Topics in Algebra: Cryptography

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universität wien

Digital currency

Two major functionalities:

- A reliable process to produce money;
- A reliable process to record transactions.

Earlier currencies: centralized architecture, transactions are secrete.

Bitcoin etc.: distributed architecture, transactions are public.

Digital currency

Three major security requirements:

- Non-repudiation of transactions: if you commit to pay you cannot later deny it.
- Integrity of the entire transaction data set: it is correct and consistent, e.g. no double spending.
- Pseudonymity of transactions: one can dissociate the identity of a holder from any transaction.

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The confidentiality of transactions is not required: they are public.

The non-repudiation is required for individual transactions.

The distributed architecture: it is easy to verify but not to modify, so the data origin authentication of a wider transaction data set is not required.

Distributed ledger

Definition: Distributed ledger=Distributed Ledger Technology (DLT)

This is a digital database that is replicated, shared and synchronized by a consensus algorithm, across multiple nodes of a peer-to-peer network.

Distributed ledgers may be unpermissioned or permissioned regarding if anyone or only approved nodes can validate transactions.

Distributed ledger

Each node replicates and saves an identical copy of the ledger and updates itself independently.

When a ledger update happens, each node constructs the new transaction, and then the nodes vote by the consensus algorithm on which copy is correct.

Once a consensus has been determined, all the other nodes update themselves with the new, correct copy of the ledger.

Blockchain

A blockchain is one example of a distributed ledger. It can be either public or private. It is an unpermissioned ledger.

It contrasts with conventional ledgers as it sets rules about a transaction that are tied to the transaction itself (not to the entire database).

Security is through cryptographic hash functions and digital signatures.

The consensus algorithm is through mining.

Block

Definition: Block

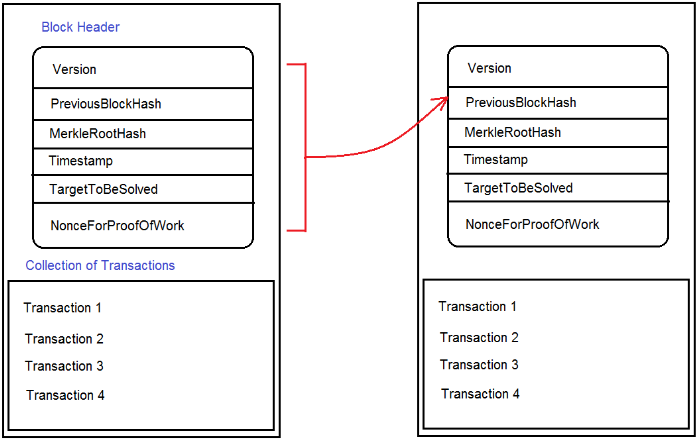
This is a data item consisting of:

- A block header, including a block hash of the block header of the previous block in the blockchain;
- A list of transactions: each accepted transaction is in a block of the blockchain.

A genesis block is the very first block, created in 2009.

Every 10 minutes, on average, a new block is appended to the blockchain through mining.

Block



A block in a blockchain. [image: blog.brakmic.com]

Blockchain

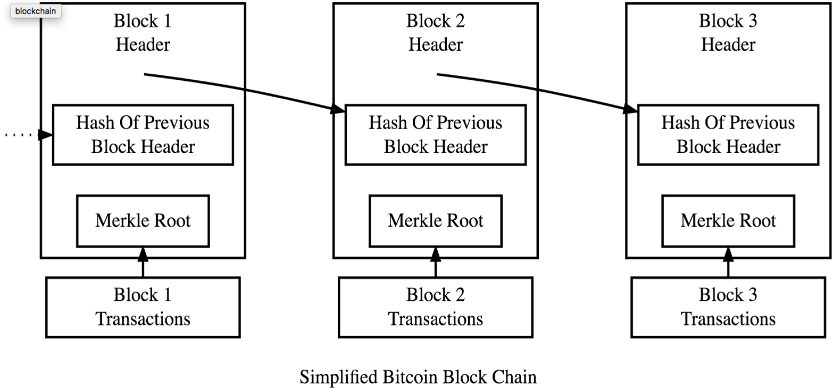
Definition: Blockchain is the distributed ledger supporting bitcoin:

It is a public record of a sequence of blocks (= block header + transactions) in chronological order.

It is used to verify the permanence of bitcoin transactions and to prevent double spending.

