Extremal basis and local estimates for the Szegö projection

by PHILIPPE CHARPENTIER

In 1990 Fefferman-Kohn-Machedon proved that if at a point of finite type z_0 of the boundary of a bounded pseudo-convex domain Ω in $\mathbb{C}^n$, the Levi form is locally diagonalizable, then, for any $\varepsilon > 0$, the Szegö projection maps the local Hölder space $\Lambda_\alpha^{\mathrm{loc}}(\partial\Omega, z_0)$ into $\Lambda_{\alpha-\varepsilon}^{\mathrm{loc}}(\partial\Omega, z_0)$. The main goal of this talk is to show that the same result is true for $\varepsilon = 0$. When Ω is of finite type and at all the points of the boundary the Levi form is locally diagonalizable, the sharp estimate of the Szegö projection was proved in 2006 by Charpentier-Dupain using a global method previously used for domains in $\mathbb{C}^2$ and for convex domains (Nagel-Rosay-Stein-Wainger, McNeal-Stein). The idea here is to use the same method for a "small" domain having a piece of the boundary in common with Ω. For this small domain the Levi form being not everywhere locally diagonalizable, a new method has to be introduced to describe it's complex geometry which is the essential tool of that method. To do that, a notion of "extremal basis" is introduced which allows to consider the problem in a more general context.