Topology on locally finite metric spaces

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Abstract: In this talk I want to introduce a way to do General Topology and Algebraic Topology on locally finite metric spaces. I will define the discrete counterparts of the classical notions of continuous function, homeomorphism and homotopic equivalence and I will define the fundamental group of a so- called path-connected locally finite metric space. I will discuss some basic but important properties, as the discretization theorem: the classical fundamental group of a compact metrizable path-connected manifold is the same as the discrete fundamental group of the natural graph of a fine enough triangulation of the manifold. As application, I will introduce the isoperimetric constant of a locally finite metric space as an invariant for this homotopy theory and I will use it to derive the apparently first known purely metric description of amenability of a finitely generated group.