Subelliptic estimates for some systems of complex vector fields : quasihomogeneous case

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Abstract

This is a common work with Makhlouf Derridj.
For about twenty five years it was a kind of folk theorem that complex vector-fields defined on $\Omega \times \mathbb{R}_t$ (with Ω open set in $\mathbb{R}^n$) by

$$L_j = \frac{\partial}{\partial t_j} + i \frac{\partial \varphi}{\partial t_j}(\mathbf{t}) \frac{\partial}{\partial x}\ ,\ j = 1, \ldots, n\ ,\ \mathbf{t} \in \Omega, x \in \mathbb{R}\ ,$$

were subelliptic as soon as they were hypoelliptic when φ was analytic. This was the case when $n = 1$ but in the case $n > 1$, an inaccurate reading of the proof given by Maire (see also Trèves) of the hypoellipticity of such systems, under the condition that φ does not admit any local maximum or minimum (through a non standard subelliptic estimate), was supporting the belief for this folk theorem. Quite recently, J.L. Journé and J.M.Trépreau show by examples that there are very simple systems (with polynomial φ's) which were hypoelliptic but not subelliptic in the standard L^2-sense. So it is natural to analyze this problem of subellipticity which is in some sense intermediate (at least when φ is C^∞) between the maximal hypoellipticity (which was analyzed by Helffer-Nourrigat and Nourrigat) and the simple local hypoellipticity (or local microhypoellipticity) and to start first with the easiest non trivial examples. The analysis presented here is a continuation of a previous work by M. Derridj and is devoted to the case of quasihomogeneous functions φ.