Horocyclic products of trees

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Abstract: This is a talk on a slightly older paper by Bartholdi, Neuhauser and myself (JEMS, 2008) about which I have not spoken before in Vienna. The horocyclic product of homogenous trees $T_1, \dots, T_d$ is obtained by taking the Busemann function with respect to a reference end (boundary point) in each tree and considering all d–tuples in the direct procut of the tree whose Busemann functions sum up to 0. Thus, we obtain a horosphere in the product of the trees. It can be equipped with a natural graph structure and becomes a vertex-transitive graph.

We describe the isometry group and address the question when the horocyclic product is a Cayley graph of a finitely generated group. This is related with the concept of finding cocompact lattices in Lie groups.

Further topics concern spectrum of simple random walk as well as the Poisson boundary.