Tensor Models in the large N limit

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For instance, in $D = 2$ how do we quantize the Polyakov string action?

$$S \sim \kappa_R \int \sqrt{g} R - \kappa_V \int \sqrt{g} + \kappa_m \int d^2\xi \sqrt{g} g^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

$$Z \sim \sum_{\text{topologies}} \int \mathcal{D}g_{(\text{worldsheet metrics})} \, \mathcal{D}X_{(\text{target space coordinates})} \, e^{-S}$$

Random Discrete Geometries

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Classical gravity = geometry. QFT = summing random configurations.

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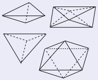
Proposal: build the geometry by gluing discrete blocks, "space time quanta".



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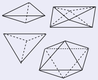

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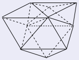
Fundamental interactions of few "quanta" lead to effective behaviors of an ensemble of "quanta".



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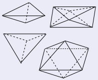

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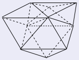

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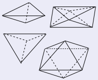

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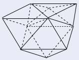

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We know the answer in two dimensions! (G. 't Hooft, E. Brezin, C. Itzykson, G. Parisi, J.B. Zuber, F. David, V. Kazakov, D. Gross, A. Migdal, M. R. Douglas, S. H. Shenker, etc.)

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A success story: Matrix Models provide a measure for random two dimensional surfaces. The theory of strong interactions, string theory, quantum gravity in $D = 2$, conformal field theory, invariants of algebraic curves, free probability theory, knot theory, the Riemann hypothesis, etc.

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- Tensor invariance $\Rightarrow$ random discretizations.

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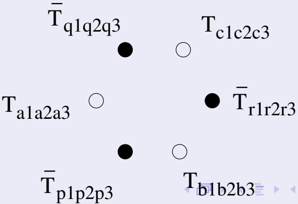

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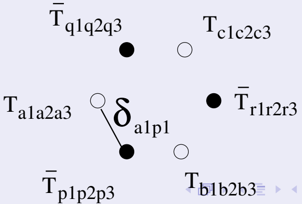

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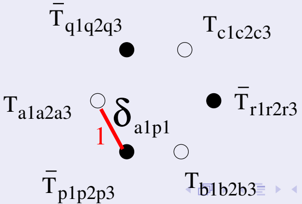

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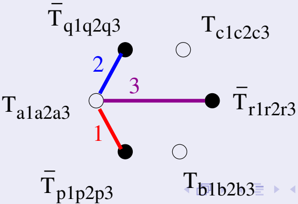

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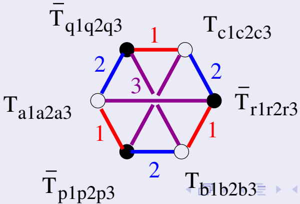

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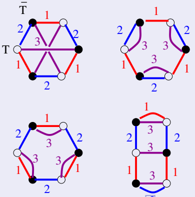

Invariant Actions for Tensor Models

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The most general single trace invariant tensor model

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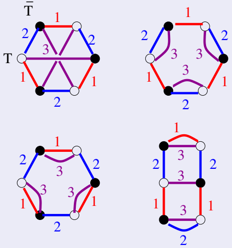

$$\int_{\bar{T},T} \quad e^{-N^{D-1}\left(\sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{a^c q^c}\right)}$$

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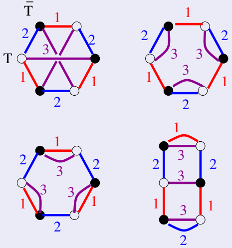

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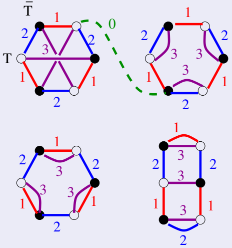
Feynman graphs: "vertices" $\mathcal{B}$. Gaussian integral: Wick contractions of T and $\bar{T}$ ("propagators") $\to$ dashed edges to which we assign the fictitious color 0.



$$\int_{\bar{T},T} \quad e^{-N^{D-1}\left(\sum T_{a^1\dots a^D}\,\bar{T}_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{a^c q^c}\right)}$$

$$\sum\left(\prod \delta_{\dots}\right) \quad \underbrace{T_{a^1a^2a^3}\,\bar{T}_{p^1p^2p^3}}_{\sim \frac{1}{N^{D-1}}\delta_{a^1p^1}\delta_{a^2p^2}\delta_{a^3p^3}} \quad \dots$$

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The most general single trace invariant tensor model

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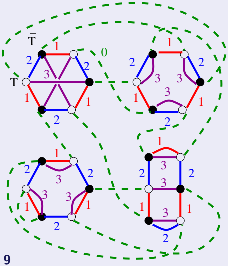

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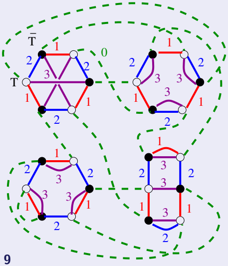

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Represent triangulated D dimensional spaces.

