Problem Set 5
Due Friday, May 9

Mathematical Logic

Math 114L, Spring Quarter 2008

1. (20 pt.) Show that that for any wff α we have $K(\alpha) = 1$, and if α' is a proper initial segment of a wff, then $K(\alpha') < 1$.
2. (20 pt.) Problem 11 of Section 2.2 in the textbook.
3. (20 pt.) Parts (a) and (b) of Problem 12 of Section 2.2 in the textbook.
4. (10 pt.) Part (a) of Problem 20 of Section 2.2 in the textbook.
5. (30 pt.) A set S of natural numbers is called a **spectrum** if there is a sentence σ in *some* first-order language such that

$$S = \big\{n \in \mathbb{N} : \text{there is a structure } \mathfrak{A} \text{ with } \mathfrak{A} \models \sigma \\ \text{whose universe } A \text{ contains exactly } n \text{ elements}\big\}.$$

Which subsets of $\mathbb{N}^{>0}$ are spectra? This problem was asked by Heinrich Scholz in 1952, and it is still unsolved. As far as I know it is even unknown whether the complement $\mathbb{N}^{>0} \setminus S$ of a spectrum S is also a spectrum. (This was asked by Günter Asser in 1955.)

Show:

(a) Every finite subset of $\mathbb{N}^{>0} = \{1, 2, 3, \dots\}$ is a spectrum.

(b) For every $m \geq 1$, the set of positive integers which are divisible by m is a spectrum.

(c) Show that the union and the intersection of two spectra is a spectrum.