Artificial general intelligence based on mathematical theory of complex systems

Paolo Giordano

Faculty of Mathematics, University of Vienna

25 - 28 July 23
9th SEAMS-UGM 2023

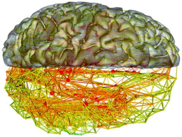
Intuitive definition of complex system

Intuitively...

A complex system (CS) is a system made by a large number of relatively simple entities that organize themselves without the intervention of an external controller, create patterns, evolve and, sometime, learn (Mitchell, 2011)

- Examples: the economy and financial markets, the immune system, road traffic, insect colonies, flocking behavior in birds or fish, pedestrian movements, urban growth and segregation, infrastructures, any non trivial software, the WWW, natural language, the brain...



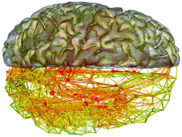



Intuitive definition of complex system

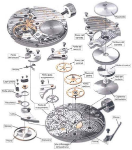
Intuitively...

A complex system (CS) is a system made by a large number of relatively simple entities that organize themselves without the intervention of an external controller, create patterns, evolve and, sometime, learn (Mitchell, 2011)

- Examples: the economy and financial markets, the immune system, road traffic, insect colonies, flocking behavior in birds or fish, pedestrian movements, urban growth and segregation, infrastructures, any non trivial software, the WWW, natural language, the brain...



Complicated but not complex: a clock



Present models and applications

Some modeling frameworks

cellular automata, agent based models, master equation based models, networked dynamical systems, neural networks, evolutionary algorithms, machine learning, complex networks, complexity measures...

Present models and applications

Some modeling frameworks

cellular automata, agent based models, master equation based models, networked dynamical systems, neural networks, evolutionary algorithms, machine learning, complex networks, complexity measures...

Some applications

epidemic diffusion, vehicular traffic and its pollution, urban growth, infrastructure management, pedestrian movements, design of emergency exists, tumor growth, population dynamics and segregation, weak points in power grids, shopping mall allocation...

The problem: a universal mathematical theory?

Pros

- Holland 95: *I do believe a useful unified theory is possible*
  - interdisciplinary perspective
  - it should encompass standard models

The problem: a universal mathematical theory?

Pros

- Holland 95: *I do believe a useful unified theory is possible*
  - interdisciplinary perspective
  - it should encompass standard models
- Crutchfield 03: *Formal theory for*
  - conceptual hygiene
  - identify tractable problems

The problem: a universal mathematical theory?

Pros

- Holland 95: *I do believe a useful unified theory is possible*
  - interdisciplinary perspective
  - it should encompass standard models
- Crutchfield 03: *Formal theory for*
  - conceptual hygiene
  - identify tractable problems
- why a math theory: common language, precise definitions and general results

The problem: a universal mathematical theory?

Pros

- Holland 95: *I do believe a useful unified theory is possible*
  - interdisciplinary perspective
  - it should encompass standard models
- Crutchfield 03: *Formal theory for*
  - conceptual hygiene
  - identify tractable problems
- why a math theory: common language, precise definitions and general results

Cons

- *Already have what we need, we simply need to apply it!*

The problem: a universal mathematical theory?

Pros

- Holland 95: *I do believe a useful unified theory is possible*
  - interdisciplinary perspective
  - it should encompass standard models
- Crutchfield 03: *Formal theory for*
  - conceptual hygiene
  - identify tractable problems
- why a math theory: common language, precise definitions and general results

Cons

- *Already have what we need, we simply need to apply it!*
- Math anxiety and *What has theory done for me lately? Nothing!*

The problem: a universal mathematical theory?

Pros

- Holland 95: *I do believe a useful unified theory is possible*
  - interdisciplinary perspective
  - it should encompass standard models
- Crutchfield 03: *Formal theory for*
  - conceptual hygiene
  - identify tractable problems
- why a math theory: common language, precise definitions and general results

Cons

- *Already have what we need, we simply need to apply it!*
- Math anxiety and *What has theory done for me lately? Nothing!*

- Other theories: *Kinetic theory for active particles* (Bellomo et al), *Memory evolutive systems* (Ehresmann, Vanbremeersh), *Universal dynamics* (Mack)... no one is universal

Interacting entities: intuitive description

Interacting entities

The system is made by *interacting entities* described by dynamical *state variables*. Intuitively, an interacting entity is everything able to send or receive signals to interact with something else.

Interacting entities: intuitive description

Interacting entities

The system is made by *interacting entities* described by dynamical *state variables*. Intuitively, an interacting entity is everything able to send or receive signals to interact with something else.

Examples

- cells of a cellular automata
- agents of an agent based model
- a vehicle, a traffic light or the piece of road between two following cars
- advertisement in a street
- goods exchanged in a market
- a whole population of individuals
- ...

Interactions: intuitive description

Interactions

An interaction i of type α is a causally directed process where a set of *agents* $a_1, \ldots, a_n$, modify the state of a *patient* p through a *propagator* r. The state space of the propagator r works as a *resource space* to change p

Interactions: intuitive description

Interactions

An interaction i of type α is a causally directed process where a set of *agents* $a_1, \ldots, a_n$, modify the state of a *patient* p through a *propagator* r. The state space of the propagator r works as a *resource space* to change p

Examples

- a particle p_1 sending a signal s to a particle p_2
- a firm (agent) sending an advertisement (propagator) to a population (patient)
- a biological entity (agent) sending a chemical signal (propagator) to another entity (patients) with suitable receptors
- an object in an object oriented program sending a message to another object
- a single neuron receives multiple connections $a_1, \ldots, a_n$ and sends an electrical signal r to its unique axon p.

Interaction spaces 1/4: interacting entities and interactions

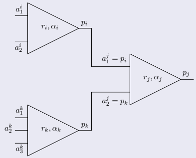
Definition

A *system of entities and interactions* $\mathcal{EI} = (E, t_{\rm st}, t_{\rm end}, \mathcal{T}, I)$ is given by

1. A set E, called the *set of interacting entities*.
2. A *time interval* $[t_{\rm st}, t_{\rm end}]$, with $0 \le t_{\rm st} < t_{\rm end} \le +\infty$.
3. A finite set $\mathcal{T}$ called the *set of types of interactions*.
4. A set I called the *set of interactions*: every interaction $i \in I$ can be written as $i = (a_1, \dots, a_n, r, \alpha, p)$ for some type of interaction $\alpha \in \mathcal{T}$, some entities $a_1, \dots a_n$, r, $p \in E$, where $n \ge 0$ depends on i;

We set $E_i := \{a_1, \dots, a_n, r, p\}$, $\mathrm{ag}(i) := \{a_1, \dots, a_n\}$, $\mathrm{pa}(i) := \{p\}$ and $\mathrm{pr}(i) := \{r\}$ to denote agents, patient and propagator of i

$$a_1, \dots, a_n \xrightarrow{r,\alpha} p$$



Activation: intuitive/formal description

Activation

Interactions occur only if at least one agent a_k is *active for that interaction* i: $\mathrm{ac}_i^{a_k}(t_i^{\mathrm{s}}) = 1$ at the *starting time* t_i^{s} of i, where $\mathrm{ac}_i^{a_k}(t) \in [0, 1]$.

