ON JOINT RECURRENCE

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Abstract. Let T be a measure-preserving and ergodic automorphism of a probability space $(X, \mathcal{S}, \mu)$. By modifying an argument in [4] we obtain a sufficient condition for recurrence of the d-dimensional stationary random walk defined by a Borel map $f\colon X \longmapsto \mathbb{R}^d$, $d \geq 1$, in terms of the asumptotic distributions of the maps $(f+fT+\cdots+fT^{n-1})/n^{1/d}$, $n \geq 1$. If $d=2$, and if $f\colon X \longmapsto \mathbb{R}^2$ satisfies the central limit theorem with respect to T (i.e. if the sequence $(f+fT+\cdots+fT^{n-1})/\sqrt{n}$ converges in distribution to a Gaussian law on $\mathbb{R}^2$), then our condition implies that the two-dimensional random walk defined by f is recurrent.

Sur la récurrence simultanèe

Résumé. Soit T un automorphisme ergodique d'un espace de probabilité $(X, \mathcal{S}, \mu)$. En modifiant un argument dans [4] on obtient une condition suffisante pour la récurrence de la marche aléatoire stationnaire définie par une fonction de Borel $f\colon X \longmapsto \mathbb{R}^d$, $d \geq 1$, en termes de la distribution asymptotique des fonctions $(f+fT+\cdots+fT^{n-1})/n^{1/d}$, $n \geq 1$. Si $d=2$, et si $f\colon X \longmapsto \mathbb{R}^2$ satisfait le théorème de la limite centrale relatif à T (c'est-à-dire si la séquence $(f+fT+\cdots+fT^{n-1})/\sqrt{n}$ converge en distribution vers une loi de Gauss sur $\mathbb{R}^2$), alors notre condition implique que la marche aléatoire à deux dimensions définie par f est récurrente.

Version abrégée

Nous étudions la récurrence de la marche stationnaire définie par une fonction mesurable f à valeurs dans $\mathbb{R}^d$ au-dessus d'un système dynamique invertible et ergodique $(X, \mathcal{S}, \mu, T)$.

Si l'on définit le cocycle $f\colon \mathbb{Z} \times X \longmapsto \mathbb{R}^d$ par (1.1) la récurrence de f (ou de la marche stationnaire associée avec f) est exprimée sous la forme (1.3).

Dans cette note nous donnons un critère de récurrence au moyen de la distribution asymptotique du cocycle f convenablement normalisée. Plus précisement, si $\sigma_k^{(d)}$ est la distribution de la fonction $f(k,\cdot)/k^{1/d}$, $k \geq 1$, alors le résultat principal de cette article affirme la récurrence de f pourvu que

$$\liminf_{\eta\to 0}\liminf_{k\to\infty}\frac{1}{k}\sum_{j=1}^{k}\sigma_j^{(d)}(B(\eta))/\lambda(B(\eta)) > 0 \tag{1}$$

(cf. Theorem 1.2).

La démonstration de cette proposition se fonde sur l'observation suivante: Si f n'est pas récurrente, alors il existe un ensemble $C \in \mathcal{S}$ avec $\mu(C) = \frac{1}{L}$,

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$L \geq 2$, et un $\varepsilon > 0$ satisfaisant (2.1) pour tous $k \in \mathbb{Z}$. Au moyen de cette ensemble nous construisons une application Borelienne $f' \colon X \longmapsto \mathbb{R}^d$ qui est cohomologue avec f telle que le cocycle $f' \colon \mathbb{Z} \times X \longmapsto \mathbb{R}^d$ défini par f' possède les propriétés suivantes μ-*p.p.*:

(a) pour tous $l \in \mathbb{Z}$, $|\{k \in \mathbb{Z} : f'(k,x) = f'(l,x)\}| = L$,
(b) si $k, l \in \mathbb{Z}$ et $f'(k,x) \neq f(l,x)$ alors $\|f'(k,x) - f(l,x)\| \geq \varepsilon$.

