We study the spectrum of the selfadjoint model operator $A[H]$ associated with a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ whose Hamiltonian $H$ is positive semidefinite and locally integrable. It is assumed that $H$ is integrable at its left endpoint, while Weyl’s limit point case prevails at the right endpoint. We address the following questions:

1. Does $A[H]$ have discrete spectrum?
2. If $\sigma(A[H])$ is discrete, what is its asymptotic distribution?

Thereby we understand the term “asymptotic distribution” in a weak sense familiar from complex analysis, having in mind the convergence exponent and the upper density of a sequence of complex numbers w.r.t. a growth of order larger than 1.

Question (1) is equivalent to a question which was posed by L.de Branges as a “fundamental problem” in 1968:

3. Which Hamiltonians $H$ are the structure Hamiltonian of some de Branges space $\mathcal{H}(E)$?

We give a — surprising and astonishingly simple — answer to these questions. It is “surprising” because it shows that the mentioned properties do not depend on the off-diagonal entry of $H$. It is “astonishingly simple” because its proof is short and elementary.

Concerning question (2), growth of order 1 is indeed a threshold; not only that our methods cannot be applied for orders $\leq 1$, in fact the actual theorems become false. One reason is the Krein-de Branges formula for exponential type.

By our method we also obtain new proofs of some results of I.S.Kac and M.G.Krein about strings with nonhomogeneous mass distribution.

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