Activation: intuitive/formal description

Activation

Interactions occur only if at least one agent a_k is *active for that interaction* i: $\mathrm{ac}_i^{a_k}(t_i^s) = 1$ at the *starting time* t_i^s of i, where $\mathrm{ac}_i^{a_k}(t) \in [0, 1]$. Agents can pass to an active state because of an interaction (endogenous or exogenous)

Activation: intuitive/formal description

Activation

Interactions occur only if at least one agent a_k is *active for that interaction* i: $\mathrm{ac}_i^{a_k}(t_i^{\mathrm{s}}) = 1$ at the *starting time* t_i^{s} of i, where $\mathrm{ac}_i^{a_k}(t) \in [0, 1]$. Agents can pass to an active state because of an interaction (endogenous or exogenous)

1. Agents activate the propagator r at the starting time t_i^{s}: $\mathrm{ac}_i^r(t_i^{\mathrm{s}}) = 1$

Activation: intuitive/formal description

Activation

Interactions occur only if at least one agent a_k is *active for that interaction* i: $\mathrm{ac}_i^{a_k}(t_i^{\mathrm{s}}) = 1$ at the *starting time* t_i^{s} of i, where $\mathrm{ac}_i^{a_k}(t) \in [0,1]$. Agents can pass to an active state because of an interaction (endogenous or exogenous)

1. Agents activate the propagator r at the starting time t_i^{s}: $\mathrm{ac}_i^r(t_i^{\mathrm{s}}) = 1$
2. If no interaction stops r, it arrives and activates the patient at the *arrival time* $t_i^{\mathrm{a}} \geq t_i^{\mathrm{s}}$: $\mathrm{ac}_i^r(t_i^{\mathrm{a}}) = 1$, $\mathrm{ac}_i^p(t_i^{\mathrm{a}}) = 1$

Activation: intuitive/formal description

Activation

Interactions occur only if at least one agent a_k is *active for that interaction* i: $\mathrm{ac}_i^{a_k}(t_i^{\mathrm{s}}) = 1$ at the *starting time* t_i^{s} of i, where $\mathrm{ac}_i^{a_k}(t) \in [0, 1]$. Agents can pass to an active state because of an interaction (endogenous or exogenous)

1. Agents activate the propagator r at the starting time t_i^{s}: $\mathrm{ac}_i^r(t_i^{\mathrm{s}}) = 1$
2. If no interaction stops r, it arrives and activates the patient at the *arrival time* $t_i^{\mathrm{a}} \geq t_i^{\mathrm{s}}$: $\mathrm{ac}_i^r(t_i^{\mathrm{a}}) = 1$, $\mathrm{ac}_i^p(t_i^{\mathrm{a}}) = 1$
3. After the arrival of the propagator we say that i is *ongoing*. We set $t_i^{\mathrm{o}}(t) = t_i^{\mathrm{a}}$ if i occurs instantaneously and $t_i^{\mathrm{o}}(t) = t$ if it occurs continuously in time

Activation: intuitive/formal description

Activation

Interactions occur only if at least one agent a_k is *active for that interaction* i: $\mathrm{ac}_i^{a_k}(t_i^{\mathrm{s}}) = 1$ at the *starting time* t_i^{s} of i, where $\mathrm{ac}_i^{a_k}(t) \in [0, 1]$. Agents can pass to an active state because of an interaction (endogenous or exogenous)

1. Agents activate the propagator r at the starting time t_i^{s}: $\mathrm{ac}_i^r(t_i^{\mathrm{s}}) = 1$
2. If no interaction stops r, it arrives and activates the patient at the *arrival time* $t_i^{\mathrm{a}} \geq t_i^{\mathrm{s}}$: $\mathrm{ac}_i^r(t_i^{\mathrm{a}}) = 1$, $\mathrm{ac}_i^p(t_i^{\mathrm{a}}) = 1$
3. After the arrival of the propagator we say that i is *ongoing*. We set $t_i^{\mathrm{o}}(t) = t_i^{\mathrm{a}}$ if i occurs instantaneously and $t_i^{\mathrm{o}}(t) = t$ if it occurs continuously in time

Examples

- only people activated for the advertised products will have a modification
- only biological entities with suitable receptors are active for interactions
- only hungry predators are active for hunting preys
- only software objects with a suitable public state can change

Goods and resources: intuitive description

We will use these notions to define a complex adaptive systems, but they are also useful for modeling

Goods and resources

When an interaction i starts, its agents probabilistically extract a quantity (called *good*) $\gamma_i(t) = \pi \in R_i$ from the state space of the propagator r of i (called *space of resources*) and send the signal (r, π) to the patient p.

Goods and resources: intuitive description

We will use these notions to define a complex adaptive systems, but they are also useful for modeling

Goods and resources

When an interaction i starts, its agents probabilistically extract a quantity (called *good*) $\gamma_i(t) = \pi \in R_i$ from the state space of the propagator r of i (called *space of resources*) and send the signal (r, π) to the patient p.

Examples

- resources exhausted before the end of i: the propagator is deactivated
- input currents of a neuron (goods) are integrated to change the output synapses
- A developer has a new house's project (good π_1) and money (good π_2). The state of the building's plot will change unless the municipal administration blocks the project

Interaction spaces 2/4: State spaces and activation

Definition

Let $\mathcal{EI} = (E, t_{\text{st}}, t_{\text{end}}, \mathcal{T}, I)$ be a system of entities and interactions. A *system of state spaces and activation maps* $\mathcal{SA} = (S, \mathfrak{S}, R, x)$ *for* $\mathcal{EI}$ is given by:

1. For every interacting entity $e \in E$, a Borel space $(S_e, \mathfrak{S}_e)$, called the *state space of the interacting entity* e
2. A *state map* x that satisfies $\forall t \in [t_{\text{st}}, t_{\text{end}}] \, \forall e \in E : \; x_e(t) \in [0,1]^I \times S_e$ (stochastic path)
3. If $i \in I$, $e \in E$, $t \in [t_{\text{st}}, t_{\text{end}}]$, the *activation map* $\text{ac}_i^e(t) := x_e(t)_{1,i} \in [0,1]$
4. If $a_1, \ldots, a_n \xrightarrow{r,\alpha} p \in I$, then $\gamma_i(t) := x_r(t)_{2,i} \in R_i$ is the *state of the goods of* i *in the space of resources* R_i

Occurrence times: intuitive description

Occurrence times

Generally speaking, interactions occur at random times. Using the previous notations, we can say that t_i^s, t_i^a, t_i^o are random times (stochastic paths) with model-depending distributions

Occurrence times: intuitive description

Occurrence times

Generally speaking, interactions occur at random times. Using the previous notations, we can say that t_i^{s}, t_i^{a}, t_i^{o} are random times (stochastic paths) with model-depending distributions

Examples

- an agent chooses a shop, based on its information about quality, prices, and goods availability, at random times
- a house leasing randomly occurs depending on the rate of birth, of marriage, of immigration, etc
- a virus infection depends randomly on the encountered hosts
- an excited electron produces a photon that changes another electron
- a program randomly starts depending on user's interaction with program's interface

Neighbourhood of an interaction: intuitive description

Neighborhood of an interaction

Occurrence and effects of an interaction i depend only on the state history of a set of entities called the *neighborhood* $\mathcal{U}_i(t)$ of the interaction.

Neighbourhood of an interaction: intuitive description

Neighborhood of an interaction

Occurrence and effects of an interaction i depend only on the state history of a set of entities called the *neighborhood* $\mathcal{U}_i(t)$ of the interaction.
Neighborhood of i ":=" all the entities where i takes the information it needs to operate (always includes active agents, patient and propagator)

Neighbourhood of an interaction: intuitive description

Neighborhood of an interaction

Occurrence and effects of an interaction i depend only on the state history of a set of entities called the *neighborhood* $\mathcal{U}_i(t)$ of the interaction. Neighborhood of i ":=" all the entities where i takes the information it needs to operate (always includes active agents, patient and propagator)

Examples

- an agent is searching for a house: only the information collected in some order in its memory will affect its decisions
- only the objects in the visual field of a pedestrian may influence its goal-oriented path
- the information collected in a graphical user interface may influence the starting of a program

Interaction spaces 3/4: Clock functions

Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T \subseteq [t_{\text{st}}, t_{\text{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1, t_k^2]$, and all accumulation points of T lie only in its subintervals.

Interaction spaces 3/4: Clock functions

Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T \subseteq [t_{\mathrm{st}}, t_{\mathrm{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1, t_k^2]$, and all accumulation points of T lie only in its subintervals.