Puisque f' est cohomologue avec f, le comportement asymptotique des distributions des applications $f'(k,\cdot)/k^{1/d}$, $k \geq 1$, est identique a celui des mesures $\sigma_k^{(d)}$, $k \geq 1$. La séparation uniforme des valeurs $f'(k,x)$, $k \in \mathbb{Z}$, μ-*p.p.* exprimée dans (a)–(b), en combinaison avec une estimation combinatoire asymptotique fournit une démonstration de (1).

Les applications les plus interessantes de (1) concernent les cas $d = 1$ et $d = 2$. Pour $d = 1$ nous renvoyons à [4]. Pour $d = 2$, (1) implique le corollaire suivant: f est récurrent si les distributions de $f(k,\cdot)/k^{1/2}$ tendent vers la loi Gaussienne le long d'une suite d'entiers k ayant une densité positive (Corollary 1.3). Ceci améliore un résultat dans [2].

1. Recurrence of d-dimensional stationary random walks

Let T be a measure preserving and ergodic automorphism of a standard probability space $(X, \mathcal{S}, \mu)$, $d \geq 1$, and let $f = (f_1, \dots, f_d) \colon X \longmapsto \mathbb{R}^d$ be a Borel map. For every $n \in \mathbb{Z}$ and $x \in X$ we set

$$f(n,x) = \begin{cases} \sum_{k=0}^{n-1} f(T^k x) & \text{if } n \geq 1, \\ 0 & \text{if } n = 0, \\ -f(-n, T^n x) & \text{if } n < 0. \end{cases} \tag{1.1}$$

The resulting map $f \colon \mathbb{Z} \times X \longmapsto \mathbb{Z}^d$ satisfies that

$$f(m, T^n x) + f(n,x) = f(m+n, x) \tag{1.2}$$

for every $m, n \in \mathbb{Z}$ and μ-a.e. $x \in X$. If $\|\cdot\|$ denotes the maximum norm on $\mathbb{R}^d$ then the map $f \colon X \longmapsto \mathbb{R}^d$ is *recurrent* (or the individual components $f_1, \dots, f_d$ of f are *jointly recurrent*) if

$$\liminf_{n \to \infty} \|f(n,x)\| = 0 \tag{1.3}$$

for μ-a.e. $x \in X$. If f is not recurrent it is called *transient* (for terminology and background we refer to [4]).

Proposition 1.1 ([4]). *Let $f \colon X \longmapsto \mathbb{R}^d$ be a Borel map. The following conditions are equivalent.*

(1) *f is recurrent;*
(2) $\mu(\{x \in X : \liminf_{|n| \to \infty} \|f(n,x)\| < \infty\}) > 0$*;*
(3) *For every $B \in \mathcal{S}$ with $\mu(B) > 0$ and every $\varepsilon > 0$,*

$$\mu(B \cap T^{-m} B \cap \{x \in X : \|f(m,x)\| < \varepsilon\}) > 0$$

for some nonzero $m \in \mathbb{Z}$.

For every $k \geq 1$ we define probability measures $\sigma_k^{(d)}$ and $\tau_k^{(d)}$ on $\mathbb{R}^d$ by setting

$$\begin{aligned} \sigma_k^{(d)}(A) &= \mu(\{x \in X : f(k,x)/k^{1/d} \in A\}), \\ \tau_k^{(d)}(A) &= \frac{1}{k}\sum_{l=1}^{k} \sigma_l^{(d)}(A) \end{aligned} \tag{1.4}$$

for every Borel set $A \subset \mathbb{R}^d$, where 1_A is the indicator function of A. In [3] and [4] it was shown that the recurrence of f can be deduced from certain properties of these probability measures. For example, if $d = 1$ and

$$\lim_{k\to\infty} \sigma_k^{(1)} = \delta_0$$

in the vague topology, where

$$\delta_0(A) = \begin{cases} 1 & \text{if } 0 \in A, \\ 0 & \text{otherwise,} \end{cases}$$

then f is recurrent by [3] or [4]. In [4] it was also shown that a map $f \colon X \longmapsto \mathbb{R}$ is recurrent whenever

$$\liminf_{\eta\to 0} \liminf_{k\to\infty} \tau_k^{(1)}([-\eta,\eta])/2\eta > 0.$$

The purpose of this paper is to prove the following extension of this result to higher dimensions.

