Lecturer: Ragnar Sigurdsson, University of Iceland.

Title: A new disc formula for the Siciak-Zahariuta extremal function.

Abstract: The lecture is a report on a joint work with Finnur Larusson to be published in Ann. Pol. Math. on envelope formulas for the The Siciak-Zahariuta extremal function V_X.

The Siciak-Zahariuta extremal function V_X of a subset X of affine space $\mathbb{C}^n$ is also called the pluricomplex Green function of X with logarithmic pole at infinity. It is defined as the supremum of the class of all plurisubharmonic functions u on $\mathbb{C}^n$ of minimal growth with $u|X \leq 0$. A plurisubharmonic function u on $\mathbb{C}^n$ is said to be of minimal growth (and belong to the Lelong class $\mathcal{L}$) if $u - \log \| \cdot \|$ is bounded above.

A disc functional on a manifold Y is a function H defined on the set $\mathcal{A}_Y$, of all (closed) analytic discs in Y, and taking values in the extended real line. A closed analytic disc is a holomorphic map f defined in some neighbourhood of the closed unit disc in the complex plane with values in Y. The envelope of H with respect to the subclass $\mathcal{B}$ of $\mathcal{A}_Y$ is a function $E_\mathcal{B} H$ defined as

$$E_\mathcal{B} H(y) = \{H(f) \, ; \, f \in \mathcal{B}, f(0) = y\}, \qquad y \in Y.$$

Our main results is that for an open connected subset X of $\mathbb{C}^n$ the function V_X is a disc envelope, where Y is the projective space $\mathbb{P}^n$, $\mathcal{B}$ is the class of all analytic discs with boundary in X, and H is a certain functional which can be expressed as a sum of values associated to each intersection of the disc f with the hyperplane at infinity.