type (G, P). The Cartan connection ω induces a linear connection ∇ on VM, so one has the twisted de–Rham sequence $(\Omega^*(M, VM), d^\nabla)$.

The bundles $\Lambda^j T^*M \otimes VM$ in the twisted de–Rham sequence correspond to the P–representations $\Lambda^j\mathfrak{p}_+ \otimes \mathbb{V}$. There are P–equivariant linear maps

$$\partial^* : \Lambda^j\mathfrak{p}_+ \otimes \mathbb{V} \to \Lambda^{j-1}\mathfrak{p}_+ \otimes \mathbb{V}$$

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$\partial^* \circ \partial^* = 0$ and the P–representations $H_j(\mathfrak{p}_+, \mathbb{V}) = \ker(\partial^*)/\operatorname{im}(\partial^*)$ are induced by representations of G_0, which are algorithmically computable by Kostant's version of the Bott–Borel–Weil theorem.

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All these constructions have geometric counterparts:

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The core of the method is the construction of invariant differential operators $L : \Gamma(\mathcal{H}_j) \to \Omega^j(M, VM)$ which split the canonical projection $\pi_H : \ker(\partial^*) \to \mathcal{H}_j$. While these operators are complicated, they are characterized by this fact and by $\partial^* \circ d^\nabla \circ L = 0$. Then the BGG operator $D^V : \Gamma(\mathcal{H}_j) \to \Gamma(\mathcal{H}_{j+1})$ is defined by $D^V := \pi_H \circ d^\nabla \circ L$.

The first operators in a BGG sequence

$\mathcal{H}_0$ always corresponds to the P–irreducible quotient of $\mathbb{V}$. The first BGG operator $D^V : \Gamma(\mathcal{H}_0) \to \Gamma(\mathcal{H}_1)$ splits according to the decomposition of $H_1(\mathfrak{p}_+, \mathbb{V})$ into irreducible components. The construction directly implies that π_H and L induce inverse bijections $\Gamma(\mathcal{H}_0) \supset \ker(D^V) \cong \{s \in \Gamma(VM) : \nabla s \in \Gamma(\mathrm{im}(\partial^*))\}$

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In the case $k = 1$:

- $\mathcal{H}_1$ is an irreducible and $\cong S^r T^*M \circledcirc \mathcal{H}_0$, the highest weight component in $S^r T^*M \otimes \mathcal{H}_0$.
- The principal symbol of D^V is induced by the projection to the highest weight subspace.
- For any irreducible bundle $E \to M$ and any $r > 0$, there is a unique irreducible $\mathbb{V}$ such that $\mathcal{H}_0 \cong E$ and $\mathcal{H}_1 \cong S^r T^*M \circledcirc E$.

Examples of first BGG operators for conformal structures ($G = SO(n+1,1)$, $G_0 = CO(n)$):

- For $E = \mathcal{E}[1]$ (densities) and $k = 2$, we obtain $\mathbb{V} = \mathbb{R}^{n+1,1}$, $\mathcal{H}_1 = S^2_0 T^*M[1]$ (weighted tracefree symmetric two–tensors), and $D^V f = (\nabla_{(a}\nabla_{b)_0} + \mathrm{P}_{(ab)_0})f$. $\ker(D^V)$ is in bijective correspondence with Einstein metrics in the conformal class.

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- For $E = TM$ and $k = 1$, one gets $\mathbb{V} = \mathfrak{g} \cong \Lambda^2\mathbb{R}^{n+1,1}$, $\mathcal{H}_1 = S^2_0 T^*M[2]$ and the conformal Killing equation. Solutions are conformal Killing fields, i.e. infinitesimal automorphisms of the conformal structure.

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- Similarly, putting $E = \Lambda^j T^*M[w]$ for appropriate w, one gets $\mathbb{V} = \Lambda^{j+1}\mathbb{R}^{n+1,1}$ and the conformal Killing equation on forms, while for $E = S^r_0 TM[w]$ with appropriate w, one gets $\mathbb{V} = \circledcirc^r\mathfrak{g}$ and the conformal Killing equation on tracefree symmetric r–tensors.

