Problem Set 5
Due Wednesday, May 9.

Real Analysis

Math 131A, Spring Quarter 2012

1. Do problems 10.7, 10.10, 11.6, 11.8, 11.10 in the textbook.
2. Suppose that for each $n \geq 1$, we are given a closed and bounded interval $[a_n, b_n]$ in $\mathbb{R}$, such that $[a_{n+1}, b_{n+1}] \subseteq [a_n, b_n]$ for all $n \geq 1$. (Such a sequence $([a_n, b_n])_{n\geq 1}$ of intervals in $\mathbb{R}$ is said to be *nested.*) Show that $\bigcap_{n\geq 1}[a_n, b_n] \neq \emptyset$, i.e.: there is a real number a such that $a \in [a_n, b_n]$ for all $n \geq 1$. (Hint: use the Bolzano-Weierstrass Theorem.)
3. Let (a_n) and (b_n) be bounded sequences of real numbers. Show that

$$\limsup a_n + \liminf b_n \leq \limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n.$$

Give an example of a *single* pair of sequences (a_n), (b_n) for which both inequalities are strict.