Smooth multiparameter perturbation of polynomials and operators

Armin Rainer
(Universität Wien)

Abstract: Let \( P(x)(z) = z^n + \sum_{j=1}^{n} (-1)^j a_j(x) z^{n-j} \) be a family of polynomials whose coefficients \( a_j \) are smooth complex valued functions defined near \( 0 \in \mathbb{R}^q \). For generic \( P \) we construct a finite collection \( \mathcal{T} \) of transformations \( \Psi : \mathbb{R}, 0 \to \mathbb{R}, 0 \), such that
\[
\bigcup \{ \text{im}(\Psi) : \Psi \in \mathcal{T} \}
\]
is a neighborhood of 0, which desingularizes the roots of \( P \), i.e., for each \( \Psi \in \mathcal{T} \), the roots of \( P \circ \Psi \) allow smooth parameterizations near 0. As a consequence we prove that generic \( P \) locally admit roots with first partial derivatives in \( L^1 \). Moreover, we deduce a simple proof of Bronshtein’s theorem (under slightly stronger conditions): For hyperbolic (not necessarily generic) \( P \) with \( C^{n(n+1)/2} \) coefficients any continuous arrangement of its roots is locally Lipschitz. There are applications to the perturbation theory of linear operators.