Colored Graphs as gluings of colored simplices

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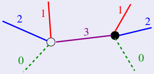
White and black $D+1$ valent vertices connected by edges with colors $0, 1 \ldots D$.



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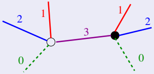

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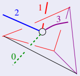
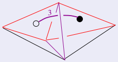


Edges $\leftrightarrow$ gluings along $D-1$ simplices respecting all the colorings



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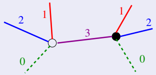

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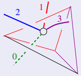

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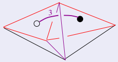

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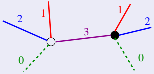

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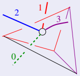

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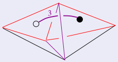

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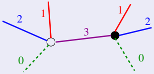

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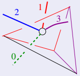

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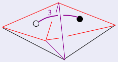

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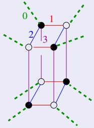

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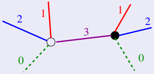

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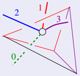

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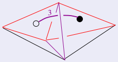

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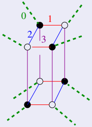

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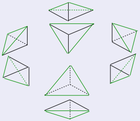
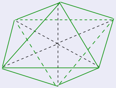


Gluing along all $D-1$ simplices except 0: "chunk" in D dimensions



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Remarks:

- The path integral yields a canonical measure over the discrete geometries.
- Weight of a triangulation: discretized EH, $B\wedge F$, etc.
- Need to take some kind of limit in order to go from discrete triangulations to continuum geometries.

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Tensor Models compute correlations

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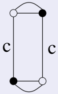

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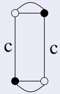

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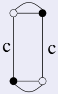

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Discretized Einstein Hilbert action on an equilateral triangulation with fixed boundary!

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The metric assigned to a combinatorial triangulation is encoded in the choice of $A^{\mathcal{G}}(\lambda, N)$.

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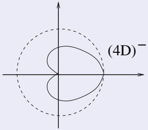
3) non perturbative: $K_2 = \frac{(1+4D\lambda)^{\frac{1}{2}} - 1}{2D\lambda} + \ldots + \mathcal{R}_N^{(p)}(\lambda)$

$\mathcal{R}_N^{(p)}(\lambda)$ analytic in $\lambda = |\lambda| e^{\imath\varphi}$ in the domain



$$|\mathcal{R}_N^{(p)}(\lambda)| \le \frac{1}{N^{p(D-2)}} \frac{|\lambda|^p}{\left(\cos\frac{\varphi}{2}\right)^{2p+2}} \, p! \, A \, B^p$$

The $N \to \infty$ limit

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$\lim_{N\to\infty} |\mathcal{R}_N^{(1)}(\lambda)| = 0$, hence

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- Change the covariance (propagator)
- Take the branched polymers seriously: a first phase transition to branched polymers can be followed by subsequent phase transitions to smoother spaces.

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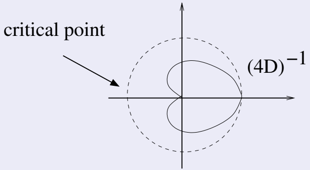

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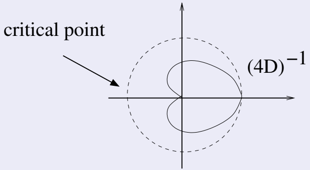

Major (nonperturbative) challenge: extend the analyticity domain of $\mathcal{R}_N^{(p)}(\lambda)$ to the disk of radius $(4D)^{-1}$ minus the negative real axis!

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In perturbative sense the graphs can be reorganized as

$$K_2 = \sqrt{(4D)^{-1} + \lambda} \sum_{p \geq 0} \frac{c_p}{\left(N^{D-2} \left[(4D)^{-1} + \lambda \right] \right)^p} + Rest$$

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At leading order in the double scaling limit an explicit family of graphs larger than the "melonic" family emerges!

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Question: What precise model in this framework describes our universe?

- we don't know hence we concentrate on universal predictions.

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A personal list of open questions:

- non perturbative results
  - extend the non perturbative treatment to other models.
  - extend the analyticity domain of the rest and study the discontinuity of the rest on the negative real axis (non perturbative cut effects are crucial for unitarity and the role of time)
- study the geometry of the space emerging under multiple scalings.
  - algebra of constraints, Hausdorff and spectral dimensions, geodesics.
- Effective field theory description of the critical regime.
- Phenomenological implications.
- . . . .