

# CAT(0) spaces with polynomial divergence of geodesics

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Abstract: We will introduce divergence of geodesics as a geometric invariant of groups, and describe examples of CAT(0) complexes with polynomial divergence of geodesics of degree greater than two, answering a question of Gersten if such CAT(0) complexes exist. Gersten posed this question after constructing a CAT(0) 2–complex with quadratic divergence and therefore showing that the expectation, which he attributes to Gromov, that geodesics diverge either linearly or exponentially in non-positively curved spaces fails for CAT(0) complexes. We construct a family of finite 2–dimensional cube complexes whose universal covers are CAT(0) and have the polynomial divergence of geodesics of desired degree. These examples are mapping tori of polynomially growing automorphisms of free groups, and our construction illustrates the ideas behind the proof that the mapping torus of a free group automorphism which grows polynomially with degree  $d$  has a polynomial divergence of degree  $d + 1$ . This result implies that the mapping tori of two polynomially growing automorphisms which grow with different degrees cannot be quasi-isometric, establishing the first step in classification of mapping tori of automorphism of free groups according to their geometric properties.