Colouring hyperplanes of CAT(0) cube complexes

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Abstract: In the talk, I will present positive and negative results on colouring contact and crossing graphs of hyperplanes of CAT(0) cube complex $X$ with uniformly bounded degrees. The crossing graph of $X$ is the intersection graph of the hyperplanes of $X$ and the contact graph of $X$ is the intersection graph of carriers of the hyperplanes of $X$. The chromatic number of the crossing graph is the minimum number of tree factors so that $X$ isometrically embeds into their Cartesian product.

I will show that for any 2-dimensional CAT(0) cube complex of maximum degree $d$, the chromatic number of its contact graph (and therefore of its crossing graph) is at most $q(d) = M d^{15}$, for a fixed constant $M$. On the other hand, I will present an example (based on Burlings construction) of a 4-dimensional (respectively, 5-dimensional) CAT(0) cube complex with uniformly bounded degrees whose contact graph (respectively, crossing graph) has infinite chromatic number. The second result answers in the negative a question raised independently by F. Haglund, G. Niblo, M. Sageev, and the speaker. Joint work with Mark Hagen.