Structure

As we have seen, the construction of the first BGG operator(s) D^V corresponding to $\mathbb{V}$ implies that there is a bijection

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To extend this to a full prolongation of D^V, one first observes that if $\nabla s = \partial^* \psi$ for some $\psi \in \Omega^2(M, VM)$, then $\partial^* \psi$ can be computed (invariantly) via iterated cross–differentiation. This leads to an equivalent equation $\nabla s + B(s) = 0$ for a higher order operator B.

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As $\mathfrak{g}$, also $\mathbb{V}$ is naturally graded $\mathbb{V} = \mathbb{V}_0 \oplus \cdots \oplus \mathbb{V}_N$ in such a way that $\mathfrak{g}_i \cdot \mathbb{V}_j \subset \mathbb{V}_{i+j}$. This gives a notion of homogeneity for VM–valued forms, which plays a crucial role in all constructions. It turns out that the higher order parts of B also raise homogeneity. Looking at $\nabla s + B(s) = 0$ homogeneity by homogeneity and inserting lower homogeneities into higher ones, one can equivalently rewrite it (non–invariantly) as $\nabla s + C(s) = 0$ for a bundle map $C : \mathcal{H}_0 \to \mathcal{H}_1$.

Ignoring the issues of invariance, a simpler version of the BGG procedure for $k = 1$ was developed in joint work with T. Branson, M. Eastwood, and R. Gover (Int. J. Math., 2006). We start with a reduction of structure group to the semisimple part G_0' of G_0, e.g. a Riemannian metric, and a fixed connection ∇ on the corresponding principal bundle, e.g. the Levi Civita connection.

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Given a bundle $E \to M$ corresponding to an irreducible representation of G_0' and $r \geq 1$, let $\mathbb{V} = V_0 \oplus \cdots \oplus V_N$ be the corresponding representation of G. Restricting this to G_0', one obtains a bundle $V \to M$ with a natural projection $\pi : V \to E$. Now one constructs

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- A linear connection $\tilde{\nabla}$ on V
- A linear differential operator $L : \Gamma(E) \to \Gamma(V)$ of order N

such that

Theorem

For any semi–linear differential operator $D:\Gamma(E)\to\Gamma(S^rT^*M\circledcirc E)$ of order r with principal symbol $S^rT^*M\otimes E\to S^rT^*M\circledcirc E$ induced by the highest weigh projection, there is a smooth map $C:E\to S^rT^*M\circledcirc E$ such that $\pi:V\to E$ and $L:\Gamma(E)\to\Gamma(V)$ induce inverse bijections

$$\{\sigma\in\Gamma(E):D(\sigma)=0\}\cong\{s\in\Gamma(V):\nabla s+C(s)=0\}.$$

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While there is an explicit procedure how to compute C, it is crucial that V and L are universal. In particular, for any D with the right principal symbol, if $D(\sigma) = 0$, then σ is uniquely determined the value of $L(\sigma)$ and hence its N–jet in a single point. Likewise, if D is linear, then $\dim(\ker(D)) \leq \dim(\mathbb{V})$. Both N and $\dim(\mathbb{V})$ can be easily computed from E and r using representation theory.

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- Results towards characterization of linear operators with principal symbol of the given type with kernel of the maximal possible dimension $\dim(\mathbb{V})$.
- Prolongation procedure for BGG operators which does not break invariance. This leads to natural connections on tractor bundles (different from the canonical tractor connections) whose parallel sections correspond to solutions of the first BGG operator. (M. Hammerl, J. Šilhan, V. Souček, P. Somberg)
- Extension of the simpler non–invariant prolongation method to cases with $k > 1$ (K. Neusser). For example, this leads to prolongation procedures for certain overdetermined systems on contact manifolds.