We say that $\tau(-)$ is a *clock function* if

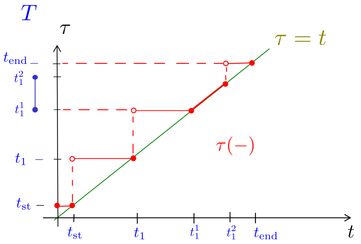
$$\exists T \text{ discr./cont.} \, \forall t \in [t_{\mathrm{st}}, t_{\mathrm{end}}] : \ \underbrace{\tau(t) = \inf \{s \geq t \mid s \in T\}}_{\textit{the next time event in } T \textit{ after } t}$$

Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T \subseteq [t_{\mathrm{st}}, t_{\mathrm{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1, t_k^2]$, and all accumulation points of T lie only in its subintervals.
We say that $\tau(-)$ is a *clock function* if

$$\exists T \text{ discr./cont.}\, \forall t \in [t_{\mathrm{st}}, t_{\mathrm{end}}] : \ \tau(t) = \inf \{s \geq t \mid s \in T\}$$

the next time event in T after t



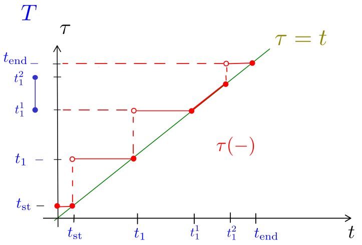
Interaction spaces 3/4: Clock functions

Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T\subseteq[t_{\mathrm{st}},t_{\mathrm{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1,t_k^2]$, and all accumulation points of T lie only in its subintervals.
We say that $\tau(-)$ is a *clock function* if

$$\exists T\text{ discr./cont.}\,\forall t\in[t_{\mathrm{st}},t_{\mathrm{end}}]:\ \tau(t)=\inf\{s\geq t\mid s\in T\}$$

the next time event in T after t



Data $\mathcal{D}=((t_i^{\mathrm{s}})_{i\in I},(t_i^{\mathrm{o}})_{i\in I},\mathcal{U})$ for the interactions: For all $t\in[t_{\mathrm{st}},t_{\mathrm{end}}]$, we ask:

- $t_i^{\mathrm{s}}(-)$, $t_i^{\mathrm{o}}(-)$ are clock functions

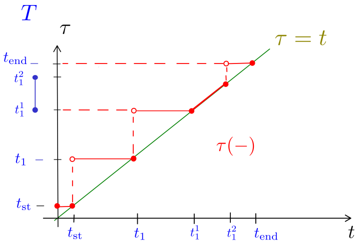
Interaction spaces 3/4: Clock functions

Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T\subseteq[t_{\mathrm{st}},t_{\mathrm{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1,t_k^2]$, and all accumulation points of T lie only in its subintervals.
We say that $\tau(-)$ is a *clock function* if

$$\exists T\text{ discr./cont.}\,\forall t\in[t_{\mathrm{st}},t_{\mathrm{end}}]:\ \tau(t)=\inf\{s\geq t\mid s\in T\}$$

the next time event in T after t



Data $\mathcal{D}=((t_i^{\mathrm{s}})_{i\in I},(t_i^{\mathrm{o}})_{i\in I},\mathcal{U})$ for the interactions: For all $t\in[t_{\mathrm{st}},t_{\mathrm{end}}]$, we ask:

- $t_i^{\mathrm{s}}(-)$, $t_i^{\mathrm{o}}(-)$ are clock functions
- If $t_i^{\mathrm{s}}(t)$, $t_i^{\mathrm{o}}(t)<+\infty$, then $t_i^{\mathrm{s}}(t)\leq t_i^{\mathrm{o}}(t)$

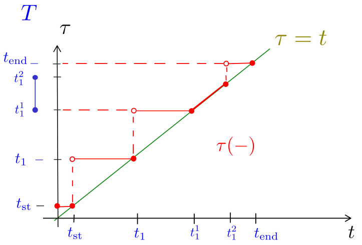
Interaction spaces 3/4: Clock functions

Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T \subseteq [t_{\text{st}}, t_{\text{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1, t_k^2]$, and all accumulation points of T lie only in its subintervals.
We say that $\tau(-)$ is a *clock function* if

$$\exists T \text{ discr./cont.}\, \forall t \in [t_{\text{st}}, t_{\text{end}}] : \ \tau(t) = \inf\{s \geq t \mid s \in T\}$$

the next time event in T after t



Data $\mathcal{D} = ((t_i^s)_{i\in I}, (t_i^o)_{i\in I}, \mathcal{U})$ for the interactions: For all $t \in [t_{\text{st}}, t_{\text{end}}]$, we ask:

- $t_i^s(-)$, $t_i^o(-)$ are clock functions
- If $t_i^s(t)$, $t_i^o(t) < +\infty$, then $t_i^s(t) \leq t_i^o(t)$
- $t_i^a(t) := t_j$ or $t_i^a(t) := t_k^1$ resp. for discr./cont. of $t_i^o(-)$

Interaction spaces 3/4: Clock functions

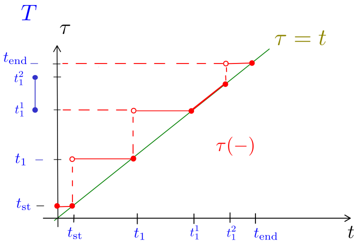
Definition

We say that T *is a set of discrete or continuous (discr./cont.) time events* if $T \subseteq [t_{\mathrm{st}}, t_{\mathrm{end}}]$ is the disjoint union of single instants t_j or of intervals $[t_k^1, t_k^2]$, and all accumulation points of T lie only in its subintervals.

We say that $\tau(-)$ is a *clock function* if

$$\exists T \text{ discr./cont.} \, \forall t \in [t_{\mathrm{st}}, t_{\mathrm{end}}] : \; \tau(t) = \inf \{ s \geq t \mid s \in T \}$$

the next time event in T after t



Data $\mathcal{D} = ((t_i^{\mathrm{s}})_{i \in I}, (t_i^{\mathrm{o}})_{i \in I}, \mathcal{U})$ for the interactions: For all $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, we ask:

- $t_i^{\mathrm{s}}(-)$, $t_i^{\mathrm{o}}(-)$ are clock functions
- If $t_i^{\mathrm{s}}(t)$, $t_i^{\mathrm{o}}(t) < +\infty$, then $t_i^{\mathrm{s}}(t) \leq t_i^{\mathrm{o}}(t)$
- $t_i^{\mathrm{a}}(t) := t_j$ or $t_i^{\mathrm{a}}(t) := t_k^1$ resp. for discr./cont. of $t_i^{\mathrm{o}}(-)$
- $\left\{ e \in E_i \mid \mathrm{ac}_e^i(t) = 1 \right\} \subseteq \mathcal{U}_i(t) \subseteq E_t$

Evolution equation: intuitive description

Transition functions

The changing of the state variables of each entity e is determined by a suitable *transition function* f_e depending on e, on all the interactions acting on e in a time interval $[t, t + \Delta_e(t)]$, and by the history of the neighborhood. Here $\Delta_e(t) \geq 0$ is model dependent

Evolution equation: intuitive description

Transition functions

The changing of the state variables of each entity e is determined by a suitable *transition function* f_e depending on e, on all the interactions acting on e in a time interval $[t, t + \Delta_e(t)]$, and by the history of the neighborhood. Here $\Delta_e(t) \geq 0$ is model dependent

Examples

- a bouncing billiard ball
- a pedestrian before interacting with other pedestrians or obstacles
- the process of building a house after its starting time and before its end
- the internal evolution of a box in a flow chart representing a computer program

Summarize of the intuitive description

- In an interaction i, active agents $a_1, \ldots, a_n$ send the propagator r and the goods $\gamma_i(t) = \pi$ as a signal to modify the state of the patient p

Summarize of the intuitive description

- In an interaction i, active agents $a_1, \ldots, a_n$ send the propagator r and the goods $\gamma_i(t) = \pi$ as a signal to modify the state of the patient p
- the modification depends on information collected from the neighborhood $\mathcal{U}_i(t)$ of that interaction

