

ANALYTICITY FOR SINGULAR SUMS OF SQUARES OF DEGENERATE VECTOR FIELDS

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ABSTRACT. Recently J.J. Kohn proved C^∞ hypoellipticity for

$$P_k = L\bar{L} + \bar{L}|z|^{2k}L = -\bar{L}^*L - (\bar{z}^k L)^*z^k L \quad \text{with} \quad L = \frac{\partial}{\partial z} + i\bar{z}\frac{\partial}{\partial t},$$

(the negative of) a singular sum of squares of complex vector fields on the complex Heisenberg group, an operator which exhibits a *loss* of $k - 1$ derivatives. Then a few months ago, M. Derridj and D. S. Tartakoff proved analytic hypoellipticity for this operator using rather different methods going back 25 years. Those methods also provide an alternate proof of Kohn's result.

In this paper, we consider the equation

$$P_{m,k} = L_m\bar{L}_m + \bar{L}_m|z|^{2k}L_m \quad \text{with} \quad L_m = \frac{\partial}{\partial z} + i\bar{z}|z|^{2m}\frac{\partial}{\partial t},$$

for which the underlying manifold is only of finite type, and prove analytic hypoellipticity using methods of Derridj and Tartakoff from 1988. This operator is also subelliptic with large loss of derivatives but the exact loss plays no role for analyticity.

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