SINGULAR MORSE INEQUALITIES

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Abstract. We give a complete metric characterisation of the volume of a holomorphic line bundle over a compact, possibly non-Kaehler, complex manifold in terms of the Monge-Ampère masses of all positive currents in its first Chern class. This result generalises Demailly's holomorphic inequalities to the case of singular fibre metrics with arbitrary singularities. Optimal metric characterisations of big line bundles and Moishezon manifolds follow. The techniques involve multiplier ideal sheaves, asymptotic estimates of singularly weighted Bergman kernels, L^2 estimates for the Cauchy-Riemann operator and Toeplitz concentration operators.