Summarize of the intuitive description

- In an interaction i, active agents $a_1, \ldots, a_n$ send the propagator r and the goods $\gamma_i(t) = \pi$ as a signal to modify the state of the patient p
- the modification depends on information collected from the neighborhood $\mathcal{U}_i(t)$ of that interaction
- the starting time t_i^s, the speed of the signal t_i^a and the duration (ongoing t_i^o) of the interaction can be stochastic

Summarize of the intuitive description

- In an interaction i, active agents $a_1, \ldots, a_n$ send the propagator r and the goods $\gamma_i(t) = \pi$ as a signal to modify the state of the patient p
- the modification depends on information collected from the neighborhood $\mathcal{U}_i(t)$ of that interaction
- the starting time t_i^{s}, the speed of the signal t_i^{a} and the duration (ongoing t_i^{o}) of the interaction can be stochastic
- interactions are local in the sense that they are affected only by entities in the neighborhood

Summarize of the intuitive description

- In an interaction i, active agents $a_1, \ldots, a_n$ send the propagator r and the goods $\gamma_i(t) = \pi$ as a signal to modify the state of the patient p
- the modification depends on information collected from the neighborhood $\mathcal{U}_i(t)$ of that interaction
- the starting time t_i^{s}, the speed of the signal t_i^{a} and the duration (ongoing t_i^{o}) of the interaction can be stochastic
- interactions are local in the sense that they are affected only by entities in the neighborhood
- their occurrence is causally constrained by logical conditions expressed by the activation $\mathrm{ac}_i^e(t)$ of the entities

Interaction spaces 4/4: Evolution equation

Definition

For all $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, we set

1. The *first arrival $\geq t$ is* $t^a(t) := \inf \{t_i^a(t) \geq t \mid i \in I\}$ is the first time of arrival of some propagator

Interaction spaces 4/4: Evolution equation

Definition

For all $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, we set

1. The *first arrival* $\geq t$ *is* $t^a(t) := \inf\{t_i^a(t) \geq t \mid i \in I\}$ is the first time of arrival of some propagator
2. $I_e(t) := \{i \in I \mid t^{\mathrm{a}}(t) \leq t_i^{\mathrm{a}}(t) \leq t^{\mathrm{a}}(t) + \Delta_e(t)\}$ all the interactions whose propagator arrives in $[t^{\mathrm{a}}(t), t^{\mathrm{a}}(t) + \Delta_e(t)]$

Interaction spaces 4/4: Evolution equation

Definition

For all $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, we set

1. The *first arrival $\geq t$ is* $t^a(t) := \inf \{t_i^a(t) \geq t \mid i \in I\}$ is the first time of arrival of some propagator
2. $I_e(t) := \{i \in I \mid t^{\mathrm{a}}(t) \leq t_i^{\mathrm{a}}(t) \leq t^{\mathrm{a}}(t) + \Delta_e(t)\}$ all the interactions whose propagator arrives in $[t^{\mathrm{a}}(t), t^{\mathrm{a}}(t) + \Delta_e(t)]$
3. $I_e^{\mathrm{pa}}(t) := \{i \in I_e(t) \mid \mathrm{pa}(i) = e\}$ all interactions in this interval acting on e

Interaction spaces 4/4: Evolution equation

Definition

For all $t \in [t_{\rm st}, t_{\rm end}]$, we set

1. The *first arrival $\ge t$ is* $t^a(t) := \inf\{t_i^a(t) \ge t \mid i \in I\}$ is the first time of arrival of some propagator
2. $I_e(t) := \{i \in I \mid t^a(t) \le t_i^a(t) \le t^a(t) + \Delta_e(t)\}$ all the interactions whose propagator arrives in $[t^a(t), t^a(t) + \Delta_e(t)]$
3. $I_e^{\rm pa}(t) := \{i \in I_e(t) \mid {\rm pa}(i) = e\}$ all interactions in this interval acting on e
4. $n_e x_t : (\tau, i, \varepsilon) \in \{(\tau, i, \varepsilon) \mid \tau \in [t_{\rm st}, t],\ i \in I_e^{\rm pa}(\tau),\ \varepsilon \in \mathcal{U}_i(\tau)\} \mapsto x_\varepsilon(\tau)$ *past state of the neighborhood* of e

Interaction spaces 4/4: Evolution equation

Definition

For all $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, we set

1. The *first arrival* $\geq t$ *is* $t^a(t) := \inf\{t_i^a(t) \geq t \mid i \in I\}$ is the first time of arrival of some propagator
2. $I_e(t) := \{i \in I \mid t^{\mathrm{a}}(t) \leq t_i^{\mathrm{a}}(t) \leq t^{\mathrm{a}}(t) + \Delta_e(t)\}$ all the interactions whose propagator arrives in $[t^{\mathrm{a}}(t), t^{\mathrm{a}}(t) + \Delta_e(t)]$
3. $I_e^{\mathrm{pa}}(t) := \{i \in I_e(t) \mid \mathrm{pa}(i) = e\}$ all interactions in this interval acting on e
4. $n_e x_t : (\tau, i, \varepsilon) \in \{(\tau, i, \varepsilon) \mid \tau \in [t_{\mathrm{st}}, t],\ i \in I_e^{\mathrm{pa}}(\tau),\ \varepsilon \in \mathcal{U}_i(\tau)\} \mapsto x_\varepsilon(\tau)$ *past state of the neighborhood* of e
5. In a *system of transition functions* $\mathcal{TF} = (f, \Omega, \mathcal{F}, P)$ *for* $\mathcal{EI}$, $\mathcal{SA}$ *and* $\mathcal{I}$ we have $(\Omega_e, \mathcal{F}_e, P_e)$ a *probability space of stochastic evolution* of e

Interaction spaces 4/4: Evolution equation

Definition

For all $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, we set

1. The *first arrival* $\geq t$ *is* $t^a(t) := \inf\{t_i^a(t) \geq t \mid i \in I\}$ is the first time of arrival of some propagator
2. $I_e(t) := \{i \in I \mid t^a(t) \leq t_i^a(t) \leq t^a(t) + \Delta_e(t)\}$ all the interactions whose propagator arrives in $[t^a(t), t^a(t) + \Delta_e(t)]$
3. $I_e^{\mathrm{pa}}(t) := \{i \in I_e(t) \mid \mathrm{pa}(i) = e\}$ all interactions in this interval acting on e
4. $n_e x_t : (\tau, i, \varepsilon) \in \{(\tau, i, \varepsilon) \mid \tau \in [t_{\mathrm{st}}, t],\ i \in I_e^{\mathrm{pa}}(\tau),\ \varepsilon \in \mathcal{U}_i(\tau)\} \mapsto x_\varepsilon(\tau)$ *past state of the neighborhood* of e
5. In a *system of transition functions* $\mathcal{TF} = (f, \Omega, \mathcal{F}, P)$ *for* $\mathcal{EI}$, $\mathcal{SA}$ *and* $\mathcal{I}$ we have $(\Omega_e, \mathcal{F}_e, P_e)$ a *probability space of stochastic evolution* of e
6. There exists $\omega \in \Omega_e$ such that if $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, $t^a(t) < +\infty$, $t^a(t) \leq s \leq t^a(t) + \Delta_e(t) \leq t_{\mathrm{end}}$, then

$$x_e(s) = f_e(\omega, s, n_e x)$$

Interaction spaces: a universal mathematical theory of CS

Definition

An *interaction space* (IS) $\mathfrak{I} = (\mathcal{EI}, \mathcal{SA}, \mathcal{I}, \mathcal{TF})$ is given by considering all the previously defined systems:

1. A system of entities and interactions $\mathcal{EI} = (E, t_{\text{st}}, t_{\text{end}}, \mathcal{T}, I)$.
2. A system of state spaces and activation maps $\mathcal{SA} = (S, \mathfrak{S}, G, x)$ *for* $\mathcal{EI}$.
3. A system of data $\mathcal{D} = ((t_i^{\text{s}})_{i\in I}, (t_i^{\text{o}})_{i\in I}, \mathcal{U})$ for the interactions of $\mathcal{EI}$ and $\mathcal{SA}$.
4. A system of transition functions $\mathcal{TF} = (f, \Omega, \mathcal{F}, P)$ for $\mathcal{EI}$, $\mathcal{SA}$ and $\mathcal{D}$.

