Representation Theory of Groups - Blatt 3

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2016

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Question 1. Let $G = C_p = \langle g | g^n = 1 \rangle$ and consider the map $\rho : G \to GL_2(\mathbb{F}_p)$ given by:

$$g \mapsto \begin{pmatrix} 1 & g \\ 0 & 1 \end{pmatrix}.$$

Check that this defines a representation of G, and show that Maschke's Theorem fails for the 1-dimensional subspace of $\mathbb{F}_p$ fixed by ρ.

Question 2. Let G be a group and H a subgroup of G. Show that any irreducible representation of G is contained in some induced irreducible representation of H.

Question 3. Let $G = S_3$ be the permutation group on the set $X = \{1, 2, 3\}$ and let $\rho : G \to GL(\mathbb{C}X)$ be the corresponding permutation representation. Use Maschke's theorem to find the projection onto the vector subspace $U := span(\delta_1 + \delta_2 + \delta_3)$, and construct the G-stable compliment of U inside $\mathbb{C}X$. What is the corresponding projection onto this compliment?

Question 4. Let $G = C_3 = \langle g \mid g^3 = 1 \rangle$ be the cyclic group of order 3. Let V be the 2 dimensional vector space on the letters v_1 and v_2. Let G act on V by extending the following formulae linearly:

$$\rho(g)(v_1) = v_2, \rho(g)(v_2) = -(v_1 + v_2).$$

a) Show that ρ defines a representation of G on V;

b) Express V as a sum of G-stable irreducible subspaces.

Question 5. Let G be a finite group and let $\rho : G \to GL(2, \mathbb{C})$ be a representation of degree 2. Prove that if there exists elements $g, h \in G$ such that $\rho(g)$ and $\rho(h)$ do not commute, then ρ is irreducible.