Canonical Systems with discrete spectrum

Harald Woracek

Institute for Analysis and Scientific Computing,
Vienna University of Technology, Wiedner Hauptstraße 8–10, Austria
http://www.asc.tuwien.ac.at/~woracek, harald.woracek@tuwien.ac.at

Joint work with Roman Romanov (St.Petersburg State University)

We study the spectrum of the selfadjoint model operator $A_{[H]}$ associated with a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ whose Hamiltonian H is positive semidefinite and locally integrable. It is assumed that H is integrable at its left endpoint, while Weyl's limit point case prevails at the right endpoint. We address the following questions:

(1) Does $A_{[H]}$ have discrete spectrum ?

(2) If $\sigma(A_{[H]})$ is discrete, what is its asymptotic distribution ?

Thereby we understand the term "asymptotic distribution" in a weak sense familiar from complex analysis, having in mind the convergence exponent and the upper density of a sequence of complex numbers w.r.t. a growth of order larger than 1.

Question (1) is equivalent to a question which was posed by L.de Branges as a "fundamental problem" in 1968:

(3) Which Hamiltonians H are the structure Hamiltonian of some de Branges space $\mathcal{H}(E)$?

We give a — surprising and astonishingly simple — answer to these questions. It is "surprising" because it shows that the mentioned properties do not depend on the off-diagonal entry of H. It is "astonishingly simple" because its proof is short and elementary.

Concerning question (2), growth of order 1 is indeed a threshhold; not only that our methods cannot be applied for orders ≤ 1, in fact the actual theorems become false. One reason is the Krein-de Branges formula for exponential type.

By our method we also obtain new proofs of some results of I.S.Kac and M.G.Krein about strings with nonhomogeneous mass distribution.

FWF Der Wissenschaftsfonds. Supported by FWF Project P-30715-N35 TU WIEN TECHNISCHE UNIVERSITÄT WIEN