Theorem

Cellular automata, agent based models, master equation based models, networked dynamical systems, neural networks and evolutionary algorithms can be faithfully embedded as IS

not stationary and Markovian IS

Definition

We say that the IS $\mathfrak{I}$ is *not stationary* if

1. All the entity $e \in E$ are always active: $\forall t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]\, \exists i \in I : \; \mathrm{ac}_i^e(t) = 1$
2. The global state space $M = \prod_{e \in E} S_e$ is at most countable
3. $\forall t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]\, \forall \sigma \in M : \; p(\sigma, t) := P^{\mathrm{g}}[X_t = \sigma] > 0$, where $(\Omega^{\mathrm{g}}, \mathcal{F}^{\mathrm{g}}, P^{\mathrm{g}})$ is product of all the prob. spaces of $\mathfrak{I}$ and $X : \Omega^{\mathrm{g}} \longrightarrow M$ is a global state RV
4. $\tau \in [t_{\mathrm{st}}, t_{\mathrm{end}}] \mapsto p(\mu, \tau \mid \sigma, t) := P^{\mathrm{g}}[X_\tau = \mu \mid X_t = \sigma]$ is differentiable at $\tau = t$
5. The IS $\mathfrak{I}$ is called *Markovian* if $p(\mu, \tau \mid \sigma, t)$ does not depend on $t \le \tau$.

Theorem

For $\sigma, \mu \in M$ and $t \in [t_{\mathrm{st}}, t_{\mathrm{end}}]$, set $w_t(\mu, \sigma) := \frac{\partial p(\mu, \tau | \sigma, t)}{\partial \tau}(t)$. If $\sum_{\mu \in M} |w_t(\sigma, \mu)| < +\infty$, then

$$\frac{\partial p}{\partial t}(\sigma, t) = \sum_{\substack{\mu \in M \\ \mu \neq \sigma}} \left[w_t(\sigma, \mu) \cdot p(\mu, t) - w_t(\mu, \sigma) \cdot p(\sigma, t) \right]$$

Complex Adaptive Systems following Zipf's idea

G.K. Zipf 1949 "Human behavior and the principle of least efforts": CAS are the result of two opposing processes: *unification* and *diversification.*

The idea

- Unification processes: decreasing in convenient *costs*
- Diversification processes: *long term changes* of suitable interactions, i.e. increasing of the *information entropy of the goods* generated by these interactions
- It is the implementation of these interactions and the most diversified exchange of fluxes of goods that enable the population to be resilient and keep a low value of costs.

Examples

- Natural language: unification processes shorten frequently used words; diversification ones make evolve the language towards longer words
- Cities development: unification brings near people so as to decrease costs of living; diversification uses all the possible living locations so as to decrease rent costs
- Natural selection: unification forces push giraffes to search for eatable trees; diversification selects all the best genetic codes that allow for a longer neck
- Companies with a longer life span not only decrease costs and increase profits (unification), but also adapt to their environment with long-term diversification processes
- Phyllotaxis: unification forces are related to energy exploitation by each primordium; diversification forces tend to uniformly distribute energy sources between old and new primordia
- Network of financial institutions: shows that this system is **not** well adapted. The centrality of certain institutions does not allow the system to be resilient to financial fails of few institutions

CAS := generalized evolution principle

Definition

Let $\mathfrak{I}$ be an IS, and let $s,\ t \in [t_{\text{st}}, t_{\text{end}}]$, $\mathcal{P} \subseteq E_s \cap E_t$, $x,\ y \in M$, $i \in I_s \cap I_t$. Then we say that *at $y,\ t$ the population $\mathcal{P}$ is better adapted than at $x,\ s$ with respect to C_i, P_{C_i} (briefly: $\mathcal{P}$ is a CAS) if*

1. $C_i : S_{\mathcal{P}} \longrightarrow \mathbb{R}_{\geq 0}$ is a random variable, where $(S_{\mathcal{P}}, \mathfrak{S}_{\mathcal{P}}) := (\prod_{e\in\mathcal{P}} S_e, \prod_{e\in\mathcal{P}} \mathfrak{S}_e)$
2. P_{C_i} is a probability on the global state space $(S_{\mathcal{P}}, \mathfrak{S}_{\mathcal{P}})$ of the population $\mathcal{P}$
3. i is an interaction of $\mathcal{P}$
4. Set $D_i(\mathrm{n}_i\, y, t) := \text{Entropy}\left(G_i(-; \mathrm{n}_i\, y, t)\right)$, then we have

$$E\left(C_i(y_t|_{\mathcal{P}})\right) \leq E\left(C_i(x_s|_{\mathcal{P}})\right) \qquad \text{unification}$$
$$D_i(\mathrm{n}_i\, y, t) \geq D_i(\mathrm{n}_i\, x, s) \qquad \text{diversification}$$

where the expected value $E(-)$ is computed using P_{C_i}

Power laws and Mandelbrot's theorem (rigorous)

Theorem

Let $y \in S_{\mathcal{P}} \subseteq \mathbb{R}^n$ be an open set and let $q_j \in \mathcal{C}^1(S_{\mathcal{P}}, \mathbb{R}_{\geq 0})$, for all $j = 1, \ldots, d \leq n$, be such that $(q_j(x))_{j=1,\ldots,d}$ is a probability $\forall x \in S_{\mathcal{P}}$. Set $D(x) := -\sum_{j=1}^{d} q_j(x_j) \cdot \log_2 q_j(x_j)$ $\forall x \in S_{\mathcal{P}}$. Let $C \in \mathcal{C}^1(S_{\mathcal{P}}, \mathbb{R}_{>0})$ be such that

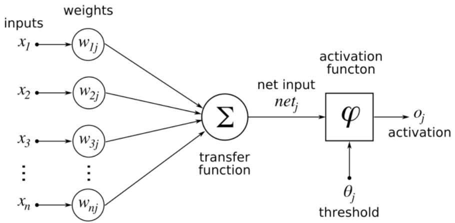
$$\forall x \in S_{\mathcal{P}} : \ 0 < \frac{C(y)}{D(y)} \leq \frac{C(x)}{D(x)}$$

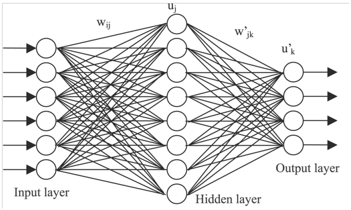
Finally assume that $q_j(x_j) = x_j$ $\forall j = 1, \ldots, d$ $\forall x \in S_{\mathcal{P}}$, $\partial_k C(y) \leq \alpha_k(y) \cdot \log_2 k$ $\forall k = 2, \ldots, d$, $\sum_{k=1}^{d} k^{-\alpha_k(y) \cdot \frac{D_i(y)}{C(y)}} =: N(y) \geq \frac{1}{q_1(y)} \geq e$, where $\alpha_k : S_{\mathcal{P}} \longrightarrow \mathbb{R}$. Then we have:

$$q_k(y) = q_1(y) \cdot k^{-\alpha_k(y) \cdot \frac{D(y)}{C(y)}} \qquad \forall k = 1, \ldots, d$$

$$q_1(y) = \frac{1}{N(y)}$$

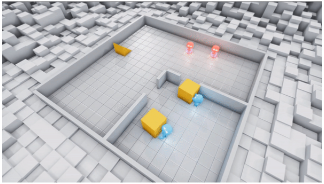
Artificial intelligence with ANN



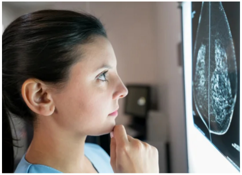


- The methodological problem: no idea about how many neurons and how to set links between artificial neurons
- ANN are universal approximators:
  - Kolmogorov theorem 1957 (13th Hilbert problem): every continuous function on $[0,1]^n$ can be written as composition of one variable continuous functions.
  - Cybenko 1989: $g(x) = \sum_{i=1}^{N} \omega_i \varphi \left(a_i^T x + b_i \right)$ are dense in $\mathcal{C}^0([0,1]^n)$

Examples



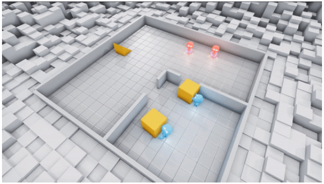
adversarial artificial neural networks trained for hide-and-seek game



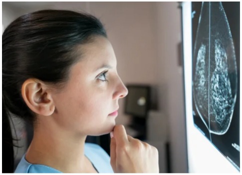
breast cancer detection (?)

