Algebraic Approach to Non-Standard Analysis and Colombeau Theory of Generalized Functions

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Abstract

We construct a chain (of infinitely many) algebras of generalized functions and show they are spacial Colombeau algebras in the sense that for each of these algebras there exists a Colombeau type of embedding of the space of Schwartz distributions (the proof is based on convolution, but also on Zorn lemma). One of these algebras is a full Colombeau algebra (with an explicit embedding). What is different from most of the similar constructions is that the scalars of these algebras form algebraically closed Cantor complete non-Archimedean fields. Our framework is the so called distributional non-standard model of the real numbers especially designed for the purpose of non-linear theory of generalized functions. No background on non-standard analysis is required by the audience; the language will be essentially algebraic: ordered fields, algebraically closed fields, maximal ideals, etc.

For supporting material we refer to Todor’s webpage http://web.me.com/ttodorov (under “Current Research Projects”), where you will find links to:


2. For a “gentle” introduction to non-standard analysis we refer to the Master Thesis of Ray Cavalcante, where the non-standard analysis is presented in terms of nets of complex numbers \( \mathbb{C}^{\mathbb{R}^+} \) (notationally similar to Colombeau theory).

3. For the more advanced introduction to distributional non-standard model of analysis we refer to the Master Thesis of Guy Berger, where the non-standard analysis is presented in terms of nets of complex numbers \( \mathbb{C}^{\mathbb{D}} \).


5. For the terminology on ordered fields we refer to any textbook on abstract algebra; in particular to S. Lang (Chapter XI).