Problem Set 2
Due Wednesday, April 18.

Real Analysis

Math 131A, Spring Quarter 2012

1. Read §2 in the textbook, then do problem 2.1.
2. Do problems 3.3 and 3.4 in the textbook.
3. Do problems 4.1–4.4 for (a), (b), (k), (u), (v), as well as 4.14, in the textbook.
4. Let K be a field, i.e., a set equipped with two maps

$$(a,b) \mapsto a+b\colon K\times K \to K, \quad (a,b)\mapsto a\cdot b\colon K\times K\to K$$

satisfying the axioms (A1)–(A4), (M1)–(M4) and (DL) stated in class.

   (a) Show that the element 0 postulated to exist in (A3) is unique, and that for each $a \in K$ the element $-a$ postulated to exist in (A4) is also unique.

   (b) Show that the element 1 postulated to exist in (M3) is unique, and that for each $a \in K$, $a \neq 0$, the element a^{-1} postulated to exist in (M4) is unique.

5. Recall that the complex numbers $\mathbb{C}$ is the set of all numbers of the form $a+bi$ where $a,b\in\mathbb{R}$ and i is a number satisfying $i^2=-1$. We add and multiply complex numbers in the natural way:

$$(a+bi)+(a'+b'i) = (a+a')+(b+b')i$$
$$(a+bi)\cdot(a'+b'i) = (aa'-bb')+(ab'+ba')i$$

   (a) Verify that using these operations, $\mathbb{C}$ becomes a field.

   (b) Show that there does *not* exist a binary relation $\leq$ on $\mathbb{C}$ so that $\mathbb{C}$ becomes an ordered field.