- The blockchain is stored and maintained by the nodes (users) in the bitcoin network;
- Some nodes store the full blockchain, while others are lightweight and only store block headers;
- Two major functionalities are guaranteed by mining;

Blockchain



Blockchain. [image: Satoshi Nakamoto]

Bitcoin address

A bitcoin address is a string of 26-35 alphanumeric characters; it starts with 1 or 3 and represents a destination for a bitcoin payment.

It is a destination ‘account’, to which the bitcoin can be transferred.

It is available to anyone from whom the user wishes to receive a transaction (like a public key).

A unique address is used for each transaction. A user can have many addresses.

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Definition: Bitcoin address

A bitcoin address is a 160-bit hash of the public portion of a public/private ECDSA keypair.

The verification key of a verification / signature key pair for the ECDSA.

The verification key is $\sim$ 512-bits, the signature key is $\sim$ 256-bits.

Bitcoin address

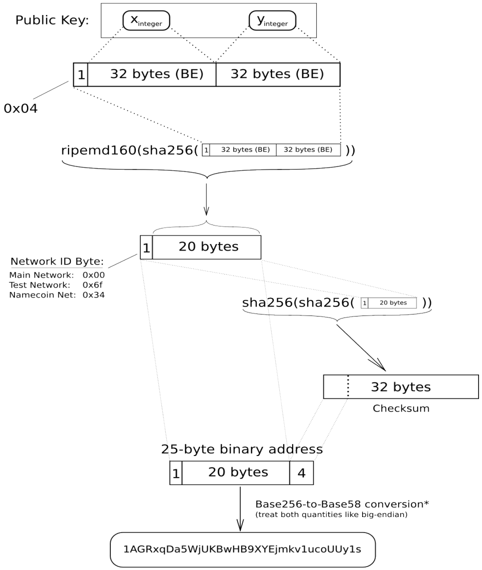
The bitcoin address is derived as follows:

1. Hash the verification key using SHA-256;
2. Hash the result using RIPEMD160;
3. Base58 encode the result (to convert binary to alphanumeric).

The verification key $\longrightarrow$ a pseudorandom string of characters.

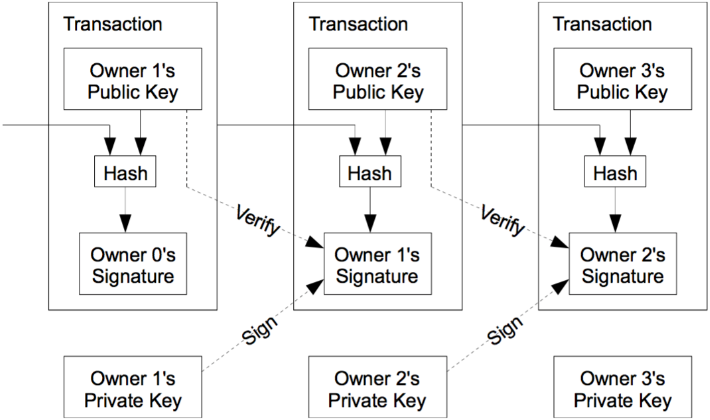
The encoding also provides a possibility for a vanity address.

Elliptic-Curve Public Key to BTC Address conversion



Bitcoin transaction

A bitcoin transaction is a digitally signed statement committing a payment to a bitcoin address.



Chaining of transactions. [image: Satoshi Nakamoto]

Bitcoin transaction

A bitcoin transaction is a digitally signed statement committing a payment to a bitcoin address.

1. Input= the UTXO from a previous transaction, with scriptSig.
2. Output= the UTXO binding (with scriptPubKey) the recipient bitcoin address to a transferred amount.
3. Total input = transferred amount + fee (goes to miners).

Unspent Transaction Output=UTXO

scriptPubKey = locking, conditions required to spend the output.

scriptSig = unlocking, conditions allowing the output to be spent.

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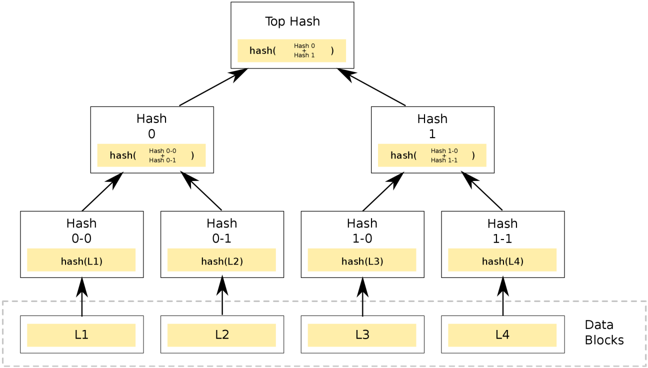
One has to provide a digital signature and the corresponding public key as the unlocking script in the transaction input in order to allow the output (from a previous transaction) to be spend.

