Representation Theory of Groups - Blatt 2

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2016

Goulnara Arzhantseva
goulnara.arzhantseva@univie.ac.at

Martin Finn-Sell
martin.finn-sell@univie.ac.at

Question 1. Let $G = C_2 \times C_4 \times C_5$, where C_n denotes the cyclic group of order n. Calculate all the irreducible representations of G.

Question 2. Construct two non-isomorphic non-faithful irreducible representation of the restricted wreath product $S_3 \wr C_2$.

Question 3. Let G be a group and H a subgroup of G. Show that any irreducible representation of G is contained in some induced irreducible representation of H.

Question 4. Let $G = C_p = \langle g | g^n = 1 \rangle$ and consider the map $\rho : G \to GL_2(\mathbb{F}_p)$ given by:

$$g \mapsto \begin{pmatrix} 1 & g \\ 0 & 1 \end{pmatrix}.$$

Check that this defines a representation of G, and show that Maschke's Theorem fails for the 1-dimensional subspace of $\mathbb{F}_p$ fixed by ρ.

Question 5. Let G be a finite group.

a) Show that for any irreducible representation ρ of G with character χ, the set

$$\ker(\chi) := \{g \in G \mid \chi(g) = \chi(1)\}$$

is a normal subgroup of G;

b) Show that each normal subgroup $N \triangleleft G$ is the intersection of subgroups of the form $\ker(\chi)$, where χ is the character of an irreducible representation.

Question 6. Let G be a finite group, X be a finite set on which G acts, ρ denote the corresponding permutation representation and χ the corresponding character. For every $g \in G$ show that $\chi(g)$ is the number of elements of X fixed by g.