Theorem 1.2. *Let T be a measure preserving and ergodic automorphism of a probability space $(X, \mathcal{S}, \mu)$, $d \geq 1$, $f \colon X \longmapsto \mathbb{R}^d$ a Borel map, and define the probability measures $\tau_k^{(d)}$, $k \geq 1$, on $\mathbb{R}^d$ by (1.4). We denote by λ the Lebesgue measure on $\mathbb{R}^d$ and set, for every $\eta > 0$, $B(\eta) = \{v \in \mathbb{R}^d : \|v\| < \eta\}$. If f is transient then*

$$\sup_{\eta>0} \limsup_{k\to\infty} \tau_k^{(d)}(B(\eta))/\lambda(B(\eta)) < \infty \tag{1.5}$$

and

$$\liminf_{\eta\to 0} \liminf_{k\to\infty} \tau_k^{(d)}(B(\eta))/\lambda(B(\eta)) = 0. \tag{1.6}$$

The interesting cases are, of course, $d = 1$ and $d = 2$. The case $d = 1$ was discussed in [4]; in order to explain the significance of Theorem 1.2 for $d = 2$ we say that a Borel map $f \colon X \longmapsto \mathbb{R}^d$ satisfies the *central limit theorem* with respect to T if the distributions of the functions $f(n,\cdot)/\sqrt{n}$, $n \geq 1$, converge to a (possibly degenerate) Gaussian probability measure on $\mathbb{R}^d$ as $n \to \infty$ (for the existence of such functions see [1] and [2]). A somewhat weaker form of the following corollary also appears in [2].

Corollary 1.3. *Let T be a measure preserving and ergodic automorphism of a probability space $(X, \mathcal{S}, \mu)$, and let $f \colon X \longmapsto \mathbb{R}^2$ be a Borel map satisfying the central limit theorem with respect to T. Then f is recurrent.*

More generally, if there exists an increasing sequence $(n_k,\ k \geq 1)$ of natural numbers with positive density in $\mathbb{N}$ such that the distributions of the functions $f(n_k,\cdot)/\sqrt{n_k}$ converge to a (possibly degenerate) Gaussian probability measure on $\mathbb{R}^d$ as $k \to \infty$, then f is recurrent.

Proof of Corollary 1.3. If the sequence $(f(n,\cdot)/\sqrt{n},\, n \geq 1)$ converges in measure to a constant, then this constant has to be zero by (1.2). This shows that, if f satisfies the central limit theorem with respect to T (with either degenerate or nondegenerate limit), then there exists a positive constant c such that $\tau_k^{(2)}(B(\eta)) > c\eta^2$ for all sufficiently large k and all sufficiently small $\eta > 0$. According to (1.6) this means that f is recurrent.

The proof of the second assertion is analogous. □

Note that Theorem 1.2 and Corollary 1.3 make no assumptions concerning the integrability of f.

2. The proof of Theorem 1.2

The proof of Theorem 1.2 differs from that of Theorem 3.6 in [4] only by avoiding the use of the total order of $\mathbb{R}$ (which is, of course, not available if $d > 1$).

Let T be a measure preserving and ergodic automorphism of a standard probability space $(X, \mathcal{S}, \mu)$, $d \geq 1$, and let $f\colon X \longmapsto \mathbb{R}^d$ be a transient Borel map. For the definition of the probability measures $\sigma_k^{(d)}, \tau_k^{(d)}$ on $\mathbb{R}^d$ we refer to (1.4).

Proposition 1.1 implies that there exist a Borel set $C \subset X$ with $\mu(C) > 0$ and an $\varepsilon > 0$ with

$$\mu(C \cap T^{-k}C \cap \{x \in X : \|f(k,x)\| < \varepsilon\}) = 0 \tag{2.1}$$

for every $k \in \mathbb{Z}$. By decreasing C, if necessary, we may assume that $\mu(C) = 1/L$ for some $L \geq 1$.