- Problem 1: they need too much data (75 million for hide-and-seek games)
- Problem 2: interpretation of their “reasoning”

Examples



adversarial artificial neural networks trained for hide-and-seek game



breast cancer detection (?)

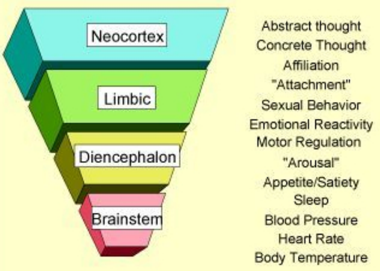
- Problem 1: they need too much data (75 million for hide-and-seek games)
- Problem 2: interpretation of their "reasoning"
- Sony "Focused Research Award" for next-generation AI: "The limitations of many current-day AI methods and techniques are evident [...] we are seeking powerful and efficient biologically-inspired AI methods that have the potential to open entirely new capabilities that are not possible with the methods in our current AI toolbox and that will support more reliable and more trustworthy AI. AI methods powerful enough to even, possibly, realize an AGI are sought-after here"

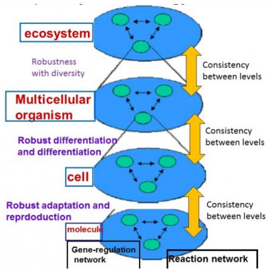
Future developments: morphisms of IS

- Using interactions $a_1, \ldots, a_m \xrightarrow{\alpha, r} p$ we can define a cause-effect relations and a corresponding multi-category
- Functors between the multi-categories generated by two IS become morphisms of IS:

$$a_1, \ldots, a_m \xrightarrow{\alpha, r} p \quad \Rightarrow \quad F(a_1), \ldots, F(a_m) \xrightarrow{F(\alpha), F(r)} F(p)$$

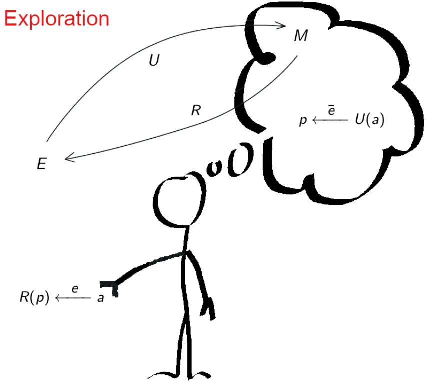
- This can be used to define hierarchies in complex systems





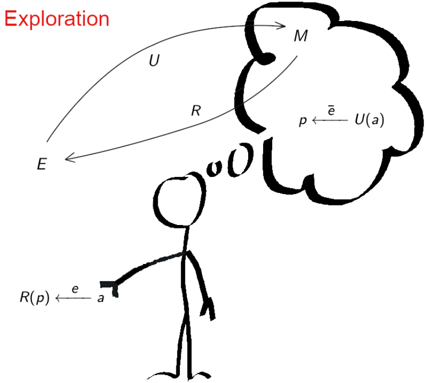
Ideas for AGI: exploration rule

- Imagine an agent M living in a virtual environment E



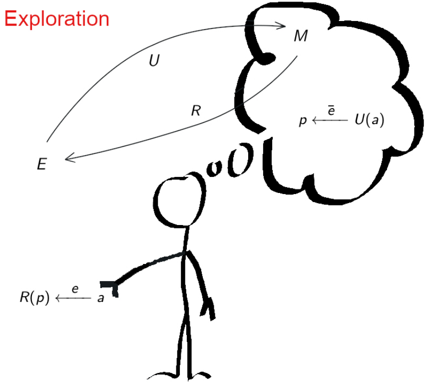
Ideas for AGI: exploration rule

- Imagine an agent M living in a virtual environment E
- Both M and E are modeled as interaction spaces

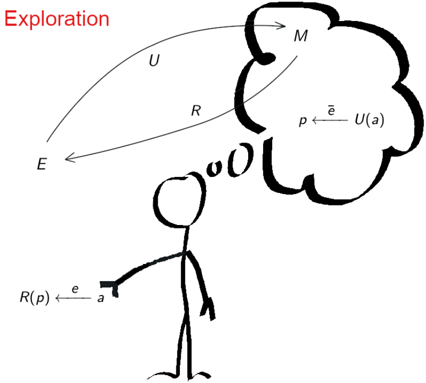


Ideas for AGI: exploration rule

- Imagine an agent M living in a virtual environment E
- Both M and E are modeled as interaction spaces
- $U : E \longrightarrow M$ forgetful functor: agent's representation of environment is necessarily simplified. It preserves cause-effect relations between agent and environment



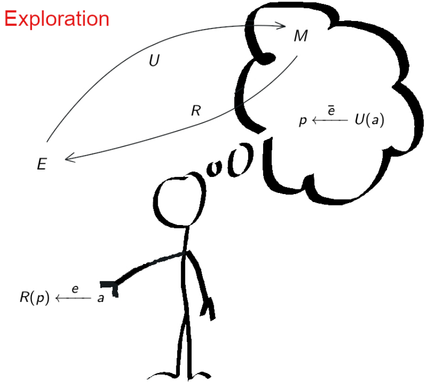
- Imagine an agent M living in a virtual environment E
- Both M and E are modeled as interaction spaces
- $U : E \longrightarrow M$ forgetful functor: agent's representation of environment is necessarily simplified. It preserves cause-effect relations between agent and environment
- $R : M \longrightarrow E$ right adjoint of U: in E the real interaction $a \xrightarrow{e} R(p)$ bijectively corresponds to the mental interaction $U(a) \xrightarrow{\bar{e}} p$ in M



Ideas for AGI: exploration rule

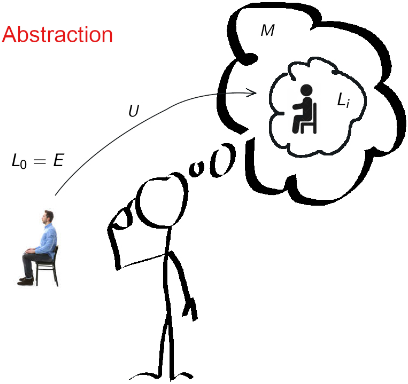
- Imagine an agent M living in a virtual environment E
- Both M and E are modeled as interaction spaces
- $U : E \longrightarrow M$ forgetful functor: agent's representation of environment is necessarily simplified. It preserves cause-effect relations between agent and environment
- $R : M \longrightarrow E$ right adjoint of U: in E the real interaction $a \xrightarrow{e} R(p)$ bijectively corresponds to the mental interaction $U(a) \xrightarrow{\bar{e}} p$ in M

What will happen to the real counterpart $R(p)$ of the patient $p \in M$ if I change the real agent $a \in E$?



