Problem Set 3
Due Friday, October 20.

Algebra

Math 110A, Fall Quarter 2017

1. Show that if $a \in \mathbb{Z}$, then $a^2 \equiv 0, 1,$ or $4 \bmod 8$. Use this to prove that there are no integers x, y, z such that $x^2 + y^2 + z^2 = 999$.
2. Do problems 2.2.3, 2.2.5, 2.2.9 in the textbook.
3. Do problems 2.3.1, 2.3.2, 2.3.3, 2.3.5 in the textbook.
4. Do problems 3.1.1, 3.1.9, 3.1.17, 3.1.18, 3.1.35 in the textbook.
5. (a) Show that $R = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ is a subring of $\mathbb{R}$. Is R a domain?

   (b) Prove or disprove: $S = \left\{\frac{1}{2}(a + b\sqrt{2}) : a, b \in \mathbb{Z}\right\}$ is a subring of $\mathbb{R}$.