RANDOM WALKS ON GROUPS, 2023 SS EXERCISES E

(1) Recall that for functions $f, g \colon \mathbb{N} \to \mathbb{N}$ we say $f \leq g$ if there are L and A such that $f(n) \leq Lg(Ln+A) + A$, and $f \approx g$ if $f \leq g$ and $g \leq f$.

Show that $n \mapsto n^d$ and $n \mapsto n^{d'}$ are not equivalent if $d \neq d'$.

Conclude that being of polynomial growth of degree d is well defined up to \approx .