Lemma 2.1. *For every $\eta > 0$ and $N \geq 1$,*

$$\begin{gathered}\limsup_{k\to\infty} \tau_k^{(d)}(B(\eta)) \leq 2^d L \varepsilon^{-d} \lambda(B(\eta)),\\ \limsup_{k\to\infty} \sum_{n=0}^{N} 2^n \tau_{2^n k}^{(d)}(B(2^{-n/d}\eta)) \leq 2^{d+1} d L^d \varepsilon^{-d} \lambda(B(\eta)).\end{gathered} \tag{2.2}$$

Proof. We modify T on a null-set, if necessary, and assume without loss in generality that $T^n x \neq x$ for every $x \in X$ and $0 \neq n \in \mathbb{Z}$ and hence that (1.2) holds for every $m, n \in \mathbb{Z}$ and $x \in X$. Denote by

$$R_T = \{(T^n x, x) : x \in X,\, n \in \mathbb{Z}\} \subset X \times X$$

the *orbit equivalence relation* of T and define a Borel map $\mathbf{f}\colon R_T \longmapsto \mathbb{R}^d$ by setting

$$\mathbf{f}(T^n x, x) = f(n, x) \tag{2.3}$$

for every $(T^n x, x) \in R_T$. Then (1.2) implies that

$$\mathbf{f}(x, x') + \mathbf{f}(x', x'') = \mathbf{f}(x, x'') \tag{2.4}$$

whenever $(x, x'), (x, x'') \in R_T$.

We denote by $[T]$ the full group of T, i.e. the group of all measure preserving automorphisms V of $(X, \mathcal{S}, \mu)$ with $Vx \in \{T^n x : n \in \mathbb{Z}\}$ for every $x \in X$. Since T is ergodic we can find, for any pair of sets $B_1, B_2 \in \mathcal{S}$ with $\mu(B_1) = \mu(B_2)$, an element $V \in [T]$ with $\mu(VB_1 \triangle B_2) = 0$. If $\{C = C_0, C_1, \dots, C_{L-1}\} \subset \mathcal{S}$ is a partition of X with $\mu(C_i) = 1/L$ for

$i = 0, \dots, L-1$, this allows us to find an automorphism $W \in [T]$ with $\mu(WC_i \triangle C_{i+1}) = 0$ for $i = 0, \dots, L-2$ and $W^L x = x$ for every $x \in X$. Put

$$m_C(x) = \begin{cases} \min \{j \geq 1 : T^j x \in C\} & \text{if this set is nonempty,} \\ 0 & \text{otherwise,} \end{cases}$$

denote by

$$T_C x = T^{m_C(x)}$$

the transformation induced by T on C, and set

$$Sx = \begin{cases} Wx & \text{if } x \in \bigcup_{k=0}^{L-2} C_k, \\ T_C W x & \text{if } x \in C_{L-1}. \end{cases}$$

There exists a T-invariant μ-null set $N \in \mathcal{S}$ with the following properties:

(i) if $C' = C \smallsetminus N$ then the sets $C'_k = S^k C'$ are disjoint for $k = 0, \dots, L-1$, and $S^L C' = C'$,

(b) $N = X \smallsetminus \bigcup_{k=0}^{L-1} C'_k$,

(c) for every $x \in C'$, the sets $\{j \geq 1 : S^j x \in C'\}$ and $\{j \geq 1 : S^{-j} x \in C'\}$ are infinite.

Then $\{S^n x : n \in \mathbb{Z}\} = \{T^n x : n \in \mathbb{Z}\}$ for every $x \in X \smallsetminus N$.

We define a Borel map $b \colon X \longmapsto \mathbb{R}^d$ by setting, for every $x \in C'$, $b(S^k x) = \mathbf{f}(S^L x, S^k x)$ for $k = 1, \dots, L$, and by putting $b(x) = 0$ for $x \in N$. The map $g(x) = \mathbf{f}(Sx, x) + b(Sx) - b(x)$ satisfies that

$$g(x) = \begin{cases} \mathbf{f}(S^L x, x) = f(m_{C'}(x), x) & \text{if } x \in C', \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, if $f'(x) = f(x) + b(Tx) - b(x)$, and if $f'(n, \cdot) : X \longmapsto \mathbb{R}^d$ and $\mathbf{f}' \colon R_T \longmapsto \mathbb{R}^d$ are defined by (1.1) and (2.3) with f' replacing f, then

$$\begin{aligned} f'(n, x) &= f(n, x) + b(T^n x) - b(x), \\ \mathbf{f}'(x, x') &= \mathbf{f}(x, x') + b(x) - b(x') \end{aligned} \tag{2.5}$$

for every $x \in X \smallsetminus N$, $n \in \mathbb{Z}$ and $x' \in \{T^k x : k \in \mathbb{Z}\} = \{S^k x : k \in \mathbb{Z}\}$.

