GROUPS ADMITTING AFFINE CRYSTALLOGRAPHIC ACTIONS

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In this talk I will discuss affine and nil-affine crystallographic actions of groups. In the seventies J. Milnor studied fundamental groups of complete affinely flat manifolds. In this context he proved that any virtually polycyclic group Γ admits a faithful and properly discontinuous action by affine transformations $x \mapsto Ax + b$ on some Euclidean space $\mathbb{R}^n$. He conjectured that Γ can act also with compact quotient in this way, i.e., that Γ admits an affine crystallographic action. This was an open problem for a long time. However, it turned out to be false in general.

In a more general setting however, every virtually polycyclic group Γ admits a so called *nil-affine* crystallographic action. We will explain this generalization and present joint work with Karel Dekimpe on this topic. This includes results on the "generalized Auslander conjecture", on simply transitive actions of a given Lie group G on some other nilpotent Lie group N; on the translation to the Lie algebra level, and on the study of associated compatible algebra structures on the Lie algebra of N.

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