Transaction verification

A lightweight node stores a block header.

Such nodes use a Merkle tree’1979 (a hash tree) to verify whether a transaction belongs to a given block.



Hash tree. [image: Wikimedia]

Transaction verification

The hash function is the application of SHA-256 twice:
$h(x) =$SHA-256(SHA-256(x)).

A lightweight node stores a block header, a 256-bit root value.

Such a lightweight node gets a verification path of intermediate hash values, sufficient to compute the root from a transaction.

Example: a bitcoin block containing 1024 transactions requires a verification path of 10 intermediate hash values.

Proof-of-work

Definition: Proof-of-work (PoW) — Dwork-Naor'1993

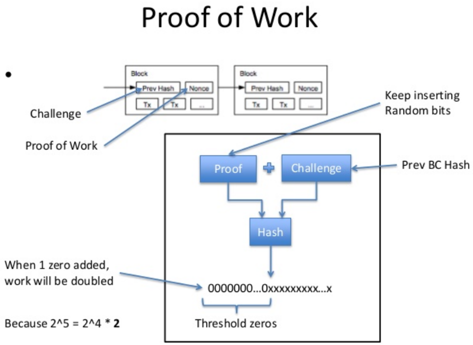
A consensus protocol, i.e. an irrefutable system to achieve agreement between various devices across a distributed network preventing exploitation.

In bitcoin, it is used to achieve both of the two major functionalities: to produce money and to validate transactions.

21 million of bitcoins in total, by 2140.

Bitcoin mining

The bitcoin mining is an ‘exhaustive’ search for preimages of the SHA-256(SHA-256(·)) hash function, trying to hash on random inputs.



Proof-of-work. [image: canardcoincoin.com]

As fo Aug. 2017, 72 of 256 hash bits of the target output must be zero.

Bitcoin mining

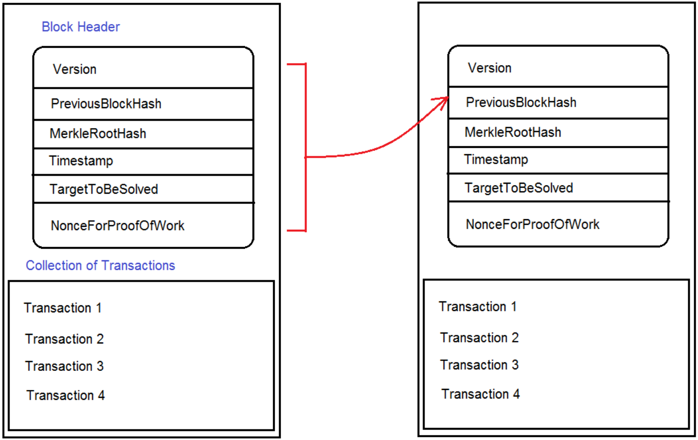
A set preimage resistance: given a set Z of potential outputs of the hash function h, it is difficult to find x with $h(x) = z$ for some $z \in Z$.

The difficulty is controlled by the size of Z.

Z is 'evenly spread' among possible outputs (the hash function provides a pseudorandom output).

Z is defined to be the set of the hash function outputs smaller than a particular target output (the order is on binary numbers).

Reminder: Block



A block in a blockchain. [image: blog.brakmic.com]

More on design

Blockchain forks are possible but they can be managed:
– the longest blockchain version is accepted;
– the transactions from a 'losing' block are returned to the pool of 'floating transactions'.

Flexibility:
– the difficulty of mining is adjusted to have a new block on average every 10min;
– 10min. is a trade-off between the speed of new block acceptance and the risks of forking.

Environmental impact!

More on security

The main security issue is in logistic:
– loss of signing key;
– malicious mining farms.

The cryptographic security is in the hash functions:
– for data integrity;
– for pseudorandomness;
– for difficulty of mining.

and the ECDSA.

Test questions

Question 20

(a) What other uses of cryptographic proofs-of-work do you know?

(b) What are (dis)advantages of deploying distributed ledgers?

Question 21

What is the length (=the number of intermediate hash values) of a verification path in the Merkle tree having n transactions? What is it for k-ary tree with n-leaves?

Question 22

Why in your opinion is the difficulty of the proof of work in Bitcoin set to 10 minutes? What would go wrong if it was changed to 60 minutes or 10 seconds?