Problem Set 3
Due Wednesday, April 25.

Real Analysis

Math 131A, Spring Quarter 2012

1. Do problems 5.1, 5.3, 5.4 in the textbook.

2. Do problems 7.4, 8.1 (b), (d), 8.2 (b), (e), 8.4, 8.7 (b), 8.9 in the textbook.

3. A subset S of $\mathbb{R}$ is said to be *closed* if whenever (s_n) is a sequence in S and (s_n) converges to a real number s, then $s \in S$.

   (a) Prove that for any $a, b \in \mathbb{R}$ with $a < b$, the interval $[a, b]$ is closed.

   (b) Give an example of an unbounded, closed set $S \subseteq \mathbb{R}$.

   (c) Suppose that $S \subseteq \mathbb{R}$ is closed and bounded from above. Prove that S has a largest element.

4. Let (s_n) be a sequence in $\mathbb{R}$ with limit $s \in \mathbb{R}$. Show that then the sequence (σ_n) with $\sigma_n = \frac{1}{n}(s_1 + \cdots + s_n)$ also converges to s.