We denote by σ'_k, τ'_k the probability measures defined by (1.4) with f' replacing f and obtain as in Lemma 3.4 in [4] that

$$\begin{aligned} &\liminf_{|k| \to \infty} (\sigma_k^{(d)}(B(\eta + \eta')) - \sigma'_k(B(\eta))) \geq 0, \\ &\liminf_{|k| \to \infty} (\sigma'_k(B(\eta + \eta')) - \sigma_k^{(d)}(B(\eta))) \geq 0 \end{aligned} \tag{2.6}$$

for all $\eta, \eta' > 0$. In particular, the inequalities (2.2) will be satisfied if

$$\begin{aligned} \limsup_{k \to \infty} \tau'_k(B(\eta)) &\leq L 2^d \eta^d \varepsilon^{-d}, \\ \limsup_{k \to \infty} \sum_{n=0}^{N} 2^n \tau'_{2^n k}(B(2^{-n/d} \eta)) &\leq d L^d 2^{d+1} \eta^d \varepsilon^{-d} \end{aligned} \tag{2.7}$$

for every $\eta > 0$ and $N \geq 1$.

The equations (2.1) and (2.5) yield that

$$C' \cap T^{-k} C' \cap \{x \in X : \|f'(k, x)\| < \varepsilon\}$$

$$= C' \cap V^{-1}C' \cap \{x \in X : Vx \neq x \text{ and } \|\mathbf{f}'(Vx,x)\| < \varepsilon\} = \varnothing$$

whenever $k \neq 0$ and $V \in [T]$. We set $Y = X \times \mathbb{R}^d$, $\nu = \mu \times \lambda$, denote by $\mathbf{S}\colon Y \longmapsto Y$ the skew product transformation

$$\mathbf{S}(x,t) = (Sx, t + \mathbf{f}'(Sx,x)) = (Sx, t + g(x)),$$

and obtain that the set

$$D = C' \times B(\varepsilon/2)$$

is *wandering* under $\mathbf{S}$, i.e. that $\mathbf{S}^m D \cap D = \varnothing$ whenever $0 \neq m \in \mathbb{Z}$. For every $x \in X \smallsetminus N$ we denote by $V_x \subset \mathbb{R}^d$ the discrete set

$$\{f'(k,x) : k \in \mathbb{Z}\} = \{\mathbf{f}'(S^k x, x) : k \in \mathbb{Z}\}$$

and observe that

$$|\{k \in \mathbb{Z} : f'(k,x) = v\}| = |\{k \in \mathbb{Z} : \mathbf{f}'(S^k x, x) = v\}| = L$$

for every $v \in V_x$ and $x \in X \smallsetminus N$, and that

$$\|v - v'\| \geq \varepsilon$$

whenever $v, v' \in V_x$ and $v \neq v'$. Hence

$$\begin{aligned} |\{0 < l \leq k : 0 < \|f'(l,x)\| &\leq l^{1/d}\eta\}| \\ &\leq |\{0 < l \leq k : 0 < \|f'(l,x)\| \leq k^{1/d}\eta\}| \\ &< (k^{1/d} + \varepsilon/\eta)^d \nu(X \times B(\eta))/\nu(D) \\ &= (k^{1/d} + \varepsilon/\eta)^d L 2^d \eta^d \varepsilon^{-d}, \end{aligned}$$

since $\mathbf{S}^{j+l} D \subset X \times B(k^{1/d}\eta + \varepsilon)$, and since the sets $\mathbf{S}^m D$, $m \in \mathbb{Z}$, are all disjoint. By integrating we obtain that

