Colouring hyperplanes of CAT(0) cube complexes

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Abstract: In the talk, I will present positive and negative results on colouring contact and crossing graphs of hyperplanes of CAT(0) cube complex X with uniformly bounded degrees. The crossing graph of X is the intersection graph of the hyperplanes of X and the contact graph of X is the intersection graph of carriers of the hyperplanes of X. The chromatic number of the crossing graph is the minimum number of tree factors so that X isometrically embeds into their Cartesian product.

I will show that for any 2–dimensional CAT(0) cube complex of maximum degree d, the chromatic number of its contact graph (and therefore of its crossing graph) is at most $q(d) = Md^{15}$, for a fixed constant M. On the other hand, I will present an example (based on Burlings construction) of a 4–dimensional (respectively, 5–dimensional) CAT(0) cube complex with uniformly bounded degrees whose contact graph (respectively, crossing graph) has infinite chromatic number. The second result answers in the negative a question raised independently by F. Haglund, G. Niblo, M. Sageev, and the speaker. Joint work with Mark Hagen.