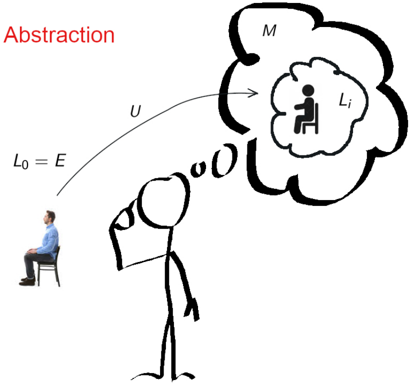
Ideas for AGI: abstraction rule

- Inside M we can have different levels L_i, $i > 0$, of representations of interacting entities and their interactions



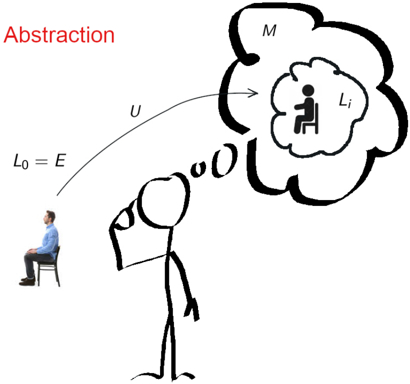
Ideas for AGI: abstraction rule

- Inside M we can have different levels L_i, $i > 0$, of representations of interacting entities and their interactions
- Each level L_i is a subspace of M but $L_0 = E$ is the environment



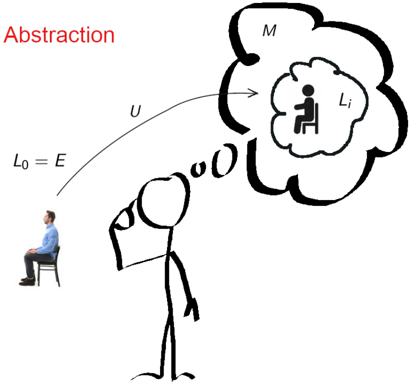
Ideas for AGI: abstraction rule

- Inside M we can have different levels L_i, $i > 0$, of representations of interacting entities and their interactions
- Each level L_i is a subspace of M but $L_0 = E$ is the environment
- Higher levels of abstraction L_{i+1} are linked to lower ones L_i by a forgetful functor $U_i : L_i \longrightarrow L_{i+1}$ $(U_0 =: U)$



Ideas for AGI: abstraction rule

- Inside M we can have different levels L_i, $i > 0$, of representations of interacting entities and their interactions
- Each level L_i is a subspace of M but $L_0 = E$ is the environment
- Higher levels of abstraction L_{i+1} are linked to lower ones L_i by a forgetful functor $U_i : L_i \longrightarrow L_{i+1}$ ($U_0 =: U$)
- Interactions of L_{i+1} are a subset of those of L_i so that if a, p are agent resp. patient in L_i, then $U_i(a)$ and $U_i(p)$ are agent resp. patient in L_{i+1}

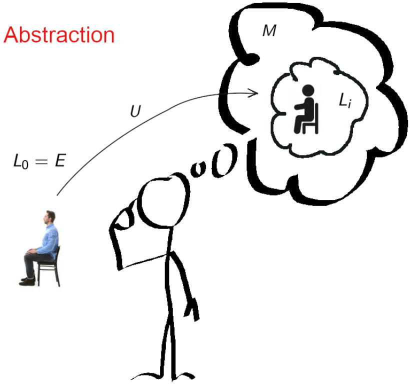


Ideas for AGI: abstraction rule

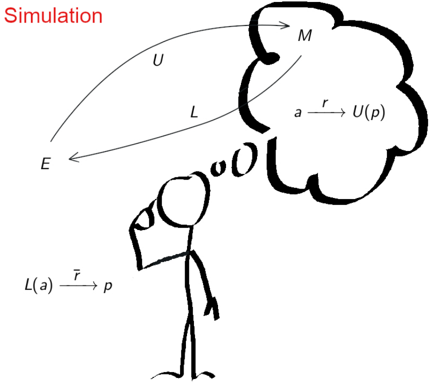
- Inside M we can have different levels L_i, $i > 0$, of representations of interacting entities and their interactions
- Each level L_i is a subspace of M but $L_0 = E$ is the environment
- Higher levels of abstraction L_{i+1} are linked to lower ones L_i by a forgetful functor $U_i : L_i \longrightarrow L_{i+1}$ ($U_0 =: U$)
- Interactions of L_{i+1} are a subset of those of L_i so that if a, p are agent resp. patient in L_i, then $U_i(a)$ and $U_i(p)$ are agent resp. patient in L_{i+1}

Note:

- U_i preserves cause-effect relations
- Interactions between different parts (entities) of objects are important
- "a chair" is what you can do with a chair + interactions between its parts

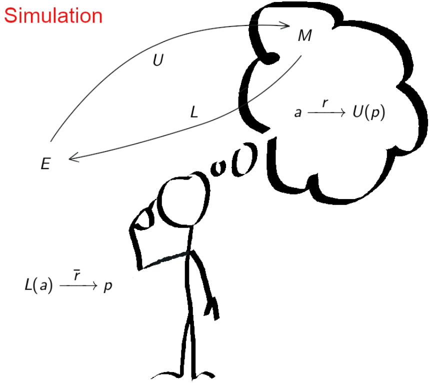


- $U : E \longrightarrow M$ forgetful functor



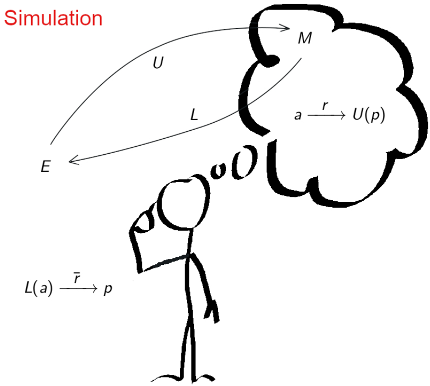
Ideas for AGI: simulation rule

- $U : E \longrightarrow M$ forgetful functor
- $L : M \longrightarrow E$ left adjoint of U: in M the mental simulation $a \xrightarrow{r} U(p)$ bijectively corresponds to the real interaction $L(a) \xrightarrow{\bar{r}} p$ in E



Ideas for AGI: simulation rule

- $U : E \longrightarrow M$ forgetful functor
- $L : M \longrightarrow E$ left adjoint of U: in M the mental simulation $a \xrightarrow{r} U(p)$ bijectively corresponds to the real interaction $L(a) \xrightarrow{\bar{r}} p$ in E
- simulations are composition (trees/chains) of cause-effect relations

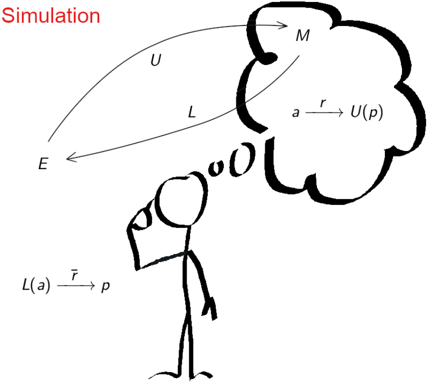


Ideas for AGI: simulation rule

- $U : E \longrightarrow M$ forgetful functor
- $L : M \longrightarrow E$ left adjoint of U: in M the mental simulation $a \xrightarrow{r} U(p)$ bijectively corresponds to the real interaction $L(a) \xrightarrow{\bar{r}} p$ in E
- simulations are composition (trees/chains) of cause-effect relations

How can I change the real patient $p \in E$ thinking at the mental agent $a \in M$ and using its real counterpart $L(a)$?

