

A mountain pass theorem (existence and bifurcation)

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Abstract. We present a *new* variational characterization of multiple critical points for *even* energy functionals corresponding to nonlinear Schrödinger equations of the following type:

$$\begin{cases} -\Delta u + V(x)u - q(x)|u|^\sigma u = \lambda u, & (x \in \mathbf{R}^N) \\ u \in H^1(\mathbf{R}^N) \setminus \{0\}. \end{cases}$$

We assume $N \geq 3$, $q(x) \in L^\infty(\mathbf{R}^N)$, $q(x) > 0$ a.e. with $\lim_{|x| \rightarrow \infty} q(x) = 0$ and $0 < \sigma < \frac{4}{N-2}$. Our results cover the following 3 cases in a *uniform* way:

1. $V(x) \equiv 0$;
2. $V(x)$ is a Coulomb potential and
3. $V(x) \in L^\infty(\mathbf{R}^N)$ with $V(x+k) \equiv V(x)$ for all $k \in \mathbf{Z}^N$.

The eigenvalue λ thereby *may or may not lie inside a spectral gap*.

Our variational characterization is “simple” and well suited for discussing *multiple bifurcation* of solutions.

References:

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