$$\begin{aligned} \tau_k'(B(\eta)) = \frac{1}{k}\sum_{l=1}^{k} \sigma_l'(B(\eta)) &= \frac{1}{k}\sum_{l=1}^{k} \mu(\{x \in X : \|f'(l,x)\| \leq l^{1/d}\eta\}) \\ &\leq \frac{L}{k} + \frac{1}{k}\int |\{0 \leq l \leq k : 0 < \|f'(l,x)\| \leq l^{1/d}\eta\}|\, d\mu(x) \\ &< \frac{L}{k} + \frac{(k^{1/d} + \varepsilon/\eta)^d}{k} \cdot L 2^d \eta^d \varepsilon^{-d}, \end{aligned}$$

and by letting $k \to \infty$ we have proved the first inequality in (2.7).

Similarly one sees that

$$\begin{aligned} \sum_{n \geq 0} &|\{0 < l \leq 2^n k : 0 < \|f'(l,x)\| \leq l^{1/d} 2^{-n/d}\eta\}| \\ &= L \cdot \sum_{0 \neq v \in V_x} |\{n \geq 0 : v = f'(l,x) \text{ for some } l \\ &\qquad\qquad \text{with } 0 < l \leq 2^n k \leq k l \eta^d / \|v\|^d\} \\ &\leq L \cdot \sum_{0 \neq v \in V_x} (|\{n \geq 0 : 1 \leq 2^n k \leq k\eta^d/\|v\|^d\}| + 1) \\ &\leq L \cdot \sum_{j \geq 1} \sum_{v \in V_x \cap (B((j+1)\varepsilon) \smallsetminus B(j\varepsilon))} (|\{n \geq 0 : 1 \leq 2^n k \leq k\eta^d / j^d \varepsilon^d\}| + 1) \\ &\leq L^d 2^{d-1} d \cdot \sum_{j=1}^{k^{1/d}\eta/\varepsilon} j^{d-1} \biggl(\frac{\log(\eta^d / j^d \varepsilon^d)}{\log 2} + 1\biggr) \end{aligned}$$

$$< L^d 2^{d-1} d \cdot 4k\eta^d/\varepsilon^d.$$

Hence

$$\sum_{n=0}^{N} 2^n \tau'_{2^n k}(B(2^{-n/d}\eta)) = \sum_{n=0}^{N}\Big(\frac{L}{k} + 2^n \tau'_{2^n k}(B(2^{-n/d}\eta) \smallsetminus \{0\})\Big) \\ \leq \frac{(N+1)L}{k} + L^d d 2^{d+1}\eta^d/\varepsilon^d,$$

and by letting $k \to \infty$ we obtain the second inequality in (2.7). Since (2.7) is equivalent to (2.2) we have proved the lemma. □

Proof of Theorem 1.2. Suppose that $f\colon X \longmapsto \mathbb{R}^d$ is transient. Lemma 2.1 yields a constant $c > 0$ such that

$$\limsup_{k\to\infty} \sum_{n=0}^{N} 2^n \tau^{(d)}_{2^n k}(B(2^{-n/d}\eta)) \leq c\lambda(B(\eta))$$

for every $\eta > 0$ and $N \geq 1$. It follows that there exists, for every $\eta > 0$ and $N \geq 1$, an integer $n \in \{0, \dots, N\}$ with

$$\liminf_{k\to\infty} \tau^{(d)}_{2^n k}(B(2^{-n/d}\eta)) \leq \frac{c}{N+1} \cdot \lambda(B(2^{-n/d}\eta)).$$

We conclude that

$$\liminf_{\eta\to 0} \liminf_{k\to\infty} \tau^{(d)}_{2^n k}(B(2^{-n/d}\eta)) \leq \frac{c}{N+1} \cdot \lambda(B(2^{-n/d}\eta))$$

for some $n \in \{0, \dots, N\}$, and hence that

$$\liminf_{\eta\to 0} \liminf_{k\to\infty} \tau^{(d)}_{k}(B(\eta)) \leq \frac{c}{N+1} \cdot \lambda(B(\eta)).$$

As $N \geq 1$ was arbitrary this proves (1.6). The inequality (1.5) is an immediate consequence of the first inequality in (2.2). □

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