Adjoint functors in AI, see: D. Ellerman, 2005-2016

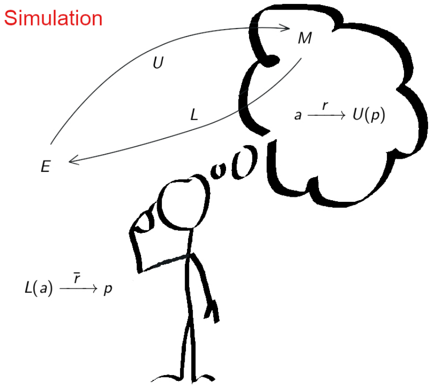


Ideas for AGI: simulation rule

- $U : E \longrightarrow M$ forgetful functor
- $L : M \longrightarrow E$ left adjoint of U: in M the mental simulation $a \xrightarrow{r} U(p)$ bijectively corresponds to the real interaction $L(a) \xrightarrow{\bar{r}} p$ in E
- simulations are composition (trees/chains) of cause-effect relations

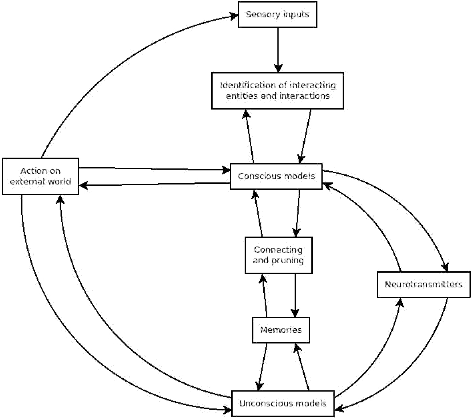
How can I change the real patient $p \in E$ thinking at the mental agent $a \in M$ and using its real counterpart $L(a)$?

Adjoint functors in AI, see: D. Ellerman, 2005-2016



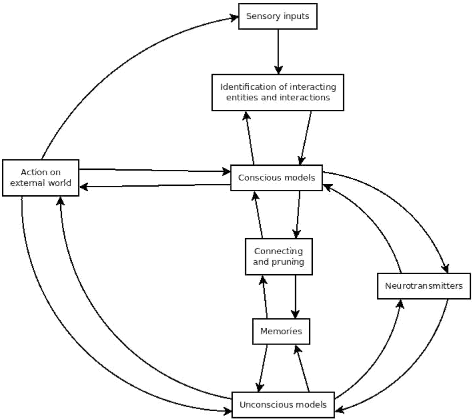
These seem to be very general learning rules

Architecture of a brain-like system



- Starting from hard-wired interactions the agent builds up new ones using the exploration and abstraction rules

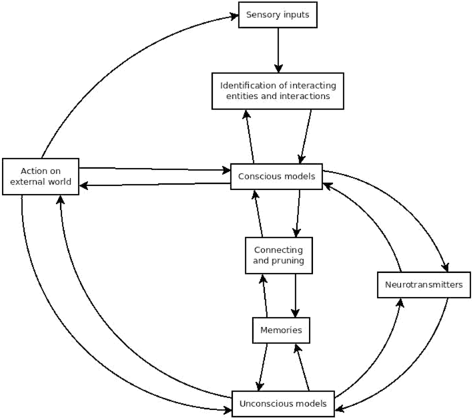
Architecture of a brain-like system



- Starting from hard-wired interactions the agent builds up new ones using the exploration and abstraction rules
- Using simulation rule the agent can try cause-effect based solutions

[J. Pearl, D. McKenzie, 2018]

Architecture of a brain-like system

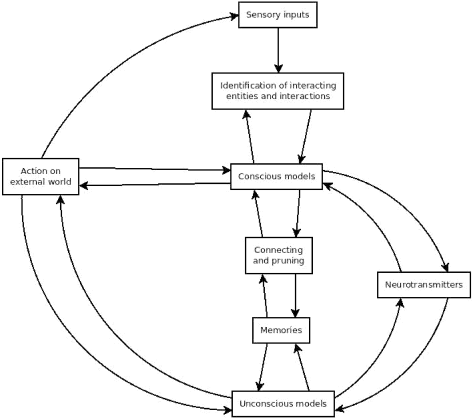


- Starting from hard-wired interactions the agent builds up new ones using the exploration and abstraction rules
- Using simulation rule the agent can try cause-effect based solutions

[J. Pearl, D. McKenzie, 2018]

- The space M must contain a simplified model of the agent itself (self consciousness)

Architecture of a brain-like system

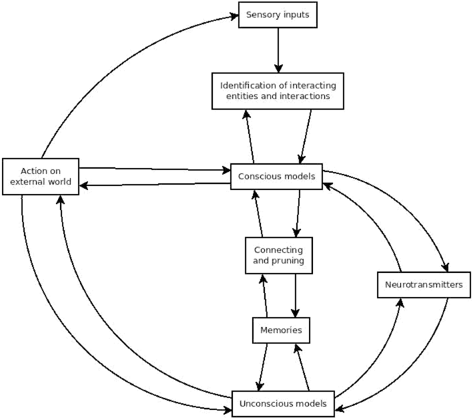


- Starting from hard-wired interactions the agent builds up new ones using the exploration and abstraction rules
- Using simulation rule the agent can try cause-effect based solutions

[J. Pearl, D. McKenzie, 2018]

- The space M must contain a simplified model of the agent itself (self consciousness)
- At the end, we get interpretable cause-effect graphs

Architecture of a brain-like system

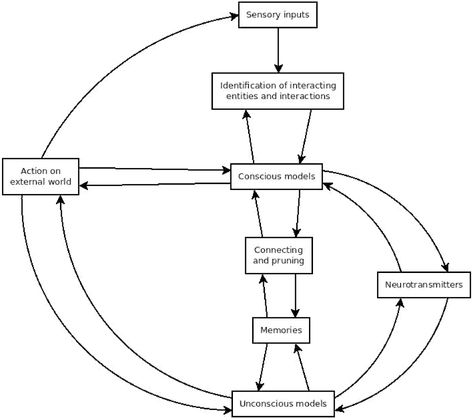


- Starting from hard-wired interactions the agent builds up new ones using the exploration and abstraction rules
- Using simulation rule the agent can try cause-effect based solutions

[J. Pearl, D. McKenzie, 2018]

- The space M must contain a simplified model of the agent itself (self consciousness)
- At the end, we get interpretable cause-effect graphs
- Suitable (positive and negative) costs and the generalized evolution principle always act

Architecture of a brain-like system



- Starting from hard-wired interactions the agent builds up new ones using the exploration and abstraction rules
- Using simulation rule the agent can try cause-effect based solutions

[J. Pearl, D. McKenzie, 2018]

- The space M must contain a simplified model of the agent itself (self consciousness)
- At the end, we get interpretable cause-effect graphs
- Suitable (positive and negative) costs and the generalized evolution principle always act
- Does the brain work in a similar way? Maybe...

Examples of applications

Doable:

- recognize when two objects are equal or not but only differently placed
- recognize and label an object using a comparable human learning set
- play hide-and-seek using a comparable human learning set
- take an object out from a given room through the door
- play chess with comparable human strategies and learning set
- help in software verification

Examples of applications

Doable:

- recognize when two objects are equal or not but only differently placed
- recognize and label an object using a comparable human learning set
- play hide-and-seek using a comparable human learning set
- take an object out from a given room through the door
- play chess with comparable human strategies and learning set
- help in software verification

Dreams for the future:

- learn from a human to do a dangerous job
- remove mines from a minefield
- help in removing ruins after a earthquake
- describe an environment to help blind people
- teaching where there are no teachers
- help elderly or disabled peoples
- ...

Conclusions

1. IS is a theory at its beginning that need simplifications and stability
2. IS are a framework where good formalizations of intuitive notions are possible
3. IS are easy to understand intuitively (useful for interdisciplinary work)
4. In this idea of AGI neurons are interacting entities and links are set using the generalized evolution principle: methodological clarity, efficiency
5. Cause-effect graphs yields explainable AGI
6. There are non trivial epistemological problems concerning validation of models of complex systems
7. Strong ethical problems concerning AGI must be considered

References:

www.mat.univie.ac.at/~giordap7/

Contact:

paolo.giordano@univie.ac.at

Thank you for your attention!