

$$\lceil \log_2(n+1) \rceil$$

Testfrage:

↑ Warum ... +1?

$$\lceil \log_2 \left( \lceil (n+1)/2 \rceil \right) \rceil$$

↑ Warum kein +1?

Seite 27 Erinnerung: Iversons Notation

$$[n \equiv 0(2)] := \begin{cases} 1 & \text{wenn } n \equiv 0 \pmod{2} \\ 0 & \text{wenn } n \not\equiv 0 \pmod{2} \end{cases}$$

$$n = 2k \quad \left\lceil \frac{n+1}{2} \right\rceil = \left\lceil \frac{2k+1}{2} \right\rceil = \left\lceil k + \frac{1}{2} \right\rceil = k+1$$

$$\frac{n+1}{2} + \frac{[n \equiv 0(2)]}{2} = \frac{2k+1}{2} + \frac{1}{2} = k+1$$

$$n = 2k+1 \quad \left\lceil \frac{n+1}{2} \right\rceil = \left\lceil \frac{2k+2}{2} \right\rceil = k+1$$

$$\frac{n+1}{2} + \frac{[n \equiv 0(2)]}{2} = \frac{2k+2}{2} + 0 = k+1$$

$$\begin{aligned} \left\lceil \frac{(n+1)}{2} \right\rceil &= \\ &\Rightarrow \\ &= \frac{n+1}{2} + \frac{[n \equiv 0(2)]}{2} \end{aligned}$$

$$\lceil x+k \rceil = \lceil x \rceil + k, \quad \forall x \in \mathbb{R}, \forall k \in \mathbb{Z} \quad \Rightarrow$$

$$1 + \lceil \log_2 \left\lceil \frac{(n+1)}{2} \right\rceil \rceil = \left\lceil 1 + \log_2 \left( \frac{n+1}{2} \cdot \frac{n+1 + [n \equiv 0(2)]}{n+1} \right) \right\rceil =$$

$$= \left\lceil \underbrace{\log_2 2}_1 + \log_2 \frac{n+1}{2} + \log_2 \left( 1 + \frac{[n \equiv 0(2)]}{n+1} \right) \right\rceil = \left\lceil \log_2 2 \cdot \frac{n+1}{2} \cdot \left( 1 + \frac{[n \equiv 0(2)]}{n+1} \right) \right\rceil$$

$\log_2 x \cdot y = \log_2 x + \log_2 y$        $\log_2 x + \log_2 y = \log_2 x \cdot y$

$$n = 2^r - 2 \mapsto r, \text{ d.h. } \lceil \log_2(2^r - 2 + 1 + 1) \rceil = r \text{ und}$$

$$\lceil \log_2(2^r - 2 + 1) \rceil = r \quad \text{↑ Warum?}$$